1

NETWORK VARIABLES AND ELEMENTS

1.1 INTRODUCTION :

Since the first discovery of electricity in the year 600 BC by Thales of Miletus, the knowledge and use of electricity has multiplied by enormous counts. All the modern day conveniences are dependent to the most extent on electricity. Without electricity the life of the modern man will come to a standstill.

Network theory is a basic discipline of electric science. It describes how the energy is transferred from one device to another. Thorough understanding of this subject is essential for every type of engineer and physicist. The objective of this text is to provide a thorough understanding and proficiency in the subject of engineering circuit analysis.

1.2 NETWORK MODEL :

Just as in every branch of science and engineering, where the behaviour of a complex physical device is studied by constructing its mathematical model, in network theory also, an idealised model for the device is constructed and its behaviour is analysed. Network theory deals with the determination of response of this network model to a stimulus or excitation as shown in Fig. 1.1. In electrical engineering it is often possible to separate the excitation and response.

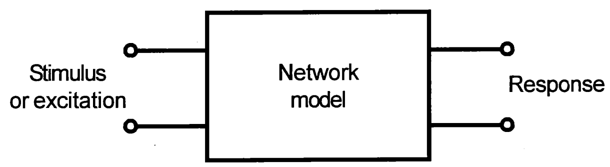


Fig. 1.1 Excitation and response

The network model itself is an interconnection of various idealised models of physical devices. A basic characteristic of many physical networks encountered in engineering practice is that the response is proportional to the excitation. If the input is doubled, the output is also doubled. This property of the network is called *linearity*. In this text we will be concerned with the study of networks which obey this property.

We are often concerned with the flow of electrical energy from one device to another and not in the internal distribution of the energy in any particular device. Thus we will treat the basic building blocks of our network models to be '*lumped*' elements rather than distributed elements. Thus their behaviour can be analysed by the effects produced at the terminals of the device, through which the energy leaves or enters the device. A simple model of an electric device is represented in Fig. 1.2.

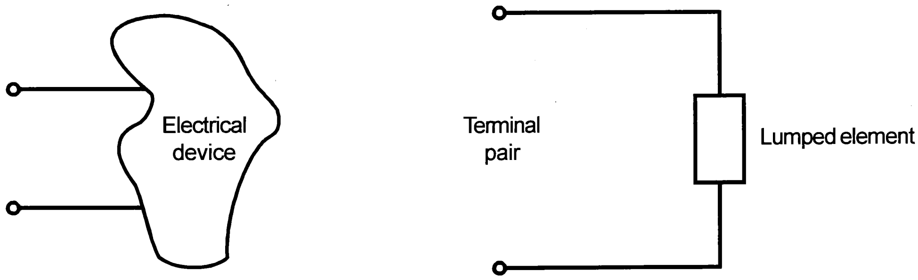


Fig. 1.2 An electrical device and its lumped model

The networks are constructed with these ideal elements interconnected by ideal connectors which absorb zero energy.

1.3 NETWORK VARIABLES :

As we are concerned with the flow of energy from one device to another in a network, we must know what are the variables that are used in the study of the flow of energy. Energy by itself is not a measurable quantity and is therefore measured by association with other measurable quantities. For example, the mechanical energy is measured in terms of force and distance.

The electrical energy can be measured by defining a measurable quantity called *electric charge*. Charges were first observed by rubbing dry substances together. Suppose a light material such as pith ball is suspended by a string and a hard rubber comb, rubbed with a woollen cloth, is brought near it, it is observed that the pith ball tends to swing away from it. If the pith ball is approached with the woollen cloth now, the pith ball is found to be attracted towards it. This phenomenon is explained by saying that there are forces acting due to electric charges present on the pith ball, the comb and the woollen cloth. Since these forces can be either attraction or repulsion as seen in the above experiment, it is postulated that there are two kinds of electrical charges : positive and negative and like charges repel and unlike charges attract. Benjamin Franklin first named the charges that are present on the comb to be negative and that on the woollen cloth, positive.

According to atomic physicists, the entire matter in the universe is made up of fundamental building blocks called *atoms* and these atoms are composed of different kinds of fundamental particles. The three most important particles are the electron, proton and the neutron. The ultimate unit of electric charge is the electron, with a mass of 9.107×10^{-31} kg. The fundamental unit of charge, called *the Coulomb*, named after Charles Coulomb, is 6.24×10^{18} electrons. These electrons move in orbits around the nucleus of an atom and many of them break loose and move at random in the interatomic spaces. An excess of electrons in a body represents negative charge and deficit of electrons represents a positive charge.

Hans Christian Oersted, in the year 1819, observed that a flow of electrical charge produced a force on a nearby magnetic compass needle and he found that the force was proportional to the rate of flow of charge. Since the measurement of this force was much easier than the measurement of forces between static charges, a new variable, current, was defined as the rate of flow of electric charge and its unit, ampere = 1 Coulomb/sec. Thus the current, rather than the charge, has come to be the basic variable of circuit analysis. Charges are represented by the symbols q or Q and the currents are represented by i or I depending on whether they are varying with respect to time or constant respectively. Thus

$$i = \frac{dq}{dt} \qquad .....(1.1)$$

We observe two important aspects of the flow of current. First, the current has direction. The number of coulombs per second has a direction and the current is said to be flowing in a circuit in a particular direction. Secondly, the current flow may be due to the flow of electrons in a particular direction or due to motion of positive charge, or due to motion of positive charge in one direction and negative charge in the other direction. Conventionally, the direction of current is taken to be the flow of positive charge in the direction of a reference arrow placed for convenience.

In Fig. 1.3, two equivalent methods are shown to represent the current in a conductor. 1A of current in Fig. 1.3(a) means a net positive charge of 1 C/sec is flowing in the direction of arrow, whereas, in Fig. 1.3(b) a net negative charge of –1 C/sec

Fig. 1.3 Refrence direction for currents

is flowing in the direction of arrow. Since the magnetic effect produced by a flow of positive charge in one direction and that due to a flow of negative charge in the opposite direction is identical, the currents are equivalent in both the cases. Hence to avoid ambiguity, the current is always considered to be an equivalent flow of positive charge. Thus a current of positive charge flowing in the direction of an arbitrary reference direction is taken to be positive. Again coming back to our lumped model of the elements, the current is measured at the terminals and no consideration is given to the finite speed of propogation of electricity through the material. Thus the current entering one terminal of the element is taken to be equal to the current leaving the other terminal and currents are measured and defined only at the terminals of the lumped element as shown in Fig. 1.4.

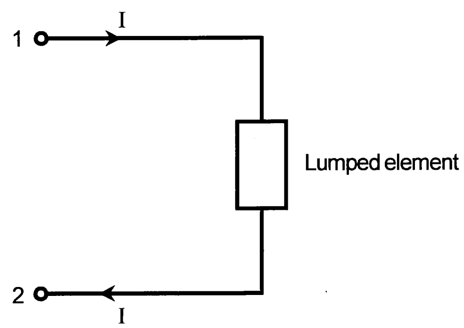


Fig. 1.4 Current in a lumped element

Coming back to the measurement of electrical energy, in addition to the concepts of charge and current, we need to define an additional variable to measure the amount of energy lost or gained by each charge as it passes through the element. This variable is called *voltage* or *potential* and is denoted by the letters e, E or v, V.

The voltage across a terminal pair of a device is a measure of the work required to move the electric charge through the device. Thus

$$v = \frac{dW}{dQ} \qquad .....(1.2)$$

where v = voltage in Volts
W = energy in Joules
Q = charge in Coulombs

From the definitions of current and voltage given in eqs. (1.1) and (1.2), it is clear that the product of voltage and current is the power associated with the device. Power is measured in Joules/sec or watts and is represented by the symbols p or P. Thus

$$p = v \cdot i \qquad .....(1.3)$$

Just as a current has direction, voltage has a polarity depending on whether energy is supplied to the device or received from it. A plus sign at the terminal where the current enters the device indicates that the energy is absorbed by the device and negative sign at that terminal indicates that the device is supplying energy. We therefore use a pair of signs, plus and minus, at the two terminals of an electric device to give a reference polarity for the voltage. Thus, placing a +ve sign at the terminal 1 and –ve sign at the terminal 2 indicates that the terminal 1 is at a higher potential than the terminal 2. As for the current, the reference marks for voltage are for algebraic reference only. If the actual voltage has the same polarity as the reference marks, the voltage is positive and it is negative if it has opposite polarity. Thus the two figures 1.5(a) and 1.5(b) are equivalent.

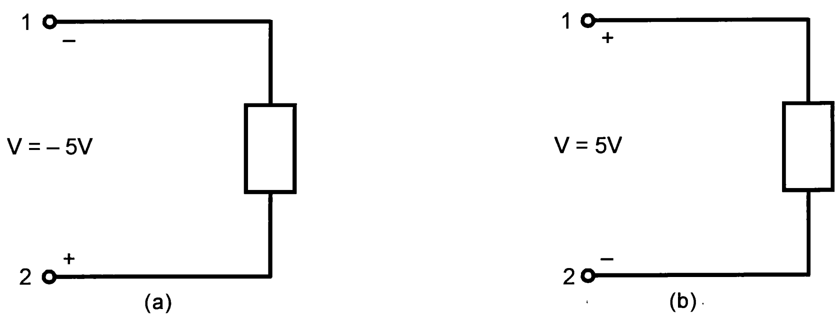


Fig. 1.5 Reference directions for voltage

Consistent signs must be assigned to the current and voltage in a device if the power is absorbed by the device or supplied by the device. First an arbitrary direction is assigned for the current and the sign of voltage is made to agree. A plus sign is assigned to the terminal at which the current enters the device. With this convention, a positive sign for power indicates that the device is absorbing power or it is a sink, whereas a negative sign for the power indicates that the device is supplying power and hence it is a source.

