



# 1

# PN Junction Diode and its Applications

*In this Chapter,*

- ◆ The basic aspects connected with Semiconductor Physics and the terms like Effective Mass, Semiconductors and their types are introduced, Intrinsic, Extrinsic, and Semiconductors are introduced.
  - ◆ Atomic Structure, Configurations, Concept of Hole, Conductivity in *p-type* and *n-type* semiconductors are given. This forms the basis to study Semiconductor Devices in the following chapters.
  - ◆ After p-type and n-type semiconductors, we shall study semiconductor devices formed using these two types of semiconductors.
  - ◆ We shall also study the physical phenomena such as conduction, transport mechanism, electrical characteristics, and applications of semiconductor devices such as p-n diode, zener diode, tunnel diode and so on.
  - ◆ Circuit applications of p-n junction diode namely Half Wave Rectifier (HWR), Full Wave Rectifier (FWR) and Bridge Rectifier circuits, for rectification applications are described.
  - ◆ Inductor, L-section and  $\pi$ -section filter circuits are explained.
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## 1.1 REVIEW OF SEMICONDUCTOR PHYSICS

### 1.1.1 ENERGY LEVELS AND ENERGY BANDS

To explain the phenomenon associated with conduction in metals and semiconductors and the emission of electrons from the surface of a metal, we have to assume that atoms have loosely bound electrons which can be removed from it.

Rutherford found that atom consists of a nucleus with electrons (each electron carrying a unit negative charge) rotating around it. The mass of the atom is concentrated in the nucleus. It consist of protons which are positively charged. In hydrogen atom, there is one positively charged nucleus (a proton) and a single electron. The charge on particle is positive and is equal to that of electron. So hydrogen atom is neutral in charge. The proton in the nucleus carries the charge of the atom, so it is immobile. The electron will be moving around it in a closed orbit. The force of attraction between electron and proton follows Coulombs Law. {directly proportional to product of charges and inversely proportional to (distance)<sup>2</sup>}.

Assume that the orbit of the electron around nucleus is a circle. We can calculate the radius of this circle, in terms of total energy 'W' of the electron. The force of attraction between the electron and the nucleus is :

$$F \propto e^2$$

( Therefore, the nucleus has the proton with electron charge equal to 'e' )

$$F \propto \frac{1}{r^2}$$

( r is the radius of the orbit )

$$\therefore F \propto \frac{e^2}{r^2}$$

$$F = \frac{e^2}{4\pi\epsilon_0 r^2}$$

$$\therefore F \propto \frac{e^2}{r^2}$$

where  $\epsilon_0$  is the permittivity of free space. Its value is  $\frac{10^{-9}}{36\pi}$  F/m.

As the electron is moving around the nucleus in a circular orbit with radius r, and with velocity v, then the force of attraction given by the above expression F should be equal to the

centripetal force  $\frac{mv^2}{r}$  (according to Newton's Second Law of Motion)

$$( F = m.a; m \text{ is the mass of electron and acceleration, } a = \frac{v^2}{r} )$$

$\therefore$  The Potential Energy of the electron =  $-\frac{e^2}{4\pi\epsilon_0 r^2} \times r = -\frac{e^2}{4\pi\epsilon_0 r}$   
 (-ve sign is because Potential Energy is by definition work done against the field )

$$\text{Kinetic Energy} = \frac{1}{2} mv^2$$

$\therefore$  Total Energy W possessed by the electron =  $\frac{1}{2} mv^2 - \frac{e^2}{4\pi\epsilon_0 r}$

But  $mv^2 = \frac{e^2}{4\pi\epsilon_0 r}$  reduces to

$$W = \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r} = -\frac{e^2}{8\pi\epsilon_0 r}$$

$\therefore$  Energy possessed by the electron  $W = -\frac{e^2}{8\pi\epsilon_0 r}$

W is the energy of the electron. Only for Hydrogen atom, W will also be the energy of atom since it has only one electron. The negative sign arises because the Potential Energy of the electron is

$$W = -\frac{e^2}{8\pi\epsilon_0 r}$$

As radius r increases, Potential Energy decreases. When r is infinity, Potential Energy is zero. Therefore the energy of the electron is negative. If r is  $< \infty$ , the energy should be less. (Any quantity less than 0 is negative).

The above equation is derived from the classical model of the electron. But according to classical laws of electromagnetism, an accelerated charge must radiate energy. Electron is having charge = e. It is moving with velocity v or acceleration  $v^2/r$  around the nucleus. Therefore this electron should also radiate energy. If the charge is performing oscillations with a frequency 'f', then the frequency of the radiated energy should also be the same. Hence the frequency of the radiated energy from the electron should be equal to the frequency with the electron orbiting round the nucleus.

But if the electron is radiating energy, then its total energy 'W' must decrease by the amount equal to the radiation energy. So 'W' should go on decreasing to satisfy the equation,

$$W = -\frac{e^2}{8\pi\epsilon_0 r}$$

If W decreases, r should also decrease. Since if  $-\frac{e^2}{8\pi\epsilon_0 r}$  should decrease, this quantity

should become more negative. Therefore, r should decrease. So, the electron should describe smaller and smaller orbit and should finally fall into nucleus. So classical model of atom is not fairly satisfactory.

### 1.1.2 THE BOHR ATOM

The above difficulty was resolved by Bohr in 1913. He postulated three fundamental laws.

1. *The atom can possess only discrete energies. While in states corresponding to these discrete energy levels, electron does not emit radiation in stationary state.*
2. *When the energy of the electron is changing from  $W_2$  to  $W_1$  then radiation will be emitted. The frequency of radiation is given by*

$$f = \frac{W_2 - W_1}{h}$$

where 'h' is Plank's Constant.

$$h = 6.626 \times 10^{-34} \text{ J - sec.}$$

*i.e., when the atom is in stationary state, it does not emit any radiation. When its energy changes from  $W_2$  to  $W_1$  then the atom is said to have moved from one stationary state to the other. The atom remains in the new state corresponding to  $W_1$ . Only during transition, will some energy be radiated.*

3. *A stationary state is determined by the condition that the angular momentum of the electron in this state must be an integral multiple of  $h/2\pi$ .*

$$\text{So } mvr = \frac{nh}{2\pi}$$

where  $n$  is an integer, other than zero.

### 1.1.3 EFFECTIVE MASS

An electron mass 'm' when placed in a crystal lattice, responds to applied field as if it were of mass  $m^*$ . The reason for this is the interaction of the electron even within lattice.

$E$  = Kinetic Energy of the Free Electron

$p$  = Momentum

$v$  = Velocity

$m$  = Mass

$p = mv$

$m^*$  = Effective mass of electron

$$E = \frac{p^2}{2m}$$

Electrons in a solid are not free. They move under the combined influence of an external field plus that of a periodic potential of atom cores in the lattice. An electron moving through the lattice can be represented by a wave packet of plane waves grouped around the same value of  $K$  which is a wave vector.

Electron velocity falls to zero at each band edge. This is because the electron wave further becomes standing wave at the top and bottom of a band i.e.,  $v_g = 0$ .

Now consider an electronic wave packet moving in a crystal lattice under the influence of an externally applied uniform electric field. If the electron has an instantaneous velocity 'v<sub>g</sub>' and moves a distance 'dx' in the direction of an accelerating force 'F', in time 'dt', it acquires energy δE where

$$\delta E = F \times \delta x = F \times v_g \times \delta t$$

$$\delta E = \frac{F}{\hbar} \times \frac{\delta E}{\delta k} \times \delta t$$

$$v_g = \frac{\delta E}{\hbar \delta k}$$

Within the limit of small increments in 'K', we can write,

$$\frac{dK}{dt} = \frac{F}{\hbar} \quad \dots\dots\dots( 1.1 )$$

But this is not the case for the electron in a solid because the externally applied force is not the only force acting on the electrons. Forces associated with the periodic lattice are also present.

Acceleration of an electronic wave packet in a solid is equal to the rate of change of its velocity.

$$\text{Acceleration of an electron in a solid} = \frac{d v_g}{d t} = \frac{d}{d t} \left( \frac{d E}{d p} \right) = \frac{d^2 E}{(d p)(d t)}$$

$$\therefore v_g = \frac{d E}{d p}$$

$$\frac{d k}{d t} = \frac{F}{\hbar}$$

$$F = \hbar \times \frac{d k}{d t}$$

But  $\frac{d k}{d t} = \frac{F}{\hbar}$  from Eq. ( 1.1 )

$$\therefore \frac{d v_g}{d t} = \left( \frac{d p}{d t} \right) \frac{d^2 E}{d p^2} = \frac{F}{\hbar^2} \cdot \frac{d^2 E}{d p^2}$$

or  $F = \hbar^2 \times \left( \frac{d^2 E}{d p^2} \right)^{-1} \times \frac{d v_g}{d t}$

This is of the form, F = ma, from Newton's Laws of Motion,

where  $m^* = \hbar^2 \times \left( \frac{d^2 E}{d p^2} \right)^{-1}$  and  $a = \frac{d v_g}{d t}$

$$\therefore F = m^* \times \frac{d v_g}{d t}$$

where m\* is the effective mass.