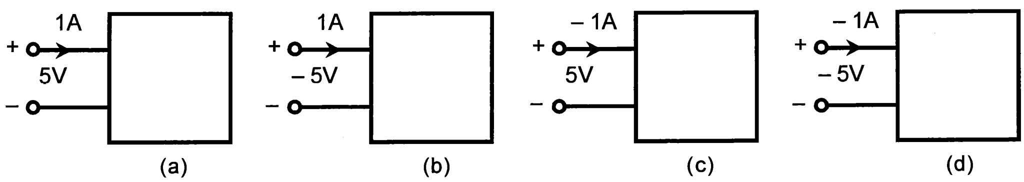


Fig. 1.6 Reference directions for voltage and current

In Fig. 1.6 four different combinations of the polarities are shown for an electric device. In Fig. 1.6(a) the power is P = 5 × 1 = 5 watts and hence it is a sink. In Fig. 1.6(b) the power is P = – 5 × 1 = – 5 W and hence it is a source. In a similar manner Fig. 1.6(c) is a source and Fig. 1.6(d) is a sink.

1.4 NETWORK ELEMENTS :

The network elements are the mathematical models of two terminal electric devices which can be characterised by their voltage and current relationship at the terminals. These network elements are interconnected to form a network and can be divided into two major groups as shown in Fig. 1.7.

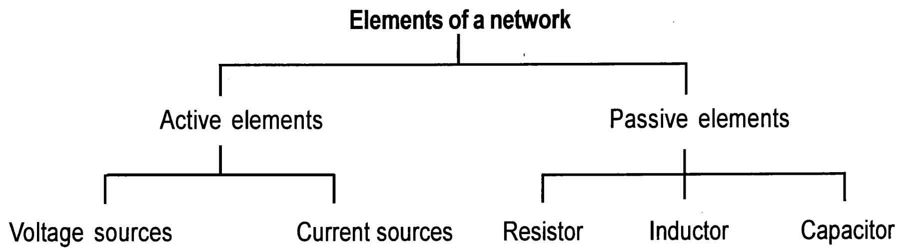


Fig. 1.7 Classification of elements of a network

1.4.1 Active elements :

Active elements are those elements which are capable of delivering energy to the networks or devices which are connected across them. There are essentially two kinds of active elements in a network.

Independent voltage source :

An independent voltage source is characterised by the property that the voltage across its terminals is independent of current passing through it. The symbolic representation and the v – i relationship of the element is given in Fig. 1.8.

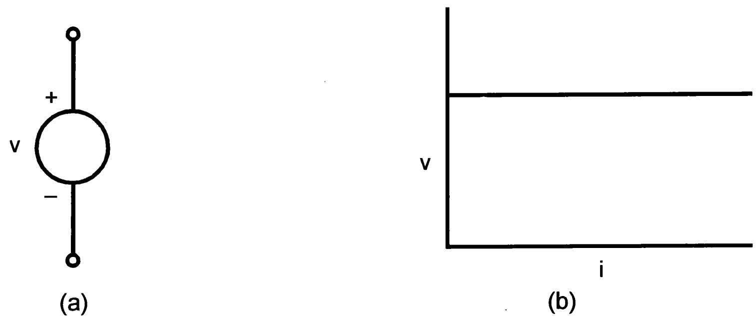


Fig. 1.8 Independent voltage source (a) its circuit representation and (b) v – i characteristic

If the voltage source has a constant voltage, it is termed as a d.c voltage source and is represented by either of the symbols shown in Fig. 1.9(a) or (b).

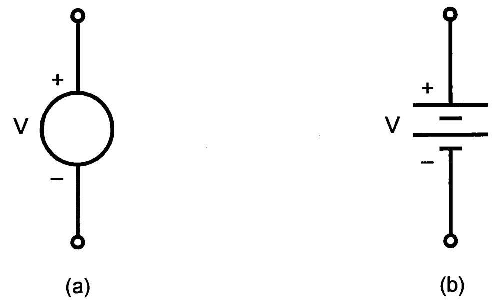


Fig. 1.9. Symbolic representation of d.c. sources

The sources described above are '*ideal*' sources and do not represent exactly any practical device encountered in physical world, because, theoretically it can deliver infinite amount of power to the circuit to which it is connected. This device is supposed to deliver unlimited number of coulombs per second, each receiving 'v' Joules as it passes through the source. This, clearly, is impossible, but there are several practical devices which can be modelled very closely with these elements. The v – i characteristic of a practical voltage source is characterised by a drooping characteristic as given in Fig. 1.10.

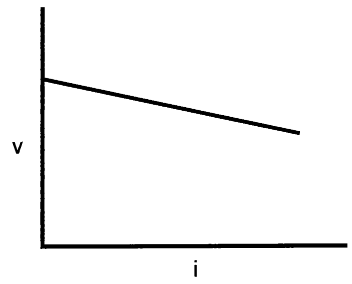


Fig. 1.10 v – i characteristic of a practical voltage source

One point worth emphasising here is about the polarity signs placed in Fig. 1.8(a). The '*plus*' sign placed at the upper terminal does not mean that it is always positive with respect to the lower terminal. In fact, it only means that the upper terminal is 'v' volts positive with respect to the other terminal at a particular instant of time. If 'v' is negative at any instant the upper terminal is actually 'v' volts negative with respect to the lower terminal at that instant.

Independent current source :

The second ideal source is an independent current source which delivers a constant current independent of the voltage across its terminals. The symbol and the v – i characteristic are given in Fig. 1.11.

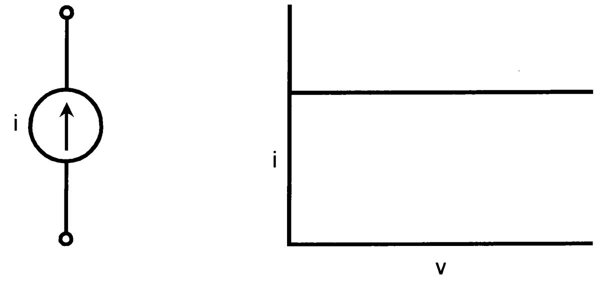


Fig. 1.11 The symbol and the v – i characteristic of an independent current source

This, again, can be considered as a close approximation to a practical device. This too, theoretically, is capable of supplying infinite power from its terminals since it will be able to deliver the same current even if the voltage across it is infinite ! However, this element is very useful in representing the behaviour of some electronic devices. The v – i characteristic of a practical device is given in Fig. 1.12 which is a drooping characteristic.

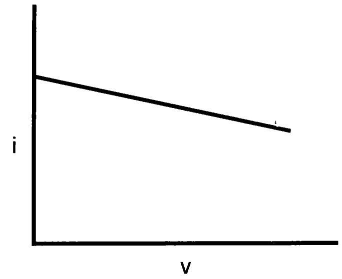


Fig. 1.12 v – i characteristic of a practical current source

The two active elements described above are called *independent sources* because their values are not affected by the currents or voltages in other elements of the network to which these elements are connected. There are other types of sources which are very useful in describing many electronic devices that are represented by their equivalent circuits. These are known as *dependent* or *controlled sources* in which the voltage or the current associated with the element is dependent on either the current or voltage in some other element in the network. In order to distinguish these sources from the independent sources, a diamond shaped symbol is used to represent them as shown in Fig. 1.13.

(a) voltage source (b) current source

Fig. 1.13 Symbols for dependent sources

There are essentially four types of controlled sources depending on whether it is a current source or a voltage source and whether it is controlled by a current or voltage in some other element of the network. Thus we have

a) Voltage controlled voltage source
b) Current controlled voltage source
c) Voltage controlled current source
and d) Current controlled current source.

1.4.2 Passive elements :

The passive elements are those elements which are capable of only receiving power. They cannot deliver power. However there are some passive elements which can store finite energy and then return this energy to external elements. Since these elements cannot deliver unlimited energy over an infinite time interval, they are treated as passive elements only. There are essentially three kinds of passive elements called *the resistor*, *the inductor* and *the capacitor* as shown in Fig. 1.7.

As described earlier, a network is an interconnection of two or more elements. If this interconnection involves atleast one closed circuit, the network is also called as *an electrical circuit.* Thus every circuit is a network but every network need not be a circuit. Also, if the network contains at least one independent source, the network is called *an active network.* If the network does not contain any active element it is called *a passive network.*

Example 1. 1

Find the power absorbed by each element in the circuit of Fig. 1.14.

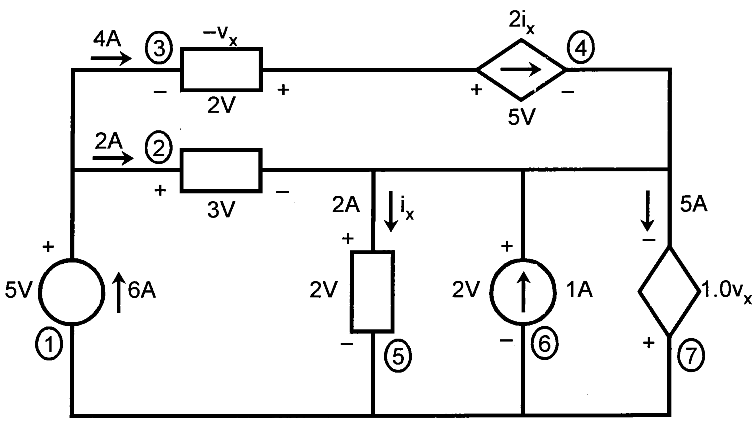


Fig. 1.14 Network for example 1.1

Solution :

The elements are identified by the encircled numbers. For any element if the current is entering the +ve terminal, the current is taken as +ve and hence the power absorbed will be positive.