If an electric field 'ε' is impressed, the electron will accelerate and its velocity and energy will increase. Hence the electron is said to have positive mass. On the other hand, if an electron is at the upper end of a band, when the field is applied, its energy will increase and its velocity decreases. So the electron is said to have negative mass.

In an atom,

$$\text{Coulombs force of attraction} = \frac{e^2}{4\pi\epsilon_0 r^2}$$

$$\text{Centripetal Force} = \frac{mv^2}{r}$$

Equating these two forces in an atom,

$$\frac{e^2}{4\pi\epsilon_0 r^2} = \frac{mv^2}{r} \quad v = \text{velocity}; r = \text{radius of Orbit}$$

$$\therefore \frac{e^2}{4\pi\epsilon_0} = \frac{mv^2}{r} \times r^2 = mv^2 r$$

$$\therefore r = \frac{e^2}{4\pi\epsilon_0 mv^2} \quad \dots\dots\dots(1.2)$$

But  $mv r = \frac{nh}{2\pi}$   $h = \text{Plank's Constant}; n = \text{Principle Quantum Number.}$

$$\therefore v = \frac{nh}{2\pi m r} \quad \dots\dots\dots(1.3)$$

Substituting the value of v in eq. (1.2),

$$r = \frac{e^2 \times 4\pi^2 m^2 r^2}{4\pi\epsilon_0 \cdot m \times n^2 h^2}$$

$$\therefore \boxed{r = \frac{n^2 h^2 \epsilon_0}{\pi m e^2}} \quad \dots\dots\dots(1.4)$$

This is the expression for radii of stable states.

Energy possessed by the atom in the stable state is

$$W = \frac{e^2}{8\pi\epsilon_0 r} \quad \dots\dots\dots(1.5)$$

Substituting the value of r in Eq. (1.4), then

$$W = - \frac{e^2 \times \pi m e^2}{8\pi\epsilon_0 n^2 h^2 \epsilon_0}$$

$$W = - \frac{me^4}{8h^2 \epsilon_0^2} \cdot \frac{1}{n^2}$$

Thus energy 'W' corresponds to only the coulombs force due to attraction between ground electron (negative charge) and proton (positive charge).

**Problem 1.1**

Determine the radius of the lowest state of Ground State.

**Solution**

$$n = 1$$

$$\therefore r = \frac{n^2 h^2 \epsilon_0}{\pi m e^2}$$

Plank's Constant,  $h = 6.626 \times 10^{-34}$  J-sec

Permittivity,  $\epsilon_0 = 10^{-9}/36\pi$

Substituting the values and simplifying,

$$\therefore r = 0.58 \text{ \AA}$$

**1.1.4 ATOMIC ENERGY LEVELS**

For different elements, the value of the free electron concentration will be different. By spectroscopic analysis we can determine the energy level of an element at different wavelengths. This is the characteristic of the given element.

The lowest energy state is called the normal level or ground level. Other stationary states are called *excited, radiating, critical* or *resonance* levels.

Generally, the energy of different states is expressed in (eV) electrovolts rather than in Joules (J), and the emitted radiation is expressed by its wavelength  $\lambda$  rather than by its frequency. This is only for convenience since Joule is a larger unit and the energy is small and is in electron volts.

$$F = \frac{W_2 - W_1}{h}$$

$$f = \frac{C}{\lambda}$$

$C =$  Velocity of Light  $= 3 \times 10^{10}$  cm / sec.

$h =$  Plank's Constant  $= 6.626 \times 10^{-34}$  J - sec.

$f =$  Frequency of Radiation

$\lambda =$  Wavelength of emitted radiation

$W_1$  and  $W_2$  are the energy levels in Joules.

$1 \text{ eV} = 1.6 \times 10^{-19} \text{ Joules}$
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$$\therefore \frac{C}{\lambda} = \frac{(E_2 - E_1) \times 1.6 \times 10^{-19}}{h}$$

$$\frac{3 \times 10^{10}}{\lambda} = \frac{(E_2 - E_1) \times 1.6 \times 10^{-19}}{6.626 \times 10^{-34}}$$

$E_1$  and  $E_2$  are energy levels in eV.

$$\text{or } \lambda = \frac{3 \times 10^{10} \times 6.626 \times 10^{-34}}{(E_2 - E_1) \times 1.6 \times 10^{-19}}$$

$$\lambda = \frac{12,400}{(E_2 - E_1)} \text{A}^\circ$$

where ' $\lambda$ ' is in Armstrong,  $1 \text{A}^\circ = 10^{-8} \text{cm} = 10^{-10} \text{m}$ , and  $E_2$  and  $E_1$  in eV.

### 1.1.5 PHOTON NATURE OF LIGHT

An electron can be in the excited state for a small period of  $10^{-7}$  to  $10^{-10}$  sec. Afterwards it returns back to the original state. When such a transition occurs, the electrons will lose energy equal to the difference of energy levels ( $E_2 - E_1$ ). This loss of energy of the atom results in radiation of light. The frequency of the emitted radiation is given by

$$f = \frac{(E_2 - E_1)}{h}$$

According to classical theory it was believed that atoms continuously radiate energy. But this is not true. Radiation of energy in the form of photons takes place only when the transition of electrons will take place from higher energy state to lower energy state, so that ( $E_2 - E_1$ ), is positive. This will not occur if the transition is from lower energy state to higher energy state. When such photon radiation takes place, the number of photons liberated is very large. This is explained with a numerical example given below.

#### Problem 1.2

For a given 50 W energy vapour lamp. 0.1% of the electric energy supplied to the lamp, appears in the ultraviolet line  $2,537 \text{A}^\circ$ . Calculate the number of photons per second, of this wavelength emitted by the lamp.

#### Solution

$$\lambda = \frac{12,400}{(E_2 - E_1)}$$

$\lambda$  = Wavelength of the emitted radiation.

$E_1$  and  $E_2$  are the energy levels in eV.

$(E_2 - E_1)$  is the energy passed by each photon in eV of wavelength  $\lambda$ . In the given problem,  $\lambda = 2,537 \text{A}^\circ$ .  $(E_2 - E_1) = ?$

$$\therefore (E_2 - E_1) = \frac{12,400}{2,537} = 4.88 \text{ eV/photon}$$

0.1% of 50W energy supplied to the lamp is,

$$\text{i.e., } \frac{0.1}{100} \times 50 = 0.05 \text{ W} = 0.05 \text{ J/Sec}$$

$$\therefore 1 \text{ W} = 1 \text{ J/sec}$$

Converting this into the electron Volts,



$$\frac{0.05 \text{ J/sec}}{1.6 \times 10^{-19} \text{ J/eV}} = 3.12 \times 10^{17} \text{ eV/sec}$$

This is the total energy of all the photons liberated in the  $\lambda = \text{Wavelength of } 2,537 \text{ \AA}^\circ$ .

$$\begin{aligned} \therefore \text{Number of photons emitted per sec} &= \frac{\text{total energy}}{\text{energy / photon}} \\ &= \frac{3.12 \times 10^{17} \text{ eV/sec}}{4.88 \text{ eV/photon}} \\ &= 6.4 \times 10^{16} \text{ photon/sec} \end{aligned}$$

The lamp emits  $6.4 \times 10^{16}$  photons / sec of wavelength  $\lambda = 2,537 \text{ \AA}^\circ$ .

**1.1.6 IONIZATION POTENTIAL**

If the most loosely bound electron ( free electron ) of an atom is given more and more energy, it moves to stable state (since it is loosely bound, its tendency is to acquire a stable state. Electrons orbiting closer to the nucleus have stable state, and electron orbiting in the outermost shells are loosely bound to the nucleus). But the stable state acquired by the electron is away from the nucleus of the atom. If the energy supplied to the loosely bound electron is enough large, to move it away completely from the influence of the parent nucleus, it becomes detached from it. **The energy required to detach an electron is called Ionization Potential.**

**1.1.7 COLLISIONS OF ELECTRONS WITH ATOMS**

If a loosely bound electron has to be liberated, energy has to be supplied to it. Consider the case when an electron is accelerated and collides with an atom. If this electron is moving slowly with less energy, and collides with an atom, it gets deflected, i.e., its direction changes. But no considerable change occurs in energy. This is called **Elastic Collision**.