Thus for element (1)

Power absorbed is

$P_1 = 5 \times (-6) = -30$ W

and for the other elements

$P_2 = 3 \times 2 \times = 6$ W

$P_3 = 2 \times (-4) = -8$ W

$P_4 = 5 \times 4 = 20$ W

$P_5 = 2 \times 2 = 4$ W

$P_6 = 2 \times (-1) = -2$ W

$P_7 = (1.0 \times -2) \times (-5) = +10$ W

1.5 THE RESISTOR :

The passive elements introduced in section 1.4 will now be described in detail. First, the idealised passive element called *linear resistor*, will be considered. The mathematical model for a linear resistor is described by the famous Ohm's law for most of the conducting materials. Ohm's law states that :

"*The voltage across any conducting material is directly proportional to the current flowing through the conductor*" or

$$v = R\,i \qquad \text{......(1.4)}$$

Where 'R', the constant of proportionality, is called *the resistance of the material.* The resistance is measured in ohms. The v – i characteristic of a resistor is shown in Fig. 1.15.

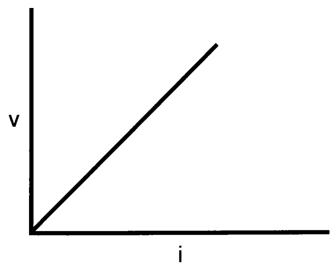


Fig. 1.15 v – i characteristic of a resistor

Since the characteristic is a linear relationship, the resistor is a linear element. The network symbol and the polarities of voltage and current for absorbing power in a resistor are shown in Fig. 1.16.

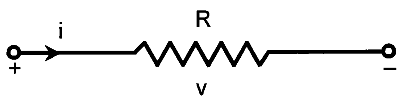


Fig. 1.16 Network model of a resistor and sign convention for voltage and current

The product v . i gives the power absorbed by the resistor and this power manifests itself as heat in the resistor. As resistor is a passive element it cannot deliver or store power. The expressions for power are

$$P = vi = i^2R = \frac{v^2}{R} \qquad .....(1.5)$$

It is pertinent to emphasise that the linear resistor is an idealised model of a physical device and the relationship of eq. (1.4) holds good for a certain range of currents only. If it exceeds a particular value, excessive heat is produced and the value of the resistance is found to change with temperature. It is no longer linear and hence the eq. (1.4) cannot be used. However, in this text, we will be concerned with the linear resistors only. The resistance of a wire of length 'l' meters, and a cross sectional area of A Sq. m. is given by

$$R = \frac{\rho l}{A}$$

where is ρ the resistivity of the wire in ohm – meters

The reciprocal of resistance is defined as conductance. Thus, conductance is the ratio of current to voltage.

$$G = \frac{1}{R} = \frac{i}{v} \qquad .....(1.6)$$

The unit of conductance is Siemens (S).

Example 1.2

Consider the resistor in Fig. 1.17(a). A voltage v(t) of waveform given in Fig. 1.17(b) is applied at its terminals. Obtain the waveform of current through it.

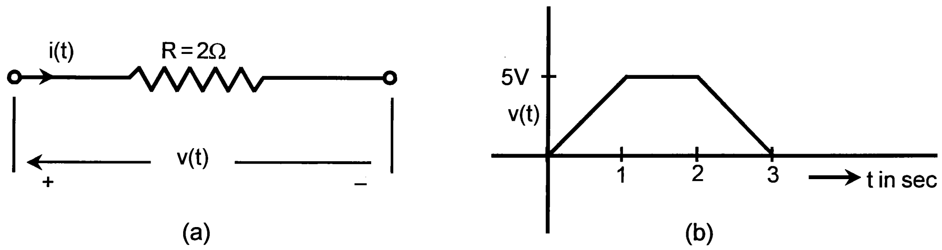


Fig. 1.17 The resistance and voltage waveform for example 1.2

Solution :

The v – i relationship for a resistor is

$$v = i R$$

or $$i = \frac{v}{R} = vG$$

$$G = \frac{1}{R} = \frac{1}{2} = 0.5S$$

For $0 \le t \le 1$ sec

$$v(t) = 5t$$

$$i(t) = 5t(0.5) = 2.5t \text{ A}$$

For $1 \le t \le 2$ sec

$$v(t) = 5 \text{ V}$$

$$\therefore \quad i(t) = 5(0.5) = 2.5 \text{ A}$$

For $2 \le t \le 3$ sec

$$v(t) = -5t + 15$$

$$i(t) = (-5t + 15)0.5 = -2.5t + 7.5 \text{ A}$$

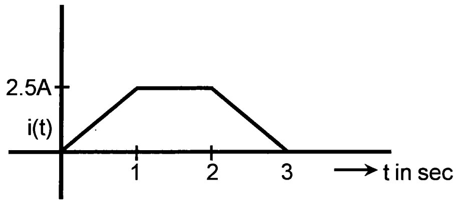


Fig. 1.17(c) Current waveform for example 1.2

The current waveform is given in Fig. 1.17(c).

Example 1.3

Through a resistor of value 2Ω a current of i(t) = 2 sin50t A is passed. What is the voltage across its terminals and what is the power consumed by it ?

Solution :

$$R = 2\ \Omega$$

$$i(t) = 2 \text{ Sin } 50t \text{ A}$$

$$v(t) = i(t) \cdot R = (2\text{Sin } 50t) \times 2 = 4\text{Sin } 50t \text{ V}$$

$$p(t) = v(t) \cdot i(t) = 4 \text{ Sin } 50t \cdot 2 \text{ Sin } 50t = 8 \text{ Sin}^2 50t \text{ Watts}$$

1.6 THE INDUCTOR :

This is another important passive element which does not consume energy but is capable of storing energy. Once it stores energy, it is then capable of supplying this energy to external devices. But unlike active source, it is not capable of supplying unlimited energy or a finite average power over an infinite interval of time.

The mathematical model for an inductor will be given purely in a network point of view, as a v – i relationship at the terminals. But it will be worth reviewing how this element was conceived as a result of magnetic effects of current. A scientist Danish Oersted showed that a current carrying conductor produced a magnetic field in the air surrounding it. Michael Faraday in the year 1831 observed a magnetic effect in the air surrounding a wire carrying current, and a magnetic needle always pointed in the direction of a circle around the wire. Faraday explained the effect in terms of magnetic lines of force in the 'aether' surrounding the wire. He postulated that the current in the wire was responsible in stretching the lines of force, like elastic bands, around the wire, and the energy thus stored in the form of magnetic energy was returned to the wire in the form of a voltage generated when the current decayed to zero.

1.6.1 Faraday's law of electromagnetic induction :

Faraday's law of electromagnetic induction states that whenever a current in a conductor changes, it produces a changing magnetic flux, and this in turn induces a voltage in the conductor which opposes the current. The magnitude of this induced voltage is proportional to the rate of change of flux. If instead of a single conductor, we have a coil of N turns, then

$$e = -\frac{d(N\phi)}{dt} \text{ volts} \qquad .....(1.7)$$

where e is the induced voltage, $\Psi = N\phi$ is the flux linkage in weber turns. The negative sign is taken to indicate that the induced voltage opposes the current producing the flux ϕ. Since N is a constant, we can write eq. (1.7) as,

$$e = -N\frac{d\phi}{dt}.$$

If a voltage v is applied to a coil of N turns, it produces a current i in the coil which in turn produces a flux ϕ. The induced voltage e, opposes the applied voltage and hence,

$$v = -e = N\frac{d\phi}{dt}.$$

1.6.2 Self inductance :

The flux linkage $\Psi = N\phi$ can be shown to be proportional to the current in the coil and the proportionality constant is termed as the self inductance, L, of the coil.

Thus,

$$v = \frac{d\psi}{dt} = \frac{d(Li)}{dt} = L\frac{di}{dt} \qquad .....(1.8)$$

Eq. (1.8) is the mathematical model of an inductor. L is measured in henries.

Practical inductors are usually made of many turns of fine wire wound in a coil or helix to increase the magnetic effects without increasing the size of the element. The inductance of a coil of a long helix of a small pitch is given by

$$L = \frac{\mu N^2 A}{l}$$

where A = cross sectional area of the coil in Sq. m.

N = number of turns of the coil

l = axial length of the helix in m

and μ = constant of the material inside the helix called *permeability* and for free space or air $\mu = \mu_o = 4\pi \times 10^{-7}$ H/m.

The network symbol and the reference directions for voltage and current are shown in Fig. 1.18 for passivity.

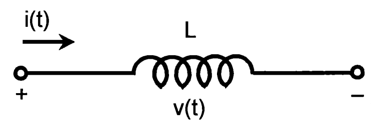


Fig. 1.18 Network symbol for inductor

From eq. (1.8) it is clear that if the current is a constant, the voltage across the inductor is zero, which means that the two terminals of the inductor are connected by a wire of zero resistance. This is called a short circuit between the two terminals. Hence we can say that the inductance behaves like a short circuit to d.c. Another characteristic we observe from eq. (1.8) is that if there is an instantaneous or sudden change in the current, the rate of change of current is infinite and the voltage across the inductor is also infinite. Thus a sudden change in the current in an inductor can occur only if the voltage across it is infinite. Hence the inductor opposes abrupt changes in the current. Further if an inductor carrying current is open circuited, for example, by opening a switch, an arc appears across the switch due to the same reason.

An alternate form of eq. (1.8) describes the relation between i and v.

Rewriting eq. (1.8) as

$$di = \frac{1}{L} v\, dt \qquad .....(1.9)$$

Integrating on both sides, and assuming that the current in the inductor was zero at the time $t = -\infty$. (The time $t = -\infty$ is a conceptual time to ensure that the current was zero at $t = -\infty$. This time could be the time at which the inductance coil was wound and obviously the current at that time was zero !)

$$\int_o^i di = \frac{1}{L}\int_{-\infty}^t v dt$$

$$i = \frac{1}{L}\int_{-\infty}^t v dt \qquad ....(1.10)$$

We observe that both the equations, eq. (1.8) and eq. (1.10), are linear and hence the inductor is a linear element.

The power entering the inductor at any instant is given by

$$p = v \cdot i = Li\frac{di}{dt} \qquad .....(1.11)$$

When the current is constant, $\frac{di}{dt} = 0$ and $p = 0$ and no additional energy is stored in the inductor. The energy associated with the lines of flux will be fixed. If the current increases, the derivative of the current is positive and the power is positive. Thus additional energy is stored in the inductor. The total energy stored in the inductor at any time is given by

$$w_L = \int_{-\infty}^t vidt = \int_{-\infty}^t iL\frac{di}{dt}\, dt$$

$$= \int_o^i Li\, di = \frac{1}{2} Li^2 \qquad .....(1.12)$$

Observe that the limits of integration are chosen appropriately in eq. (1.12) to suit the variable of integration. Note also that the energy stored in the inductor at any instant depends on the value of the current at that instant only and not on its past history.

Example 1.4

An inductor shown in Fig. 1.19(a) is supplied with a current waveform given in Fig. 1.19(b). Draw the waveforms for the voltage and energy in the inductor.

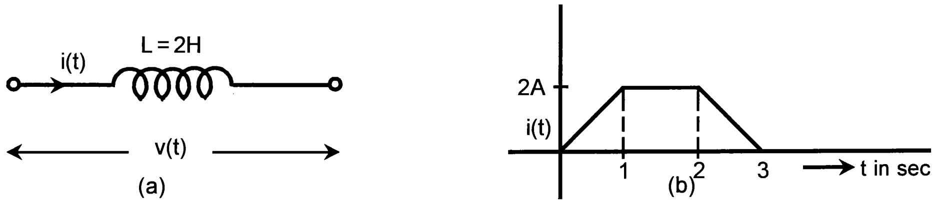


Fig. 1.19 An inductor and its associated current waveform for example 1.4

Solution :

For $0 \le t \le 1$ sec

$$i(t) = 2t \text{ A}$$

$$\therefore \quad v(t) = L\frac{di}{dt} = 2 \times 2 = 4V \qquad w_L(t) = \frac{1}{2} L i^2 = \frac{1}{2} \times 2 \times (2t)^2 = 4t^2 \text{ J}$$

For $1 \le t \le 2$ sec

$$i(t) = 2 \text{ A} \qquad w_L(t) = \frac{1}{2} \times 2 \times 2^2 = 4 \text{ J}$$

$$v(t) = 0$$

For $2 \le t \le 3$ sec

$$i(t) = -2t + 6 \text{ A} \qquad w_L(t) = \frac{1}{2} \times 2 \times (-2t + 6)^2 = 4t^2 - 24t + 36 \text{ J}$$

$$v(t) = 2 \times -2 = -4V$$

The resulting waveform of voltage and energy are given in Fig. 1.19(c).

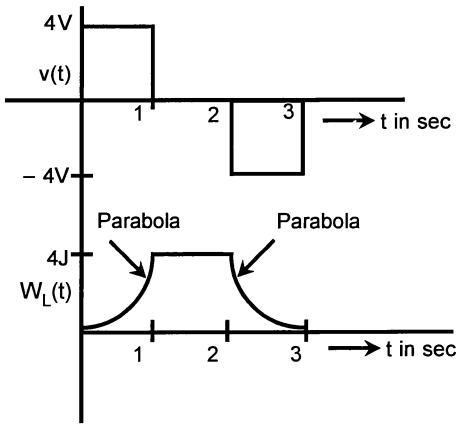
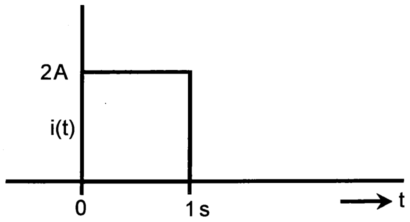


Fig. 1.19(c) Solution of example 1.4

Example 1.5

A waveform of current shown in the figure is applied to an inductor of value 0.5 H. Obtain the waveform of voltage across it.



Current wave for example 1.5

Solution :

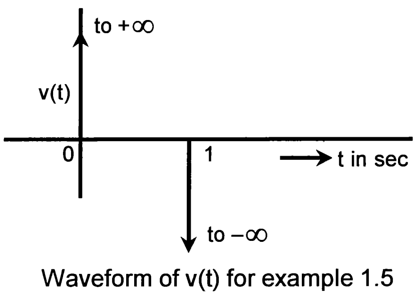
At t = 0 and t = 1s there is an abrupt change in the current. Here $\frac{di}{dt} = \infty$.

Hence at t = 0 and t = 1s we have infinite voltage spikes appearing across the inductor terminals. As discussed earlier, these infinite spikes are required to allow the current to change suddenly. These spikes are called *impulses*, which are mathematically defined later. These spikes are physically not possible as, a finite time, however small it may be, is required for the current to rise to a given value. However these spikes can be of very large magnitude.

For $0 < t < 1$ (observe that $t \neq 0$ or 1s)

$$i(t) = 2A \quad \text{and} \quad v(t) = L\frac{di}{di} = 0v$$

Thus the waveform of voltage across inductor consists of two infinite spikes called *impulses* at t = 0 and 1 sec and is zero for 0 < t < 1s. The waveform is shown in the figure.



Waveform of v(t) for example 1.5

Example 1.6

Consider a waveform of voltage given in Fig. 1.20(a) applied to an inductor of value 2 mH. Obtain the waveforms of current and energy in the inductor. Assume that at t = 0 the energy and thus current in it to be zero.

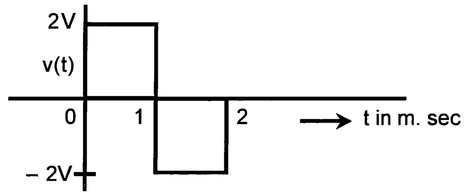


Fig. 1.20(a) waveform of voltage for example 1.6

Solution :

For $0 \leq t \leq 1$ ms

$v(t) = 2$ V

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(t)dt = \frac{1}{L}\int_{-\infty}^{0} v(t)dt + \frac{1}{L}\int_{0}^{t} v(t)dt \qquad .....(1.13)$$

The first term in eq. (1.13) represents the current in the inductor at t = 0. Thus

$$i(t) = i(0) + \frac{1}{L}\int_{0}^{t} 2dt = 0 + \frac{1}{2\times10^{-3}} 2t\Big|_{0}^{t} = 10^3 t \text{ A} \qquad .....(1.14)$$

$$w_L(t) = \frac{1}{2}Li^2 = \frac{1}{2} \times 2 \times 10^{-3} \times 10^6 t^2 = 10^3 t^2 \text{ J}$$

For $1 \leq t \leq 2$ ms

$v(t) = -2$V

Again, $$i(t) = \frac{1}{L}\int_{-\infty}^{t} vdt$$

$$= \frac{1}{L}\int_{-\infty}^{1\times10^{-3}} vdt + \int_{1\times10^{-3}}^{t}(-2)dt$$

$$= i(1\times10^{-3}) + \frac{1}{2\times10^{-3}}[-2t]_{1\times10^{-3}}^{t}$$

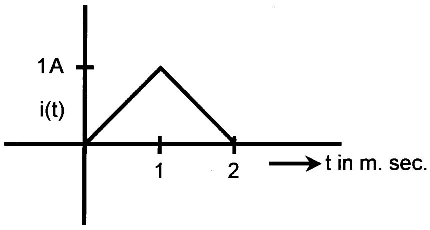
From eq. (1.14) at t = 1 m sec.

$$i(1 \times 10^{-3}) = 10^3 \times 1 \times 10^{-3} = 1\text{A}$$

$$i(t) = 1 + 10^3 [-t + 10^{-3}] = -10^3 t + 2 \text{ A}$$

$$w_L(t) = \frac{1}{2} \times 2 \times 10^{-3} (-10^3 t + 2)^2 = 10^3 t^2 - 4t + 4 \times 10^{-3} \text{ J}$$

The waveform of current and energy are shown in Fig. 1.20(b).



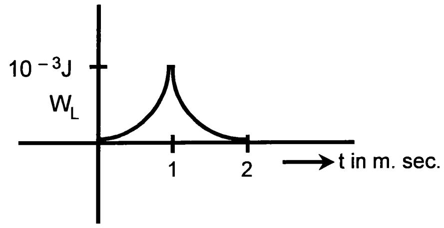


Fig. 1.20(b) Waveform of current and energy for example 1.6

1.6.3 Mutual inductance :

The inductor was defined in terms of the magnetic field produced by a current carried in a coil. The voltage produced in the coil due to the change in the magnetic flux surrounding the coil is essentially due to flux caused by the change in current in itself. Thus this inductance is appropriately called as *self inductance*. The time varying magnetic flux which is produced by a changing current in one coil, may also cause a voltage in the vicinity of a second coil. In 1831, Michael Faraday discovered this property and termed the voltage thus induced as mutually induced voltage and the property of the coils which causes this voltage as mutual inductance. The voltage induced in the second coil was found to be proportional to the time rate of change of current in the first coil. The circuit symbol for mutual inductor is shown in Fig. 1.21.

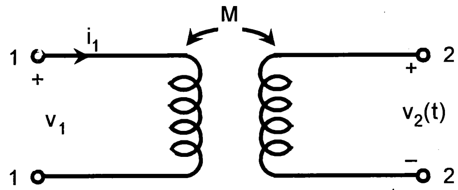


Fig. 1.21 Circuit symbol for mutual inductor

The v – i relationship is given by eq. (1.15).

$$v_2 = M_{21} \frac{di_1}{dt} \quad .....(1.15)$$

Unlike self inductance where the voltage and current signs are given considering the passivity, in mutual inductor, the signs of current and voltage have to be given on different considerations as they are associated with different terminal pairs 1, 1 and 2, 2. The symbol M_{21} indicates that the current is applied at terminals of coil 1 and the voltage is measured at coil 2. If current is applied at coil 2 and voltage is measured at coil 1, we can write

$$v_1 = M_{12} \frac{di_2}{dt} \quad .....(1.16)$$

On considerations of energy in the two cases one can easily prove that $M_{21} = M_{12}$. Hence we will use only one coefficient of mutual inductance $M_{12} = M_{21} = M$. The two coils with mutual inductance between them are said to be magnetically coupled coils. Mutual inductance is also measured in *henrys*.