If the electron is having much energy, then this electron transfers its energy to the loosely bound electron of the atom and may remove the electron from the atom itself. So another free electron results. If the bombarding electron is having energy greater than that required to liberate a loosely bound electron from atom, the excess energy will be shared by the bombarding and liberated electrons.

**Problem 1.3**

Argon resonance radiation, corresponding to an energy of 11.6 eV falls upon sodium vapor. If a photon ionizes an unexcited sodium atom, with what speed is the electron ejected? The ionization potential of sodium is 5.12 eV.

**Solution**

Ionization potential is the minimum potential required to liberate an electron from its parent atom. Argon energy is 11.6 eV. Ionization Potential of Na is 5.12 eV.

$\therefore$  The energy possessed by the electron which is ejected is

$$11.6 - 5.12 = 6.48 \text{ eV}$$

or Potential,  $V = 6.48 \text{ volts}$  (  $\because$  1eV energy, potential is 1V )

Its velocity  $v = \sqrt{\frac{2eV}{m}} = 5.93 \times 10^5 \sqrt{6.48} = 1.51 \times 10^6 \text{ m/sec.}$

**Problem 1.4**

With what speed must an electron be travelling in a sodium vapor lamp in order to excite the yellow line whose wavelength is  $5,893 \text{ \AA}$ .

**Solution**

$$E_e \geq \frac{12,400}{5,893} \geq 2.11 \text{ eV} \quad (\text{Corresponding to energy of } 2.11 \text{ eV, the potential is } 2.11 \text{ Volts})$$

$$V = 2.11 \text{ Volts}$$

$$\therefore \text{ Velocity, } v = \sqrt{\frac{2eV}{m}} = 5.93 \times 10^5 \sqrt{2.11} = 8.61 \times 10^5 \text{ m/sec.}$$

**Problem 1.5**

A radio transmitter radiates 1000 W at a frequency of 10 MHz.

- What is the energy of each radiated quantum in eV ?
- How many quanta are emitted per second ?
- How many quanta are emitted in each period of oscillation of the electromagnetic field ?

**Solution**

$$(a) \text{ Energy of each radiated quantum} = E = hf$$

$$f = 10 \text{ MHz} = 10^7 \text{ Hz, } h = 6.626 \times 10^{-34} \text{ Joules / sec}$$

$$\therefore E = 6.626 \times 10^{-34} \times 10^7 = 6.626 \times 10^{-27} \text{ Joules / Quantum}$$

$$= \frac{6.626 \times 10^{-27}}{1.6 \times 10^{-19}} = 4.14 \times 10^{-8} \text{ eV / Quantum}$$

$$(b) 1 \text{ W} = 1 \text{ Joule/sec}$$

$$1000 \text{ W} = 1000 \text{ Joules/sec} = \text{Total Energy}$$

$$\text{Energy possessed by each quantum} = 6.626 \times 10^{-27} \text{ Joules/Quantum.}$$

$$\therefore \text{ Total number of quanta per sec, } N = \frac{1000}{6.626 \times 10^{-27}} = 1.5 \times 10^{29} / \text{sec}$$

$$(c) \text{ One cycle} = 10^{-7} \text{ sec}$$

$$\therefore \text{ Number of quanta emitted per cycle} = 10^{-7} \times 1.51 \times 10^{29} \\ = 1.51 \times 10^{22} \text{ per cycle}$$

**Problem 1.6**

- What is the minimum speed with which an electron must be travelling in order that a collision between it and an unexcited Neon atom may result in ionization of this atom? The Ionization Potential of Neon is 21.5 V.
- What is the minimum frequency that a photon can have and still be able to cause Photo-Ionization of a Neon atom?

**Solution**

(a) Ionization Potential is 21.5 V

$$\therefore \text{Velocity} = \sqrt{\frac{2eV}{m}} = 5.93 \times 10^5 \sqrt{21.5} = 2.75 \times 10^6 \text{ m/sec}$$

Wavelength of radiation,

(b) 
$$\lambda = \frac{12,400}{E_2 - E_1} = \frac{12,400}{21.5} = 577 \text{ \AA}$$

Frequency of radiation,

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{577 \times 10^{-10}} = 5.2 \times 10^{15} \text{ Hz}$$

**Problem 1.7**

Show that the time for one revolution of the electron in the hydrogen atom in a circular path around the nucleus is

$$T = \frac{m^{1/2} e^2}{4\sqrt{2} \epsilon_0 (-W)^{3/2}}$$

**Solution**

$$v = \sqrt{\frac{e^2}{4\pi m \epsilon_0 r}}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi r(4\pi m \epsilon_0 r)^{1/2}}{e} = \frac{2\pi r^{3/2} (4\pi m \epsilon_0)^{1/2}}{e}$$

But radius, 
$$r = \frac{e^2}{4\pi \epsilon_0 W}$$

$$\therefore T = \frac{m^{1/2} e^2}{4\sqrt{2} \epsilon_0 (-W)^{3/2}}$$

**Problem 1.8**

A photon of wavelength 1,400 Å is absorbed by cold mercury vapor and two other photons are emitted. If one of these is the 1,850 Å line, what is the wavelength λ of the second photon ?

**Solution**

$$(E_2 - E_1) = \frac{12,400}{\lambda} = \frac{12,400}{1400} = 8.86 \text{ eV}$$

1850 Å line is from 6.71 eV to 0 eV.

∴ The second photon must be from 8.86 to 6.71 eV.

So,  $\Delta E = 2.15 \text{ eV}$ .

$$\lambda = \frac{12,400}{(E_2 - E_1)}$$

$$\lambda = \frac{12,400}{2.15} = 5767 \text{ \AA}$$

### 1.1.8 METASTABLE STATES

An atom may be elevated to an excited energy state by absorbing a photon of frequency ' $f$ ' and thereby move from the level of energy  $W_1$  to the higher energy level  $W_2$  where  $W_2 = W_1 + hf$ . But certain states may exist which can be excited by electron bombardment but not by photo excitation (absorbing photons and raising to the excited state). **Such levels are called metastable states.** A transition from a metastable level to a normal state with the emission of radiation has a very low probability of occurrence. Transition from higher level to a metastable state are permitted, and several of these will occur.

An electron can be in the metastable state for about  $10^{-2}$  to  $10^{-4}$  sec. This is the mean life of a metastable state. Metastable state has a long lifetime because they cannot come to the normal state by emitting a photon. Then if an atom is in metastable state how will it come to the normal state? It cannot release a photon to come to normal state since this is forbidden. Therefore an atom in the metastable state can come to normal state only by colliding with another molecule and giving up its energy to the other molecule. Another possibility is the atom in the metastable state may receive additional energy by some means and hence may be elevated to a higher energy state from where a transition to normal state can occur.

### 1.1.9 WAVE PROPERTIES OF MATTER

An atom may absorb a photon of frequency ' $f$ ' and move from the energy level ' $W_1$ ' to the higher energy level ' $W_2$ ' where  $W_2 = W_1 + hf$ .

Since a photon is absorbed by only one atom, the photon acts as if it were concentrated in one point in space. So wave properties can not be attributed to such atoms and they behave like particles.

Therefore according to 'DeBroglie' hypothesis, dual character of wave and particle is not limited to radiation alone, but is also exhibited by particles such as electrons, atoms, and molecules. He calculated that a particle of mass ' $m$ ' travelling with a velocity ' $v$ ' has a wavelength ' $\lambda$ ' given

by  $\lambda = \frac{h}{mv} = \frac{h}{p}$  ( $\lambda$  is the wavelength of waves consisting of these particles).

Where ' $p$ ' is the momentum of the particle. Wave properties of moving electrons can be made use to explain Bohr's postulates. A stable orbit is one whose circumference is exactly equal to the wavelength ' $\lambda$ ' or ' $n\lambda$ ' where ' $n$ ' is an integer other than zero.

Thus,  $2\pi r = n\lambda$   $r$  = radius of orbit,  $n$  = Principle Quantum Number.

But according to DeBroglie,

$$\lambda = \frac{h}{mv}$$

$$\therefore 2\pi r = \frac{nh}{mv}$$

This equation is identical with Bohr's condition,  $mvr = \frac{nh}{2\pi}$

### 1.1.10 SCHRODINGER EQUATION

Schrodinger carried the implications of the wave nature of electrons. A branch of physics called **Wave Mechanics** or **Quantum Mechanics** was developed by him. If deBroglie's concept of

wave nature of electrons is correct, then it should be possible to deduce the properties of an electron system from a mathematical relationship called the **Wave Equation** or **Schrodinger Equation**. It is

$$\nabla^2\phi - \frac{1}{v^2} \cdot \frac{\partial^2\phi}{\partial t^2} = 0, \quad \dots\dots\dots(1.6)$$

where 
$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

‘ $\phi$ ’ can be a component of electric field or displacement or pressure. ‘ $v$ ’ is the velocity of the wave, and ‘ $t$ ’ is the time.