1.6.4 Dot convention :

As explained earlier, the sign of the voltage induced in the second coil can not be decided by passivity considerations. For a given direction of current in one coil, the voltage induced may be positive or negative in the second coil depending on the sense of direction of the winding of the coil. Often, to increase the mutual inductance of the coils, they are wound on iron core so that the flux produced by the coil will be concentrated in the core and it links with the other coil wound on the same core. To indicate the sense of winding of these coils on the circuit symbol of mutual inductor, a dot convention is developed.

Consider two coils wound on a magnetic core as shown in Fig. 1.22.

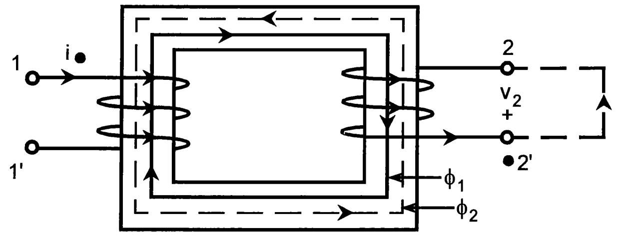


Fig. 1.22 Dot convention for mutual inductor

Place a dot at the terminal of coil 1 where the current i is entering, that is the current is increasing positively. This current produces a magnetic flux ϕ_1 in the core in a direction indicated by the arrow. This flux links with the second coil and induces a voltage at the terminals 2, 2' as per Faraday's laws of electromagnetic induction. The polarity of this voltage is given by Lenz's law. According to this, the voltage produced at terminals 2, 2' should have a polarity so as to produce a flux opposing the flux produced by coil 1, ϕ_1, if coil 2 were short circuited. This means that when coil 2 is short circuited, a current flows in the coil and the current in turn produces a flux ϕ_2 opposing the flux ϕ_1. The flux ϕ_2 should have a direction as shown in Fig. 1.21. In order to produce a flux in the indicated direction the current in the second coil must flow from left to right, or from terminal 2' to 2 in the short circuit across them. Thus the terminal 2' must be positive with respect to terminal 2 and a dot is placed at that terminal as indicated in Fig. 1.22. If the sense of winding in coil 2 is reversed as shown in Fig. 1.23, the dot must be placed at the terminal 2.

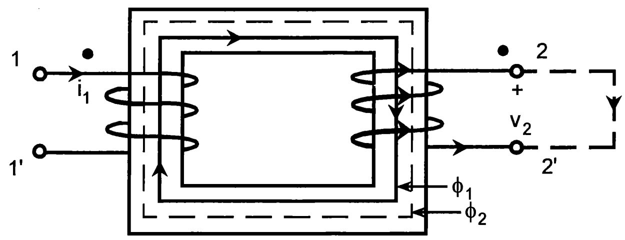


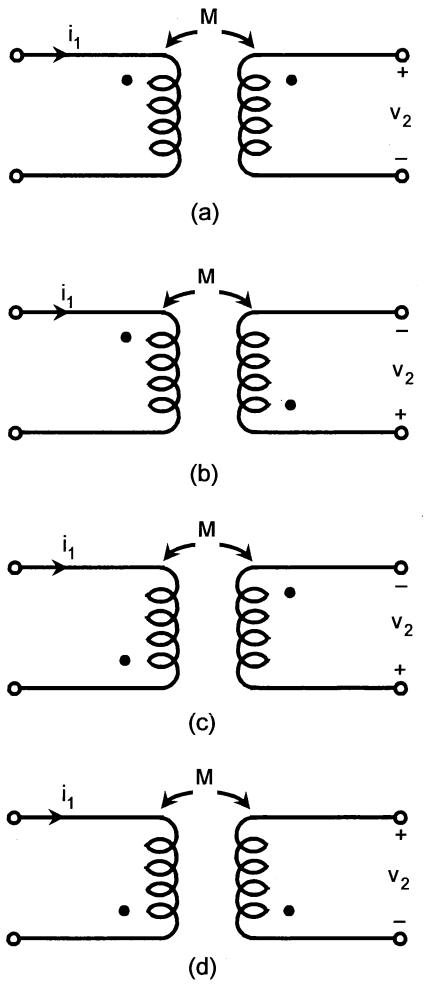
Fig. 1.23 Mutual inductor with sense of winding of coil 2 changed

It is very inconvenient to show the sense of winding for all the coils with mutual inductance in a network representation. Hence the dot convention helps to overcome this problem. The two coils are shown with a dot placed at one terminal of each coil to indicate the sense of winding and hence the polarity of the voltage induced. The following rule establishes the polarity of the mutually induced voltage.

" *If the current enters the dotted terminal of one coil, the voltage induced in the second coil will have a reference direction such that its dotted terminal is +ve* ".

In the same way, the current entering at the undotted terminal (or equivalently current leaving the dotted terminal) of first coil, produces a voltage in the second coil such that the undotted terminal is positive.

The two rules are illustrated in Fig. 1.24.



(a) $$v_2 = M\frac{di_1}{dt}$$

(b) $$v_2 = M\frac{di_1}{dt}$$

(c) $$v_2 = M\frac{di_1}{dt}$$

(d) $$v_2 = M\frac{di_1}{dt}$$

Fig. 1.24 Different possibilities of applying dot convention rules

Thus far, we had considered only the voltage produced in a open circuited coil, due to a change of current in another coil. If currents flow in each of these coil in addition to mutually induced voltages, self induced voltages will also be present. The mutual voltage induced in the coil is independent of the self induced voltage and will be in addition to it. Hence for the coils shown in Fig. 1.25, with reference direction of current and voltages as indicted, the v – i equation are

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \quad .....(1.17)$$

$$v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \quad .....(1.18)$$

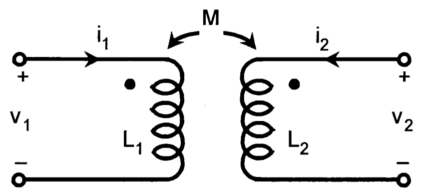


Fig. 1.25 Circuit with both self and mutual induction voltages

If the reference direction for i_2 is reversed in Fig. 1.25, the voltage due to self induction and the voltage due to mutual induction will have opposite sign and thus

$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \quad .....(1.19)$$

$$v_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} \quad .....(1.20)$$

1.6.5 Energy consideration in the magnetically coupled coils :

Consider two mutually coupled coil as shown in Fig. 1.26. Let us change the current i_1 from 0 to some value $i_1(t_1)$, keeping the second coil open. Also assuming

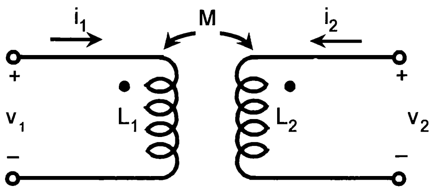


Fig. 1.26 Energy in coupled coils

zero energy storage in the coils to start with, the power delivered to the circuit is

$$P = v_1 i_1 + v_2 i_2 = L_1 i_1 \frac{di_1}{dt} + 0$$

The energy stored in the coils when $i_1 = i_1 (t_1)$ is

$$\int_0^{t_1} v_1 i_1 dt = \int_0^{i_1(t_1)} L_1 i_1 di_1 = \frac{1}{2} L_1 [i_1(t_1)]^2$$

Now, if the current i_1 is held constant at $i_1 (t_1)$ and the current i_2 is changed from 0 to $i_2 (t_2)$, then the power in the two coils will be

$$p = i_2 L_2 \frac{di_2}{dt} + M_{12} \frac{di_2}{dt} \cdot i_1$$

and the energy stored between the instants t_1 and t_2 is

$$\int_{t_1}^{t_2} (v_2 i_2 + v_1 i_1) dt = \int_{t_1}^{t_2} L_2 \frac{di_2}{dt} . i_2 dt + \int_{t_1}^{t_2} M_{12} \frac{di_2}{dt} . i_1 dt$$

$$= L_2 \int_0^{i_2(t_2)} i_2 di_2 + M_{12} i_1 \int_0^{i_2(t_2)} di_2$$

$$= \frac{1}{2} L_2 [i_2(t_2)]^2 + M_{12} i_2(t_2) i_1(t_1)$$

The total energy in the coils at $t = t_2$ is given by

$$w_{Total} = \frac{1}{2} L_1 [i_1(t_1)]^2 + \frac{1}{2} L_2 [i_2(t_2)]^2$$
$$+ M_{12}\, i_1 (t_1)\, i_2 (t_2) \qquad .....(1.21)$$

If the process is repeated by first changing the current i_2 form 0 to $i(t_2)$ with $i_1 = 0$ and then changing i_1 from 0 to $i(t_1)$, we get

$$w_{Total} = \frac{1}{2} L_1 [i_1(t_1)]^2 + \frac{1}{2} L_2 [i_2(t_2)]^2$$
$$+ M_{21}\, i_1(t_1)\, i_2 (t_2) \qquad .....(1.22)$$

Note that the only difference in eq. (1.21) and eq. (1.22) is the mutual inductance term, namely, instead of M_{12} we have M_{21}.