The variable can be eliminated in the equation by assuming a solution.

$$\phi(x, y, z, t) = \psi(x, y, z)e^{j\omega t}$$

This represents the position of a particle at ‘ $t$ ’ in 3-D Motion.

$$\omega = \text{Angular Frequency} = 2\pi f$$

$\omega$  is regarded as constant, while differentiating.

$\phi$  is a function of  $x, y, z$  and  $t$ .

But  $\psi$  is a function of  $x, y$  and  $z$  only.

To get the independent **Schrodinger Equation**

$$\frac{\partial \phi}{\partial t} = j\omega\psi(x, y, z)e^{j\omega t}$$

$$\frac{\partial^2 \phi}{\partial t^2} = -\omega^2\psi(x, y, z)e^{j\omega t}$$

But  $\omega = 2\pi f$

So,  $\omega^2 = 4\pi^2 f^2$

Substituting this in the original **Schrodinger’s Equation**,

$$e^{j\omega t} \left\{ \nabla^2\psi + \frac{1}{v^2} \times 4\pi^2 f^2 \psi \right\} = 0$$

But  $\lambda = \frac{v}{f}$ ; Wavelength =  $\frac{\text{Velocity}}{\text{Frequency}}$

$$\therefore \nabla^2\psi + \frac{4\pi^2}{\lambda^2}\psi = 0 \quad \dots\dots\dots(1.7)$$

But  $\lambda = \frac{h}{mv} = \frac{h}{p}$   $v = \text{Velocity.}$

$\therefore p = mv$

This is deBroglie’s Relationship.

$$\frac{1}{\lambda^2} = \frac{p^2}{h^2}$$

$$\begin{aligned}
 \text{But} \quad & p^2 = m^2 v^2 = 2 (\text{Kinetic Energy}) m \quad \therefore \text{K.E.} = \frac{1}{2}mv^2 \\
 & \text{Kinetic Energy} = \text{Total Energy (W)} - \text{Potential Energy (U)} \\
 \therefore \quad & p^2 = 2 (W - U) \cdot m \\
 \therefore \quad & \frac{p^2}{h^2} = \frac{2m}{h^2} (W - U) \quad \dots\dots\dots(1.8)
 \end{aligned}$$

Substituting Eq. (1.8) in Eq. (1.7) we get,

$$\nabla^2 \Psi + \frac{8\pi^2 m}{h^2} (W - U) \Psi = 0.$$

This is the Time Independent Schrodinger Equation  $\Psi$  is a function of 't', But this equation as such is not containing the term 't'.

### 1.1.11 WAVE FUNCTION

' $\Psi$ ' is called as the wave function, which describes the behavior of the particle. ' $\Psi$ ' is a quantity whose square gives the probability of finding an electron.  $|\Psi|^2 (dx \times dy \times dz)$  is proportional to the probability of finding an electron in volume  $dx, dy$  and  $dz$  at point  $P(x, y, z)$ .

Four quantum numbers are required to define the wave function. They are :

1. **The Principal Quantum Number 'n' :**

It is an integer 1, 2, 3, .... This number determines the total energy associated with a state. It is same as the quantum number 'n' of Bohr atom.

2. **The Orbital Angular Momentum Quantum Number l :**

It takes values 0, 1, 2...1... (n - l)

The magnitude of this angular momentum is  $\sqrt{l(l+1)} \times \frac{h}{2\pi}$

It indicates the shape of the classical orbit.

3. **The orbital magnetic number  $m_l$  :**

This will have values 0,  $\pm 1, \pm 2 \dots \pm l$ . This number gives the orientation of the classical orbit with respect to an applied magnetic field.

The magnitude of the Angular Momentum along the direction of magnetic

$$\text{field} = m_l \left( \frac{h}{2\pi} \right).$$

4. **Electron Spin :**

It was found in 1925 that in addition to assuming that electron orbits round the nucleus, it is also necessary to assume that electron also spins around itself, in addition to orbiting round the nucleus. This intrinsic electronic angular momentum is called **Electron Spin**.

When an electron system is subjected to a magnetic field, the spin axis will orbit itself either parallel or anti-parallel to the direction of the field. The electron

angular momentum is given by  $m_s \left( \frac{h}{2\pi} \right)$  where the spin quantum  $m_s$  number may have values  $+\frac{1}{2}$  or  $-\frac{1}{2}$ .

**1.1.12 ELECTRONIC CONFIGURATION**

**PAULI'S EXCLUSION PRINCIPLE**

No two electrons in an electron system can have the same set of four quantum numbers  $n$ ,  $l$ ,  $m_l$  and  $m_s$ .

Electrons will occupy the lower most quantum state.

**ELECTRONIC SHELLS ( PRINCIPLE QUANTUM NUMBER )**

All the electrons which have the same value of 'n' in an atom are said to belong to the same electron shell. These shells are identified by letters K, L, M, N corresponding to  $n = 1, 2, 3, 4, \dots$ . A shell is subdivided into sub shells corresponding to values of  $l$  and identified as s, p, d, f, g, h, for  $l = 0, 1, 2, 3, \dots$  respectively. This is shown in Table 1.1.

**Table 1.1**

Shell .....	K		L			M			N			
n .....	1		2			3			4			
l .....	0	0	1	0	1	2	0	1	2	3		
subshell .....	s	s	p	s	p	d	s	p	d	f		
No. of	2	2	6	2	6	10	2	6	10	14		
electron	2	8		18			32					

**ELECTRON SHELLS AND SUBSHELLS**

Number of Electrons in a sub shell =  $2(2l + 1)$

$n = 1$  corresponds to K shell

$l = 0$  corresponds to s sub shell

$\therefore l = 0, \dots, (n - 1)$  if  $n = 1$ ,

$l = 0$  is the only possibility

Number of electrons in K shell =  $2(2l + 1) = 2(0 + 1) = 2$  electrons.

$\therefore$  K shell will have 2 electrons.

This is written as  $1s^2$  pronounced as "one s two" ( 1 corresponds to K shell,  $n = 1$ ; s is the sub shell corresponds to  $l = 0$  number of electron is 2. Therefore,  $1s^2$ )

If  $n = 2$ , it is 'l' shell

If  $l = 0$ , it is 's' sub shell

If  $l = 1$ , it is 'p' sub shell

Number of electron in 's' sub shell ( i.e.,  $l = 0$ ) =  $2 \times (2l + 1) = 2(0 + 1) = 2$

Number of electron is 'p' sub shell (i.e.,  $l = 1$ ) =  $2[(1 \times 2) + 1] = 6$

In l shell there are two sub shells, s and p.

$\therefore$  Total number of electron in l shell =  $2 + 6 = 8$

This can be represented as  $2s^2 2p^6$

If  $n = 3$ , it is M shell

It has 3 sub shell s, p, d, corresponding to  $l = 0, 1, 2$

In 's' sub shell number of electrons ( $l = 0$ ) =  $2(\because l = 0) 2(2l + 1)$

In 'p' sub shell number of electrons ( $l = 1$ ) =  $2(2 + 1) = 6$

In 'd' sub shell number of electrons ( $l = 2$ ) =  $2(2 \times 2 + 1) = 10$

$\therefore$  Total number of electrons in M shell =  $10 + 6 + 2 = 18$

This can be represented as  $3s^2 3p^6 3d^{10}$

$\therefore 1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10}$

### ELECTRONIC CONFIGURATION

Atomic number 'z' gives the number of electrons orbiting round the nucleus. So from the above analysis, electron configuration can be given as  $1s^2 2s^2 2p^6 3s^1$ . First k shell ( $n = 1$ ) is to be filled. Then l shell ( $n = 2$ ) and so on.

In 'K' shell there is one sub shell (s) ( $l = 0$ ). This has to be filled.

In 'L' shell there are two sub shells and p. First - s and then p are to be filled and so on.

The sum of subscripts  $2 + 2 + 6 + 1 = 11$ . It is the atomic number represented as Z.

For Carbon, the Atomic Number is 6., i.e.,  $Z = 6$ .

$\therefore$  The electron configuration is  $1s^2 2s^2 2p^2$

For the Ge  $Z = 32$ . So the electronic configuration of Germanium is,

$\therefore 1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^2$

For the Si  $Z = 14$ . So the electronic configuration of Silicon is,

$\therefore 1s^2 2s^2 2p^6 3s^2 3p^2$

#### 1.1.13 TYPES OF ELECTRON EMISSION

Electrons at absolute zero possess energy ranging from 0 to  $E_F$  the fermi level. It is the characteristic of the substance. But this energy is not sufficient for electrons to escape from the surface. They must possess energy  $E_B = E_F + E_W$  where  $E_W$  is the work function in eV.

$E_B$  = Barrier's Energy

$E_F$  = Fermi Level

$E_W$  = Work Function.

Different types of Emission by which electrons can emit are

- (1) *Thermionic Emission*
- (2) *Secondary Emission*
- (3) *Photoelectric Emission*
- (4) *High field Emission.*

#### 1. THERMIONIC EMISSION

Suppose, the metal is in the form of a filament and is heated by passing a current through it. As the temperature is increased, the electron energy distribution starts in the metal changes. Some electrons may acquire energy greater than  $E_B$  sufficient to escape from the metal.



$E_w$  : *Work function of a metal. It represents the amount of energy that must be given for the electron to be able to escape from the metal.*

It is possible to calculate the number of electrons striking the surface of the metal per second with sufficient energy to be able to surmount the surface barriers and hence escape. Based upon that, the thermionic current is,

$$I_{th} = S \times A_o T^2 e^{-E_w/KT} \dots\dots\dots ( 1.9 )$$

It is also written as

$$\frac{I_{th}}{S} = J = A_o T^2 e^{-\frac{E_w}{kT}} = AT^2 e^{-\frac{B}{T}}$$

- where  $A = A_o$  and  $B = \frac{E_w}{k}$
- where  $S =$  Area of the filament in  $m^2$  ( Surface Area )
- $A_o =$  Constant whose dimensions are  $A/m^2 \text{ } ^\circ K$
- $T =$  Temperature in  $^\circ K$
- $K =$  Boltzman's constant  $eV/^\circ K$
- $E_w =$  Work function in  $eV$

This equation is called *Thermionic Emission Current* or *Richardson - Dushman Equation*.  $E_w$  is also called as latent heat of evaporation of electrons similar to evaporation of molecules from a liquid.

Taking logarithms

$$\log I_{th} = \log (S A_o) - \frac{E_w}{kT} \log e + \log T^2$$

$$\log I_{th} - 2 \log T = \log S A_o - 0.434 \left( \frac{E_w}{kT} \right)$$

$$\therefore \log e = 0.434$$

So if a graph is plotted between  $(\log I_{th} - 2 \log T) V_S \frac{1}{T}$ , the result will be a straight line

having a slope  $= -0.434 \left( \frac{E_w}{kT} \right)$  from which  $E_w$  can be determined.

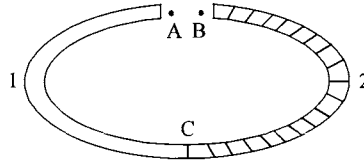
$\therefore I_{th}$  and  $T$  can be determined experimentally.

$I_{th}$  is a very strong function of  $T$ . For Tungsten,  $E_w = 4.52 \text{ eV}$ .

**CONTACT POTENTIAL**

Consider two metals in contact with each other forming junction at 'C' as in Fig 1.1 . The contact difference of potential is defined as the Potential Difference  $V_{AB}$  between a point A, just out side metal1 and a point B just outside metal2. The reason for the difference of potential is, when two metals are joined, electrons will flow from the metal of lower Work Function, say 1 to the metal of higher Work Function say 2. (  $\therefore E_w = E_B - E_F$  electrons of lower work function means  $E_w$  is small or  $E_F$  is large ). Flow of electrons from metal 1 to 2 will continue till metal 2 has acquired

sufficient negative charge to repel extra new electrons. 'E<sub>B</sub>' value will be almost same for all metals. But E<sub>F</sub> differs significantly.



**Fig 1.1 Contact Potential**

In order that the fermi levels of both the metals are at the same level, the potential energy difference  $E_{AB} = E_{W2} - E_{W1}$ , that is, **the contact difference of potential energy between two metals is equal to the difference between their work functions.**

If metals 1 and 2 are similar then contact potential is zero. If they are dissimilar, the metal with lower work function becomes positive, since it loses electrons.

#### ENERGIES OF EMITTED ELECTRONS

At absolute zero, the electrons will have energies ranging from zero to E<sub>F</sub>. So the electrons liberated from the metal surface will also have different energies. The minimum energy required is the barrier potential to escape from the metal surface, But the electrons can acquire energy greater than E<sub>B</sub> to escape from the surface. This depends upon the initial energy, the electrons are possessing at room temperature.

Consider a case where anode and cathode are plane parallel. Suppose the voltage applied to the cathode is lower, and the anode is indirectly heated. Suppose the minimum energy required to escape from the metal surface is 2 eV. As collector is at a potential less than 2V, electrons will be collected by the collector. If the voltage of cathode is lower, below 2V, then the current also decreases exponentially, but not abruptly. If electrons are being emitted from cathode with 2eV energy, and the voltage is reduced below 2V, then there must be abrupt drop of current to zero. But it doesn't happen This shows that electrons are emitted from surface of the emitter with different velocities. The decrease of current is given by  $I = I_{th} \cdot e^{-V_T}$  where V<sub>T</sub> is the retarding potential applied to the collector and V<sub>T</sub> is Volt equivalent of temperature.

$$V_T = \frac{kT}{e} = \frac{T}{11,600} \quad \text{..... (1.10)}$$

T = Temperature in °K

K = Boltzman's Constant in J/°K

#### SCHOTTKY EFFECT

If a cathode is heated, and anode is given a positive potential, then there will be electron emission due to thermionic emission. There is accelerating field, since anode is at a positive potential. This accelerating field tends to lower the Work Function of the cathode material. It can be shown that under the condition of accelerating field E is

$$I = I_{th} e^{+0.44 \epsilon l / 2T}$$

where I<sub>th</sub> is the zero field thermionic current and T is cathode temperature in 0 °K.

The effect that thermionic current continues to increase as E is increased ( even though T is kept constant ) is known as **Schottky Effect**.

**2. SECONDARY EMISSION**

This emission results from a material (metal or dielectric) when subjected to electron bombardment.

It depends upon,

1. *The energy of the primary electrons.*
2. *The angle of incidence.*
3. *The type of material.*
4. *The physical condition of surface ; whether surface is smooth or rough.*

Yield or secondary emission ratio S is defined as the ratio of the number of secondary electrons to primary electrons. It is small for pure metals, the value being 1.5 to 2. By contamination or giving a coating of alkali metal on the surface, it can be improved to 10 or 15.

**3. PHOTO ELECTRIC EMISSION**

Photo-Electric Emission consists of liberation of electrons by the incidence of light, on certain surfaces. The energy possessed by photons is 'hf' where 'h' is Plank's Constant and 'f' is frequency of incident light. When such a photon impinges upon the metal surface, this energy hf gets transferred to the electrons close to the metal surface whose energy is, very near to the barrier potential. Such electrons gain energy, to overcome the barrier potential and escape from the surface of the metal resulting in photo electric emission.

For photoelectric emission to take places, the energy of the photon must at least be equal to the work function of the metal. That is  $hf \geq e\phi$  where  $\phi$  is the voltage equivalent Work Function, (i.e. Work Function expressed in Volts). **The minimum frequency that can cause photo-electric emission is called threshold frequency** and is given by

$$f_T = \frac{e\phi}{h}$$

The wavelength corresponding to threshold frequency is called the **Threshold Wavelength**.

$$\lambda_t = \frac{c}{f_t} = \frac{hc}{\phi e}$$

If the frequency of radiation is less than  $f_T$ , then additional energy appears as kinetic energy of the emitted electron

$$\therefore hf = \phi e + \frac{1}{2} m v^2$$

$\phi$  is the Volt equivalent of Work Function.

$$v = \sqrt{\frac{2eV}{m}}$$

v is the Velocity of electrons in m/sec.

$$\therefore \boxed{hf = \phi e + eV} \quad \dots\dots\dots ( 1.11 )$$

**LAWS OF PHOTO ELECTRIC EMISSION**

1. *For each photo sensitive material, there is a threshold frequency below which emission does not take place.*
2. *The amount of photo electric emission (current) is proportional to intensity.*

3. Photo electric emission is instantaneous. (But the time lag is in nano-sec ).
4. Photo electric current in amps/watt of incident light depends upon 'f'.

#### 4. HIGH FIELD EMISSION

Suppose a cathode is placed inside a very intense electric field, then the Potential Energy is reduced. For fields of the order of  $10^9$  V/m, the barrier may be as thin as  $100 \text{ \AA}$ . So the electron will travel through the barrier. This emission is called as **High Field Emission** or **Auto Electronic Emission**.