The two values of energy given by eq. (1.21) and eq. (1.22) must be same since the initial and final conditions are same in both cases. Hence we can conclude that

$$M_{12} = M_{21} = M \qquad .....(1.23)$$

Thus the energy stored in the coil is given by

$$W = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2. \qquad .....(1.24)$$

If one current enters the dot and the other leaves the dot then

$$W = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 - M i_1 i_2. \qquad .....(1.25)$$

Thus the energy stored in a pair of mutually coupled coils is given by

$$W = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 \pm M i_1 i_2. \qquad .....(1.26)$$

Since the energy is stored in a passive network, it must be positive. The only way it can be negative is if the energy due to the mutual inductance is negative i.e..

$$W = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 - M i_1 i_2. \qquad .....(1.27)$$

This may be written as

$$W = \frac{1}{2} \left(\sqrt{L_1}\, i_1 - \sqrt{L_2}\, i_2\right)^2 + \sqrt{L_1 L_2}\, i_1 i_2 - M i_1 i_2 \qquad .....(1.28)$$

Since this can not be – ve. It follows that

$$M \le \sqrt{L_1 L_2}$$

or $$M = K \sqrt{L_1 L_2} \qquad .....(1.29)$$

Where K lies between 0 and 1. It has a zero value when the two coils have no magnetic link between them and is equal to 1 when the two coils are perfectly coupled. Thus the maximum value of mutual inductance is equal to the geometric mean of the self inductances of the two coils. The degree of coupling is given by the factor K, which is known as the *coefficient of coupling.*

Example 1. 7

Place the dots at the appropriate terminals of the three coils shown in Fig. 1.27.

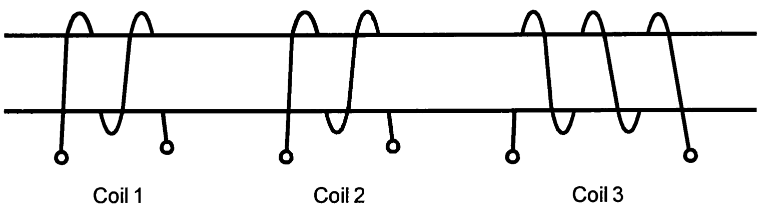


Fig. 1.27(a) The three coils on a common magnetic core

Solution :

First place a dot at the left terminal of coil 1 and assume that a current i_1 enters the dot.

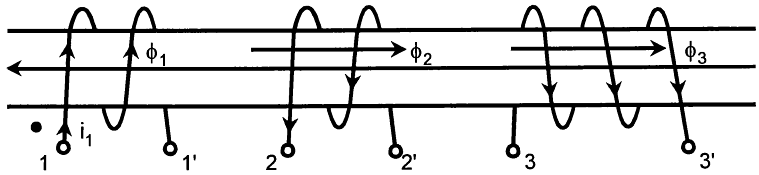


Fig. 1.27(b) Coils showing directions of currents and fluxes

The flux ϕ_1 will have a direction from right to left. The flux produced by coil 2 must oppose this flux. Thus the current direction in coil 2 must be as shown and hence this current must leave the terminal 2. Hence a dot must be placed at terminal 2 for coil 2.

Similarly, for coil 3, a current in the direction as shown in Fig. 1.27(b) must flow to produce a flux ϕ_3, opposing the flux ϕ_1. This current must leave the terminal 3' and hence a dot must be placed at 3' of the coil 3. Hence the circuit representation of these coils is given in Fig. 1.27(c).

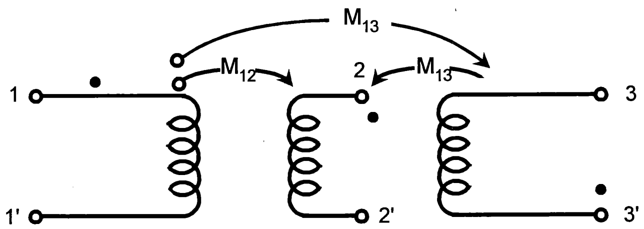


Fig. 1.27(c) Coils with dots placed to indicate polarity

When more than two coils are magnetically coupled, different types of dots (● ▲ ■) are used to indicate the polarities between various coils. In the above examples we can use ● for coils 1 and 2, ▲ for coils 1 and 3 and ■ for coils 2 and 3. The circuit symbol with dot convention is shown in Fig. 1.27(d).

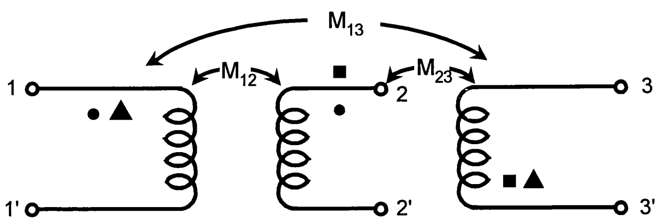


Fig. 1.27(d) Representation of coils with mutual inductance indicating polarities

1.7 THE CAPACITOR :

The capacitor is the third network element which is a passive element, but, like inductor, it can also store energy. Once it stores energy, it is capable of supplying this energy to external devices. It cannot provide unlimited energy or a finite average power over an infinite time interval. Hence it is not an active element like an ideal voltage or current source.

The inductor stores energy because of the current carried by it. In a similar manner, the capacitor is a device which stores energy by virtue of the voltage across it. Historically the capacitor was the first element to be discovered. In 1745 Van Mussenbrock devised an experiment to store static electricity by placing on insulator between two metal sheets, and then charged it by rubbing. Cunaeus, one of his friends, touched this device and received a violent shock ! Thus a device to store electricity was discovered.

Later in the year 1812 Simeon Poisson, gave a mathematical explanation for the energy storage on a capacitor. He compared the forces between charge on the plates to the gravitational force between a mass and earth and the energy associated with these stored charges to the potential energy of a mass at rest above the earth. He also showed that the energy is proportional to the area of the plates and inversely proportional to the spacing between them. Thus, capacitance, which is the property of a capacitor by which it can store energy, is given by

$$C = \frac{\epsilon A}{d}$$

where A is the area of the plates in sq. m.
d is the distance between the plates in m.
and ϵ is the permittivity of the insulating material between the two conductors

For air or vacuum,

$$\epsilon = \epsilon_0 = 8.854 \text{ pF/m.}$$

Coming back to the circuit model of a capacitor, it is described by the v – i relationship

$$i = C\frac{dv}{dt} \qquad .....(1.30)$$

The two commonly used symbols and the polarities of voltage and current for passivity requirement are shown in Fig. 1.28.

Fig. 1.28 circuit models of capacitor

An alternate form of eq. (1.30) can be derived as follows :

$$dv = \frac{1}{C}\ i\, dt \qquad .....(1.31)$$

Integrating on both sides of eq. (1.31) and assuming that the voltage across the capacitor was zero at t = – ∞, a conceptual time when the capacitor was manufactured and hence the voltage across it was zero, we get

$$\int_0^v dv = \int_{-\infty}^t \frac{1}{C} i\, dt$$

$$v = \frac{1}{C}\int_{-\infty}^t i\, dt \qquad .....(1.32)$$

From eq. (1.30), it is clear that, if the voltage across the capacitor is a constant, the current through it would be zero. But if the voltage changes suddenly, the current would by infinite. Thus a capacitor opposes any sudden change in voltage across it. An infinite current is required to be passed through it if the voltage has to change suddenly by a finite value. If the two terminals of a charged capacitor are shorted, a spark is produced because of this infinite current.

The two eqs. (1.30) and (1.32) describing the behaviour of a capacitor, are linear and hence the capacitor is a linear element. The power entering the capacitor at any instant is given by

$$P = v \cdot i$$

$$= v \cdot c\frac{dv}{dt} \qquad .....(1.33)$$

When $\frac{dv}{dt} = 0$ or v(t) is a constant, p = 0 and no additional energy is stored in the capacitor. If the voltage is increasing, the power is positive and additional energy is stored in the capacitor. The total energy in the capacitor is given by

$$w_C = \int_{-\infty}^t vi\,dt = \int_{-\infty}^t C\, v\frac{dv}{dt}\, dt$$

$$= C\int_0^v v\, dv = \frac{1}{2} Cv^2 \qquad .....(1.34)$$

Thus the energy stored in the capacitor at any instant depends on the value of the voltage at that instant only and not on the past history.

Example 1.8

The capacitor in Fig 1.29(a) is supplied with a voltage waveform shown in Fig. 1.29(b). Obtain the current and energy waveforms in the capacitor.

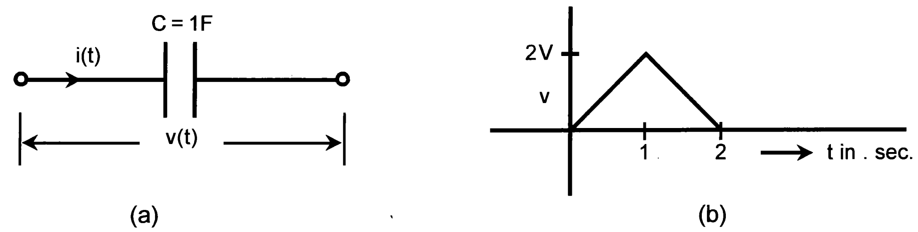


Fig. 1.29 The capacitor and the voltage waveform for example 1.8

Solution :

For $0 \leq t \leq 1$

$$v(t) = 2t \ \text{V}$$

$$i = C\frac{dv}{dt} = 1.2 = 2 \ \text{A}$$

$$w_C = \frac{1}{2}Cv^2 = \frac{1}{2} \cdot 1 \times 4t^2 = 2t^2 \ \text{J}$$

Thus the current is a constant during the interval $0 \leq t \leq 1$.

For $1 \leq t \leq 2$

$$v(t) = -2t + 4 \ \text{V}$$

and $$i(t) = 1 \,.\, (-2) = -2 \ \text{A}$$

The energy stored in the capacitor is

$$w_C = \frac{1}{2}Cv^2 = \frac{1}{2} \times 1 \times (-2t + 4)^2 = 2t^2 - 8t + 8 \ \text{J}$$

The waveforms of current and energy are given in Fig. (1.30).