##### Problem 1.9

Estimate the percentage increase in emission from a tungsten filament when its temperature is raised from 2400 to 2410 °K.

$$A_0 = 60.2 \times 10^4 \text{ A/m}^2 / \text{°K}^2$$

$$B = 52,400 \text{ °K}$$

A and B are constants in the equation for current density J

##### Solution

$$JS_1 = A T_1^2 = 1142 \text{ A / m}^2$$

$$JS_2 = A T_2^2 e^{-B/T} = 1261 \text{ A / m}^2.$$

$$\text{Percentage Increase} = \frac{1261 - 1142}{1142} \times 100 = 10.35\%$$

##### Problem 1.10

A photoelectric cell has a cesium cathode. When the cathode is illuminated with light of  $\lambda = 5500 \times 10^{-10}$  m, the minimum anode voltage required to inhibit built anode current is 0.55 V. Calculate

(a) The work function of cesium

(b) The longest  $\lambda$  for which photo cell can function.

by applying  $-0.55\text{V}$  to anode the emitted electrons are repelled. So the current can be inhibited

##### Solution

$$(a) \quad hf = e\phi + eV \quad V = 0.55 \text{ volts} \quad \phi = \text{Work Function (WF)} = ?$$

$$f = \frac{C}{\lambda}$$

$$\therefore \frac{hC}{\lambda} = e(\phi + V)$$

$$\text{Plank's Constant, } h = 6.63 \times 10^{-34} \text{ J sec.}$$

$$\text{Charge of Electron, } e = 1.6 \times 10^{-19} \text{ C}$$

$$\text{Velocity of Light, } C = 3 \times 10^8 \text{ m/sec}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{5500 \times 10^{-10}} = 1.6 \times 10^{-19} (\phi + 0.55)$$

$$\phi = 1.71 \text{ Volts}$$

(b) **Threshold Wavelength :**

$$\lambda_o = \frac{Ch}{e\phi}$$

$$\therefore \lambda_o = \frac{12,400}{\phi} = \frac{12,400}{1.71} = 7,250 \text{ \AA}$$

**Problem 1.11**

If the temperature of a tungsten filament is raised from 2300 to 2320 °K, by what percentage will the emission change ? To what temperature must the filament be raised in order to double its energy at 2300 °K.  $E_w$  for Tungsten = 4.52 eV. Boltzman's Constant  $K = 8.62 \times 10^5 \text{ eV/}^\circ\text{K}$ .

**Solution**

(a)  $I_{th} = S.A_o T^2 e^{-E_w/kT}$

Taking Logarithms,

$$\ln I_{th} - 2 \log T = \ln S A_o - \frac{E_w}{kT}$$

Differentiating,  $\left(\frac{2}{T} + \frac{E_w}{KT^2}\right) \frac{dT}{T^2}$

$$\frac{dI_{th}}{I_{th}} = \left(2 + \frac{E_w}{kT}\right) \frac{dT}{T} = \left(2 + \frac{4.52}{(8.62 \times 10^{-5})2310}\right) \frac{20}{2310} = 21.4\%$$

(b)  $I_{th} = S.A_o (2300)^2 e^{\frac{-4.52}{8.62 \times 10^{-5} \times 2300}}$

$$2I_{th} = S A_o (T)^2 e^{\frac{-4.52}{8.62 \times 10^{-5}}}$$

Ratio of these two equation is

$$2 = \left(\frac{T}{2300}\right)^2 e^{\frac{-52,400}{T} + 22.8}$$

Taking log to the base 10,

$$\log 2 = 2 \log \left(\frac{T}{2300}\right) - \left(\frac{52,400}{T} + 22.8\right) \log e$$

$$\log 2 = 2 \log \left(\frac{T}{2300}\right) - \frac{52400}{T} \times 0.434 - 22.8 \times 0.434$$

$$9.6 + 2 \log \left(\frac{T}{2300}\right) = \frac{22,800}{T}$$

This is solved by Trail and Error Method to get  $T = 2370^\circ\text{K}$

**Problem 1.12**

In a cyclotron, the magnetic field applied is 1 Tesla. If the ions (electrons) cross the gap between the D shaped discs dees twice in each cycle, determine the frequency of the R.F. voltage. If in each passage through the gap, the potential is increased by 40,000 volts how many passages are required to produce a 2 million volts particle? What is the diameter of the last semicircle?

**Solution**

$$B = 1 \text{ Tesla} = 1 \text{ Wb/m}^2$$

$$\text{Time taken by the particle to describe one circle is } T = \frac{35.5 \mu \text{ sec}}{B}$$

$$T = \frac{35.5 \times 10^{-6}}{1} \text{ sec,}$$

when it describes one circle, the particle passes through the gap twice.

∴ The frequency of the R.F voltage should be the same.

$$\therefore f = \frac{1}{T} = \frac{1}{35.5 \times 10^{-6}} = 2.82 \times 10^{10} \text{ Hz}$$

**Number of passages required :**

In each passage it gains 40,000 volts.

$$\text{To gain 2 million electron volts} = \frac{2 \times 10^6 \text{ V}}{40 \times 10^3 \text{ V}} = 50$$

**Diameter of last semicircle :**

The initial velocity for the 50<sup>th</sup> revolution is the velocity gained after 49<sup>th</sup> revolution.

∴ Accelerating potential  $V_0 = 49 \times 40,000 = 1,960 \text{ KV}$  49<sup>th</sup> revolution

$$\text{Radius of the last semicircle} = R = \frac{3.37 \times 10^{-6}}{B} \sqrt{V_0}$$

$$R = \frac{3.37 \times 10^{-6}}{1} \sqrt{1960 \times 10^3} = 47.18 \times 10^{-4} \text{ m}$$

$$\text{Diameter} = 2R = 94.36 \times 10^{-4} \text{ m}$$

**Problem 1.13**

The radiated power density necessary to maintain an oxide coated filament at 110 °K is found to be  $5.8 \times 10^4 \text{ W/m}^2$ . Assume that the heat loss due to conductor is 10% of the radiation loss. Calculate the total emission current and the cathode efficiency  $\eta$  in ma/w.

Take

$$E_w = 1.0 \text{ eV}$$

$$A_0 = 100 \text{ A/m}^2 / \text{ }^\circ\text{K}^2$$

$$S = 1.8 \text{ cm}^2$$

$$\bar{K} = \text{Boltzman's Constant in eV / }^\circ\text{K} = 8.62 \times 10^{-5} \text{ eV / }^\circ\text{K}$$

**Solution**

$$\text{Power Density} = 5.8 \times 10^4 \text{ W/m}^2$$

If 10% is lost by conductor, then the total input power density is  $1.1 \times 5.8 \times 10^4 = 6.38 \text{ W/m}^2$  and the total input power is  $6.38 \times 10^4 \times \text{Area} (1.8 \times 10^{-4}) = 11.5 \text{ W}$ .

$$\begin{aligned} I_{th} &= S A_o T^2 e^{-E_w/kT} \\ &= 1.8 \times 10^{-4} \times 100 \times (1100)^2 e^{-\frac{1}{8.62 \times 10^{-5} \times 1100}} \\ I_{th} &= 2.18 \times 10^4 e^{-10.55} \\ I_{th} &= 0.575 \text{ A} \end{aligned}$$

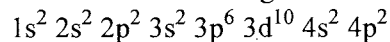
$$\text{Cathode Efficiency, } \eta = \frac{0.575}{11.5} = 50 \text{ mA / } ^\circ\text{K}$$

## 1.2 n-TYPE AND p-TYPE SEMICONDUCTORS

### 1.2.1 n-TYPE OR DONOR TYPE SEMICONDUCTORS

Intrinsic or pure semiconductor is of no use since its conductivity is less and it can not be charged much. If a pure semiconductor is doped with impurity it becomes extrinsic. Depending upon impurity doped, the semiconductor may become *n-type, where electrons are the majority carriers or donor type*, since it donates an electron. On the other hand *if the majority carriers are holes, it is p-type or acceptor type semiconductor*, because it accepts an electron to complete the broken covalent bond.

Germanium atom with its electrons arranged in shells will have configuration as



Ge is tetravalent (4). 'Ge' becomes *n-type* if a pentavalent (5), impurity atoms such as Phosphorus (P), or Arsenic are added to it.