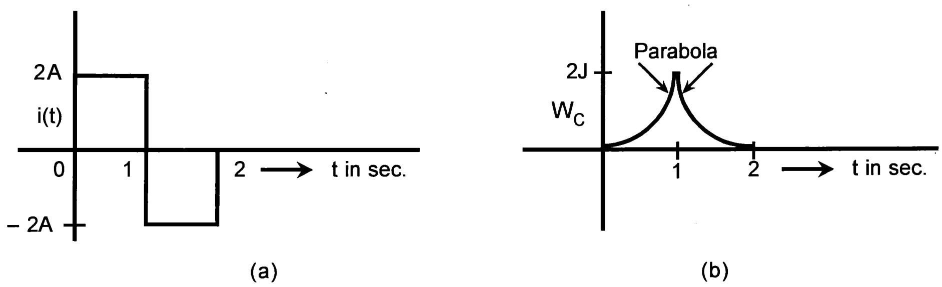


Fig. 1.30 Waveforms of i and W_C for example 1.8

Example 1.9

Consider the capacitor in Fig. 1.31(a) and a waveform of voltage shown in Fig. 1.31(b) applied across it. Find the waveform of current and energy in the capacitor.

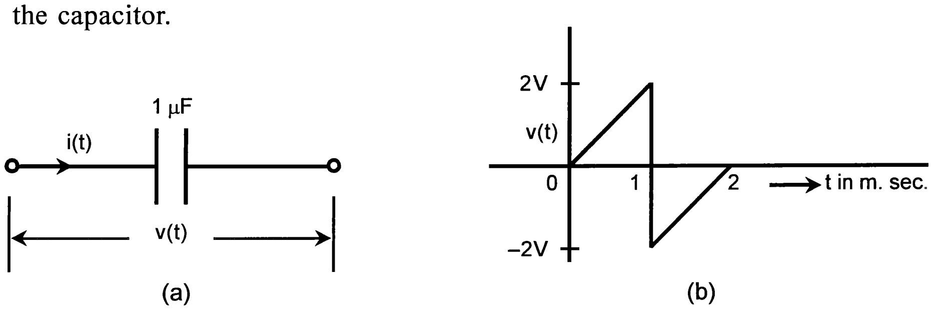


Fig. 1.31 The capacitor and the voltage waveform for example 1.9

Solution :

For $0 \leq t < 1$ m.sec.

$$v(t) = \frac{2}{1\times10^{-3}}\,t = 2\times10^3 t$$

$$i(t) = C\frac{dv}{dt} = 1\times10^{-6}\times2\times10^3 = 2 \text{ mA}$$

$$w_C = \frac{1}{2}Cv^2 = \frac{1}{2}\times1\times10^{-6}\times4\times10^6 t^2 = 2t^2 \text{ J}$$

At t = 1 m.sec the voltage across the capacitor instantaneously changes from +2V to −2V thus making $\frac{dv}{dt}$ infinite. Hence an impulse of current will be produced at t = 1 m.sec.

For $1 < t \leq 2$ m. sec.

$$v(t) = 2\times10^3\,t - 4$$

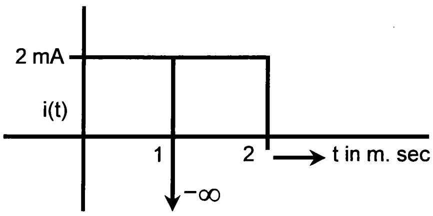
$$i(t) = C\frac{dv}{dt} = 1\times10^{-6}\times2\times10^3 = 2 \text{ mA.}$$

The energy during this interval is

$$W_C(t) = \frac{1}{2}\,C\,v^2 = \frac{1}{2}\times1\times10^{-6}\times(2\times10^3 t-4)^2$$

$$= 2\,t^2 - 8\times10^{-3}\,t + 8\times10^{-6} \text{ J}$$

The waveform of current and energy are given in Fig. 1.32.



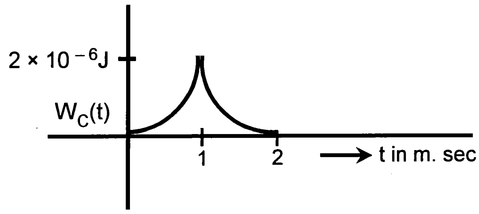


Fig. 1.32 The current and energy waveforms for example 1.8

Example 1.10

A current of waveform shown in Fig. 1.33(a) is applied to a capacitor of value 2 μ F. Find the voltage waveform.

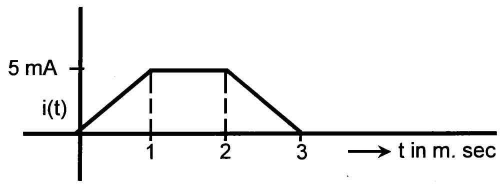


Fig. 1.33(a) Current waveform for example 1.10

Solution :

For $0 \leq t \leq 1$ m sec.

$$i(t) = \frac{5 \times 10^{-3}}{1 \times 10^{-3}} t = 5t \text{ A}$$

$$v(t) = \frac{1}{C} \int_{-\infty}^{0} i(t)dt$$

Assuming no voltage to be present across the capacitor at t = 0

$$v(t) = \frac{1}{C} \int_{-\infty}^{0} i(t)dt + \frac{1}{C} \int_{0}^{t} i(t)dt$$

$$= v(0) + \frac{1}{2 \times 10^{-6}} \int_{0}^{t} 5t \, dt$$

$$= 0 + \frac{5}{2 \times 10^{-6}} \left. \frac{t^2}{2} \right|_0^t$$

$$= 1.25 \times 10^6 t^2 \text{ V} \quad \text{.....(1.35)}$$

For $1 \leq t \leq 2$ m sec

$$i(t) = 5 \times 10^{-3}\ \text{A}$$

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)\, dt$$

$$= \frac{1}{C}\int_{-\infty}^{1\times10^{-3}} i(t)\, dt + \frac{1}{C}\int_{1\times10^{-3}}^{t} 5\times10^{-3}\, dt \qquad .....(1.36)$$

The first integral in the above expression is the voltage across the capacitor at $t = 1$ m sec. From eq. (1.35).

$$v(1 \times 10^{-3}) = 1.25 \times 10^{6} \times 1 \times 10^{-6} = 1.25\ \text{V}$$

Eq. (1.36) becomes

$$v(t) = 1.25 + \frac{1}{2\times10^{-6}}\int_{1\times10^{-3}}^{t} 5\times10^{-3}\, dt$$

$$= 1.25 + 2.5 \times 10^{3} t \Big|_{1\times10^{-3}}^{t}$$

$$= 2.5 \times 10^{3}\, t - 1.25 \qquad .....(1.37)$$

For $2 \leq t \leq 3$

$$i(t) = -5\, t + 15 \times 10^{-3}\text{A}$$

$$v(t) = \frac{1}{C}\int_{-\infty}^{2\times10^{-3}} i(t) dt + \frac{1}{2\times10^{-6}}\int_{2\times10^{-3}}^{t} \left(-5t + 15\times10^{-3}\right) dt$$

$$= v(2 \times 10^{-3}) + \frac{1}{2\times10^{-6}}\left[\frac{-5t^{2}}{2} + 15\times10^{-3}\, t\right]_{2\times10^{-3}}^{t}$$

From eq. (1.37),

$$v\,(2 \times 10^{-3}) = 3.75\ \text{V}$$

$$v(t) = 3.75 - 1.25 \times 10^{6}\, t^{2} + 7.5 \times 10^{3} t - 15 + 5$$

$$= -\,1.25 \times 10^{6} t^{2} + 7.5 \times 10^{3} t - 6.25\ \text{V}$$

The waveform of voltage is shown in Fig. 1.33(b).

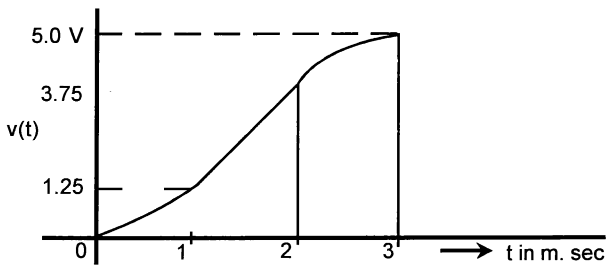


Fig. 1.33(b) Waveform of voltage across the capacitor for example 1.10

Problems

1.1 Ten Coulombs of positive charge per second are passing through a wire in a direction from 1 to 2.

(a) What is the current if the assumed reference direction is from 1 to 2 ?

(b) What is the current if the assumed reference direction is from 2 to 1 ?

(c) What would be the answer for (a) and (b) if the charge is negative instead of positive ?

1.2 A Coulomb of charge changes its energy by 30 Joules in moving from point 1 to 2. What is the voltage of point 1 with respect to point 2 if

(a) the charge is positive and the energy is lost.

(b) the charge is negative and the energy is lost.

(c) the charge is positive and the energy is gained

and (d) the charge is negative and the energy is gained.

1.3 A 2 terminal device has a positive current of 15 A entering at terminal 1 and leaving at terminal 2. What is the power absorbed in the device ? When

(a) the voltage at 1 is 5 V positive with respect to 2.

(b) the voltage at 1 is 5 V negative with respect to 2

and (c) the voltage at 2 is – 10 V with respect to 1.

1.4 Identify the source in the devices shown in Fig. P.1.1.

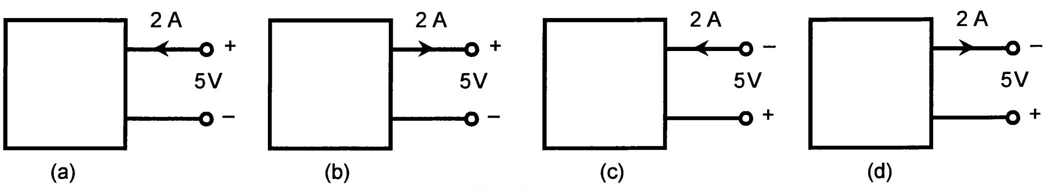


Fig. P. 1.1

1.5 A resistor of value 2 Ω is connected across a voltage whose waveform is shown in Fig. P. 1.2. Draw the waveforms of current and power.