The impurity atoms have size of the same order as that of Ge atoms. Because of the energy supplied while doping, the impurity atom dislodges one from its normal position in the crystal lattices takes up that position. But since the concentration of impurity atoms is very small (about 1 atom per million of Ge atoms), the impurity atom is surrounded by Ge atoms. The impurity atom is pentavalent. That is, it has 5 electrons in the outermost orbit (5 valence electrons). Now 4 of these are shared by Ge atoms, surrounding the impurity atom and they form covalent bonds. So one electron of the impurity atom is left free. The energy required to dislodge this fifth electron from its parent impurity atom is very little of the order of 0.01 eV to 0.05 eV. This free electron is in excess to the free electrons that will be generated because of breaking of covalent bonds due to thermal agitation. Since an excess electron is available for each impurity atom, or it can *denote an electron it is called n-type, or donor type semiconductor*.

### 1.2.2 p-TYPE OR ACCEPTOR TYPE SEMICONDUCTORS

An intrinsic semiconductor when doped with trivalent (3) impurity atoms like Boron, Gallium Indium, Aluminium etc., becomes p-type or acceptor type.

Because of the energy supplied while doping, the impurity atom dislodges one Ge atom from the crystal lattice. The doping level is low, i.e., there is one impurity atom for one million Ge atoms, the impurity atom is surrounded by Ge atom. Now the three valence electrons of impurity atom are shared by 3 atoms. The fourth Ge atom has no electron to share with the impurity atom. So the covalent bond is not filled or a hole exists. The impurity atom tries to steal one electron from the neighboring Ge atoms and it does so when sufficient energy is supplied to it. So hole moves.

There will be a natural tendency in the crystal to form 4 covalent bonds. The impurity atom (and not just 3) since all the other Ge atoms have 4 covalent bonds and the structure of Ge semiconductor is crystalline and symmetrical. The energy required for the impurity atom to steal one Ge electron is 0.01 eV to 0.08 eV. This hole is in excess to the hole created by thermal agitation.

**1.3 MASS ACTION LAW**

In an intrinsic Semiconductor number of free electrons  $n = n_i =$  No. of holes  $p = p_i$   
 Since the crystal is electrically neutral,  $n_i p_i = n_i^2$ .

Regardless of individual magnitudes of  $n$  and  $p$ , the product is always constant,

$$\therefore np = n_i^2$$

$$n_i = AT^{\frac{3}{2}} e^{\frac{-E_{G0}}{2KT}} \dots\dots\dots (1.12)$$

This is called *Mass Action Law*.

**1.4 CONTINUITY EQUATION**

Thus Continuity Equation describes how the carrier density in a given elemental volume of crystal varies with time.

If an intrinsic semiconductor is doped with n-type material, electrons are the majority carriers. Electron - hole recombination will be taking place continuously due to thermal agitation. So the concentration of holes and electrons will be changing continuously and this varies with time as well as distance along the semiconductor. We now derive the differential. equation which is based on the fact *that charge is neither created nor destroyed*. This is called *Continuity Equation*.

Consider a semiconductor of area  $A$ , length  $dx$  ( $x + dx - x = dx$ ) as shown in Fig. 1.2. Let the average hole concentration be 'p'. Let  $E_p$  is a factor of  $x$ . that is hole current due to concentration is varying with distance along the semiconductor. Let ' $I_p$ ' is the current entering the volume at ' $x$ ' at time ' $t$ ', and  $(I_p + dI_p)$  is the current leaving the volume at  $(x + dx)$  at the same instant of time ' $t$ '. So when only  $I_p$  colombs is entering,  $(I_p + dI_p)$  colombs are leaving. Therefore effectively there is a decrease of  $(I_p + dI_p - I_p) = dI_p$  colombs per second within the volume. Or in other words, since more hole current is leaving than what is entering, we can say that more holes are leaving than the no. of holes entering the semiconductor at ' $x$ '.

If  $dI_p$  is rate of change of total charge that is

$$dI_p = d\left(\frac{n \times q}{t}\right)$$

$\frac{dI_p}{q}$  gives the decrease in the number of holes per second with in the volume  $A \times dx$ . Decrease in holes per unit volume ( hole concentration ) per second due to  $I_p$  is

$$\frac{dI_p}{A \times dx} \times \frac{1}{q}$$

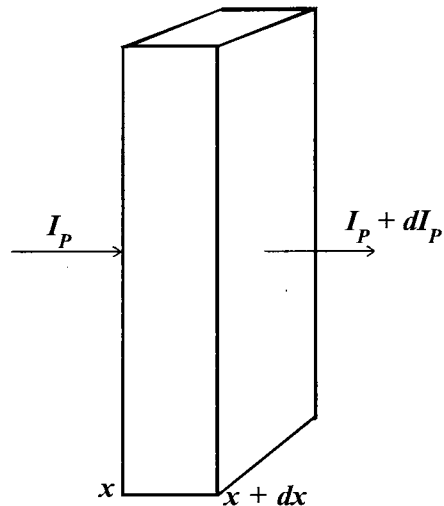


Fig 1.2 Charge flow in semiconductor



But 
$$\frac{dI_p}{A} = \text{Current Density}$$

$$= \frac{1}{q} \times \frac{dJ_p}{dx}.$$

But because of thermal agitation, more number of holes will be created. If 'p<sub>0</sub>' is the thermal equilibrium concentration of holes, ( the steady state value reached after recombination ), then, the increase per second, per unit volume due to thermal generation is,

$$g = \frac{p}{\tau_p}$$

Therefore, increase per second per unit volume due to thermal generation,

$$g = \frac{p_0}{\tau_p}$$

But because of recombination of holes and electrons there will be decrease in hole concentration.

The decrease 
$$= \frac{p}{\tau_p}$$

Charge can be neither created nor destroyed. Because of thermal generation, there is increase in the number of holes. Because of recombination, there is decrease in the number of holes. Because of concentration gradient there is decrease in the number of holes.

So the net increase in hole concentration is the algebraic sum of all the above.

$$\frac{\partial p}{\partial t} = \frac{p_0 - p}{\tau_p} - \frac{1}{q} \times \frac{\partial J_p}{\partial x}$$

Partial derivatives are used since 'p' and 'J<sub>p</sub>' are functions of both time t and distance x.

$\frac{dp}{dt}$  gives the variation of concentration of carriers with respect to time 't'.

If we consider unit volume of a semiconductor ( *n-type* ) having a hole density p<sub>n</sub>, some holes are lost due to recombination. If p<sub>no</sub> is equilibrium density, ( i.e., density in the equilibrium condition when number of electron = holes ).

The recombination rate is given as  $\frac{p_n - p_{no}}{\tau}$ . ***The expression for the time rate of change in carriers density is called the Continuity Equation.***

$$\text{Recombination rate } R = \frac{dp}{dt}$$

Life time of holes in n-type semiconductors

$$\tau_p = \frac{\Delta P}{R} = \frac{p_n - p_{no}}{dp/dt}$$

or

$$\frac{dp}{dt} = \left( \frac{p_n - p_{no}}{\tau_p} \right)$$

where  $p_n$  is the original concentration of holes in n-type semiconductors and  $p_{no}$  is the concentration after holes and electron recombination takes place at the given temperature. In other word  $p_{no}$  is the thermal equilibrium minority density. Similarly for a p-type semiconductors, the life time of electrons

$$\tau_n = \frac{n_p - n_{po}}{dx/dt} ; \frac{dx}{dt} = \frac{n_p - n_{no}}{\tau_n}$$

### 1.5 THE HALL EFFECT

If a metal or semiconductor carrying a current  $I$  is placed in a perpendicular magnetic field  $B$ , an electric field  $E$  is induced in the direction perpendicular to both  $I$  and  $B$ . This phenomenon is known as the **Hall Effect**. It is used to determine whether a semiconductor is p-type or n-type. By measuring conductivity  $\sigma$ , the mobility  $\mu$  can be calculated using **Hall Effect**.

In the Fig. 1.3 current ' $I$ ' is in the positive ' $X$ '-direction and ' $B$ ' is in the positive ' $Z$ '-direction. So a force will be exerted in the negative ' $Y$ '-direction. If the semiconductor is n-type, so that current is carried by electrons, these electrons will be forced downward toward side 1. So side 1 becomes negatively charged with respect to side 2. Hence a potential  $V_H$  called the **Hall Voltage** appears between the surface 1 and 2.

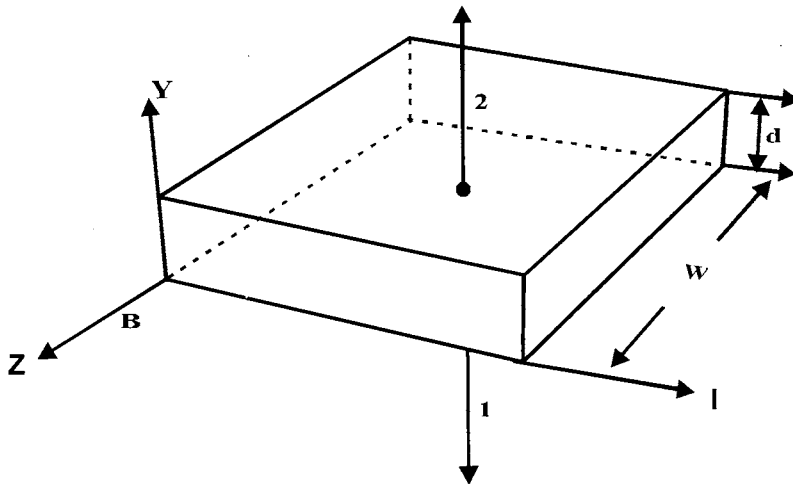


Fig 1.3 Hall effect.