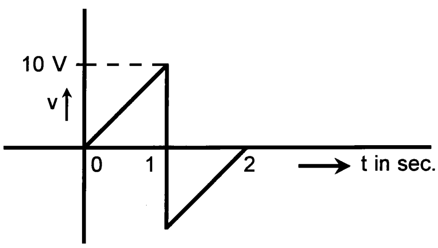


Fig. P. 1.2

1.6 The volt – ampere characteristic of a device is shown in Fig. P. 1.3. What type of device is it and what is its value ?

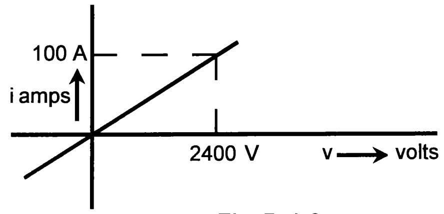


Fig. P. 1.3

1.7 A current $i = 10\ e^{-t}$ is applied to

(a) a 3Ω resistor (b) a 2H inductor and (c) a 0.1 F capacitor.

What are the respective voltages ? Write down the expressions for power in each case.

1.8 A number of waveforms are shown in Fig. P. 1.4(a). Similarly a number elements are shown in Fig. P. 1.4(b). Write down the waveforms of the response for each of these elements for each of the waveforms in Fig. P. 1.4(a) where

(a) f(t) is a current and the response is the voltage and

(b) f(t) is voltage and the response is the current.

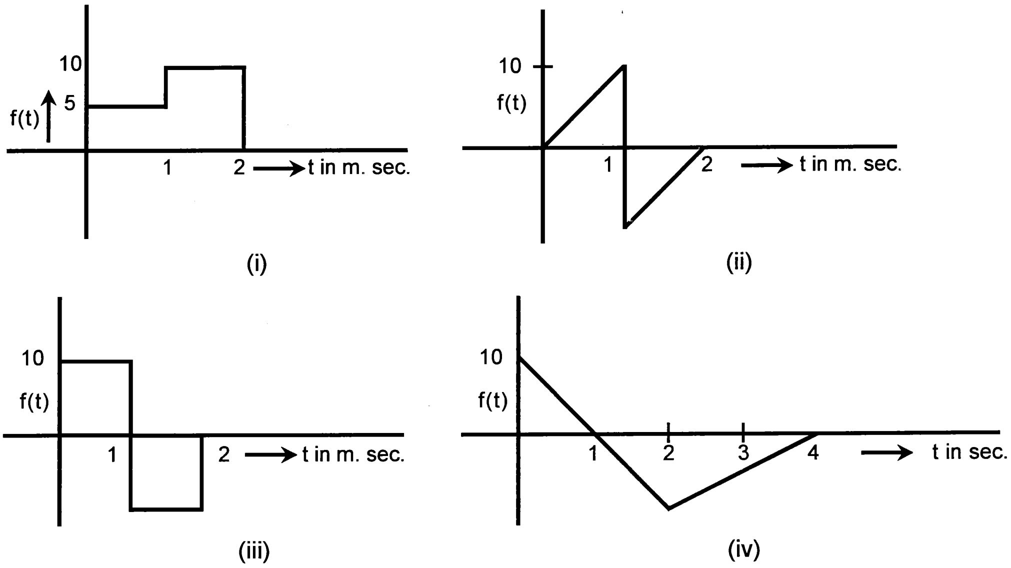


Fig. P. 1.4(a)

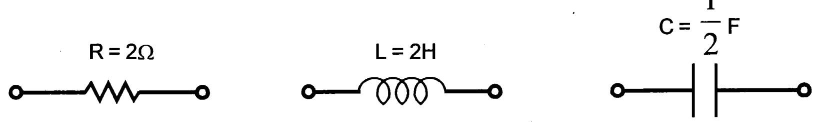


Fig. P. 1.4(b)

1.9 A voltage $v(t) = 200 \text{ Sin } 100\ \pi t$ volts is applied across a capacitor of value $C = 0.05$ farads. Find the expression for the current i(t) and the energy stored in the capacitor.

1.10 A voltage $v(t) = 25 \text{ Sin } 1000\ t$ volts is applied across a 5 mH inductor at $t = 0$ when the current in it is 2 A. Find the expression for the current i(t) for $t \geq 0$.

1.11 The current in a coil is changing at a rate of 2 A/sec. The voltage across another coil placed very near to it was found to be 10 m V. What is the value of the mutual inductance between the two coils ?

1.12 Two coils with self inductances 0.3 mH and 0.2 mH are coupled together so that the mutual inductance between them is 0.22 mH. A current waveform shown in Fig. P. 1.5 is applied to the first inductor. Draw the waveform of the self induced emf's in the first coil and the mutually induced emf in the second coil.

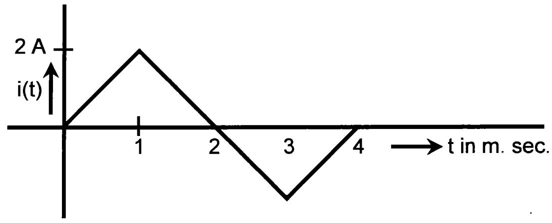


Fig. P. 1.5

1.13 For the magnetically coupled coils shown in Fig. P.1.6 establish the polarity markings using different kinds of dots.

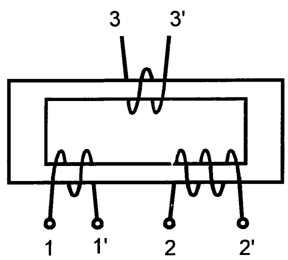


Fig. P. 1.6

1.14 Three coils have terminals 1, 1' ; 2, 2' and 3, 3'. Place these coils on a common core with proper winding sense such that

(a) The terminals 1 and 2, 2 and 3 and 3 and 1 have same polarity mark.

(b) 1 and 2' ; 2 and 3' ; 3 and 1 have same polarity mark.

1.15 Two inductors have self inductance of 0.1 mH and 0.4 mH and a mutual inductance of 0.15 mH. What is the value of the coefficient of coupling between them ? If a current

$$i(t) = 3 \text{ Sin } t + 1.5 \text{ Sin } 2t$$

is passed through the first inductor what is the expression for the voltage induced in the second coil ?

1.16 If a voltage v is applied to a coil, the quantity $\int_{-\infty}^{t} v\,dt$ is called the *flux linkage*, ψ, in the coil at time 't'. A current given by $i(t) = (1 - e^{-2t})$ A, $t > 0$ is flowing through a coil of inductance 0.5 H. The current has a value 0.865 A at a certain time. At this time

(a) What is the rate of change of current ?

(b) What is the value of flux linkages ?

(c) What is the rate of change of flux linkages ?

(d) What is the value of voltage across the coil ?

and (e) What is the energy stored in the inductor ?

1.17 The quantity $\int_{-\infty}^{t} i\,dt$ in a capacitor is called the *charge* 'q'. In a capacitor of value $\frac{1}{2}$ F and charged to a voltage of 1 V, a current of the from $i(t) = e^{-2t}$, $t > 0$ is flowing. What is the value of the current at t = 1 sec. ? At t = 1 sec.

(a) What is the total charge accumulated in the capacitor ?

(b) What is the rate of change of voltage across the capacitor ?

(c) What is the voltage across the capacitor ?

and (d) At what rate is energy being taken from the electric field of the capacitor ?

1.18 The voltage induced in the inductor is given by

$$v = \frac{d}{dt}(Li) = \frac{d\psi}{dt}$$

where ψ is the flux linkages. A voltage is induced in a coil if L is constant and i is changing with respect to time. If the inductance value changes with time a constant current also produces an emf in the coil. When a dc current of 1 A is passed through a time varying inductor whose inductance changes as shown in Fig. P. 1.7 what is the waveform of voltage across the inductor ?

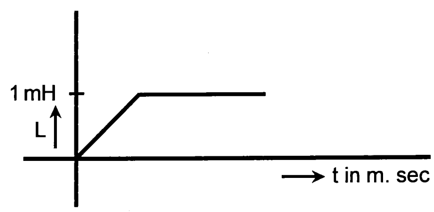


Fig. P. 1.7

1.19 A nonlinear capacitor has its v Vs q characteristic shown in Fig. P. 1.8. If a voltage v(t) = 10 Sin 2t is applied to such a capacitor what is the expression for the current through it ?

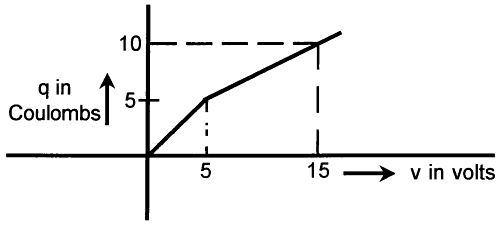


Fig. P. 1.8

1.20 Fig. P. 1.8 represents characteristic of a nonlinear inductor if the x and y axis variables are current and flux linkages respectively. If the current in the inductance is given by the waveform shown in Fig. P. 1.9 sketch the waveform of voltage across the inductor.

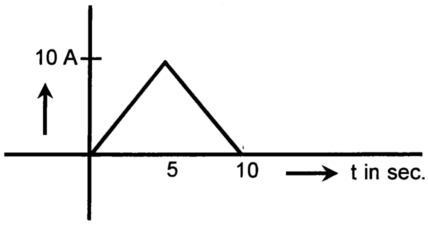


Fig. P. 1.9