In the equilibrium condition, the force due to electric field intensity ' $E$ ', because of Hall effect should be just balanced by the magnetic force or

$$eE = B ev$$

$$v = \text{Drift Velocity of carriers in m / sec}$$

$$B = \text{Magnetic Field Intensity in Tesla ( wb/m}^2 \text{)}$$

or

$$E = Bv$$

..... ( a )

But  $\epsilon = V_H/d$   
 where  $V_H$  = Hall Voltage  
 $d$  = Thickness of semiconductors.  
 $J = nev$  or  $J = \rho v$   
 $\rho$  = charge density.  
 $J$  = Current Density ( Amp / m<sup>2</sup> )

or  $J = \frac{I}{\omega.d}$   
 $\omega$  = width of the semiconductor;  $\omega d$  = cross sectional area  
 $I$  = current

$\therefore J = \text{Current Density} = \frac{I}{\omega d}$

or  $\epsilon = V_H/d$   
 $V_H = \epsilon d$   
 But  $\epsilon = Bv$  From Equation ( a )  
 $\therefore V_H = B \times v \times d$  But  $v = J/\rho$

$= \frac{B.J.d}{\rho}$  But  $J = \frac{I}{\omega d}$

$V_H = \frac{B.I.d}{\rho.\omega.d} = \frac{B.I}{\rho\omega}$

$V_H = \frac{B.I}{\rho\omega}$  ..... ( 1.13 )

If the semiconductor is n-type, electrons the majority carriers under the influence of electric field will move towards side 1, side 2 becomes positive and side 1 negative. If on the other hand terminal 1 becomes charged positive then the semiconductor is p-type.

or  $\rho = n \times e$  ( For n - type semiconductor )  
 or  $\rho = p \times e$  ( for p-type semiconductor )  
 and  $\rho = \text{Charge density.}$

$\therefore V_H = \frac{B.I}{\rho\omega}$

$\rho = \frac{B.I}{V_H.\omega}$

$\therefore R_H = \frac{V_H.\omega}{BI}$  ..... ( 1.14 )

The Hall Coefficient,  $R_H$  is defined as  $R_H = \frac{1}{\rho}$ . Units of  $R_H$  are  $\text{m}^3 / \text{coulombs}$

If the conductivity is due primarily to the majority carriers conductivity,  $\sigma = ne\mu$  in n-type semiconductors.

$$n.e = \rho = \text{charge density.}$$

$$\therefore \sigma = \rho \times \mu$$

But  $\frac{1}{\rho} = R_H$

$$\therefore \sigma = \frac{1}{R_H} \times \mu$$

or  $\mu = R_H \times \sigma = \frac{V_H \cdot \omega}{B \cdot I} \times \sigma$

We have assumed that the drift velocity 'v' of all the carriers is same. But actually it will not be so. Due to the thermal agitation they gain energy, their velocity increases and also collision with other atoms increases. So for all particles v will not be the same. Hence a correction has to be

made and it has been found that satisfactory results will be obtained if  $\frac{1}{R_H}$  is taken as  $\frac{3\pi}{8\rho}$ .

$$\therefore \mu = \left( \frac{8\sigma}{3\pi} \right) R_H \quad \dots\dots\dots (1.15)$$

Multiply  $R_H$  by  $\frac{8}{3\pi}$ . Then it becomes **Modified Hall Coefficient**. Thus mobility of carriers ( electrons or holes ) can be determined experimentally using Hall Effect.

The product  $(Bev)$  is the **Lorentz Force**, because of the applied magnetic field B and the drift velocity 'v'. So the majority carriers in the semiconductors, will tend to move in a direction perpendicular to B. But since there is no electric field applied in that particular direction, there will develop a Hall voltage or field which just opposes the Lorentz field.

So with the help of Hall Effect, we can experimentally determine

1. *The mobility of Electrons or Holes.*
2. *Whether a given semiconductor is p-type or n-type ( from the polarity of Hall voltage  $V_H$  )*

#### **Problem 1.14**

The Hall Effect is used to determine the mobility of holes in a p-type Silicon bar. Assume the bar resistivity is  $200,000 \Omega\text{-an}$ , the magnetic field  $B_z = 0.1 \text{ Wb/m}^2$  and  $d = w = 3\text{mm}$ . The measured values of the current and Hall voltage are  $10\text{mA}$  and  $50 \text{ mv}$  respectively. Find  $\mu_p$  mobility of holes.

**Solution**

$$B = 0.1 \text{ Wb / m}^2 \text{ (or Tesla)}$$

$$V_H = 50 \text{ mv.}$$

$$I = 10 \text{ mA;}$$

$$\rho = 2 \times 10^5 \Omega \text{ - cm ;}$$

$$d = w = 3\text{mm} = 3 \times 10^{-3} \text{ meters}$$

$$\frac{1}{R_H} = \frac{B.I}{V_H \cdot w} = \frac{0.1 \times 10 \times 10^{-3}}{50 \times 10^{-2} \times 3 \times 10^{-3}} = \frac{1}{150} = 0.667.$$

$$\text{Conductivity} = \frac{1}{\rho} = \frac{1}{2 \times 10^5 \times 10^{-2}} = \frac{1}{2000} \text{ mhos / meter.}$$

$$\mu = \sigma \times R_H$$

$$\mu_p = \frac{1}{0.667} \times \frac{1}{2000} = 750 \text{ cm}^2 / \text{V - sec}$$

**1.6 FERMI LEVEL IN INTRINSIC AND EXTRINSIC SEMICONDUCTORS**

**1.6.1 FERMI LEVEL**

Named after *Fermi*, it is the Energy State, with 50% probability of being filled if no forbidden band exists. In other words, it is the mass energy level of the electrons, at 0°K.

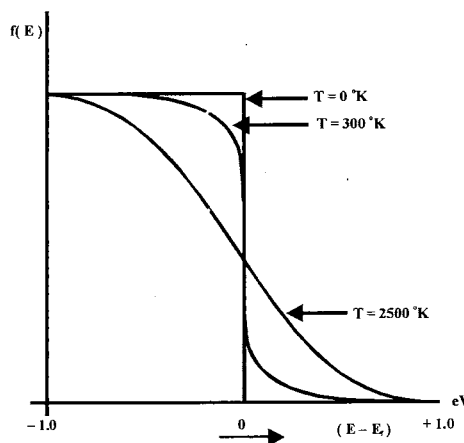
If  $E = E_f$ ,

$$f(E) = \frac{1}{2}$$

If a graph is plotted between  $(E - E_F)$  and  $f(E)$ , it is shown in Fig. 1.4

At  $T = 0^\circ\text{K}$ , if  $E > E_F$  then,  $f(E) = 0$ .

That is, there is no probability of finding an electron having energy  $> E_F$  at  $T = 0^\circ\text{K}$ . Since fermi level is the max. energy possessed by the electrons at  $0^\circ\text{k}$ .  $f(E)$  varies with temperature as shown in Fig. 1.4.



**Fig. 1.4 Fermi level variation with temperature.**

## 1.6.2 THE INTRINSIC CONCENTRATION

$$f(E) = \frac{1}{1 + e^{(E-E_F)/KT}} \quad \dots\dots\dots (1.16)$$

$$1 - f(E) = \frac{e^{(E-E_F)/KT}}{1 + e^{(E-E_F)/KT}} \simeq e^{-(E_F-E)/KT}$$

$$e^{(E-E_F)/KT} \left[ \frac{1}{1 + e^{(E-E_F)/KT}} \right]^{-1} \simeq e^{-(E_F-E)/KT}$$

Fermi function for a hole =  $1 - f(E)$ .

$(E_F - E) \gg KT$  for  $E \leq E_V$

the number of holes per  $m^3$  in the Valence Band is,

$$p = \int_{-\infty}^{E_V} \gamma(E_V - E)^{3/2} e^{-(E_F-E)/KT} \times dE$$

$$= N_V \times e^{-(E_F-E_V)/KT}$$

where

$$N_V = 2 \left( \frac{2\pi m_p \bar{K}T}{h^2} \right)^{3/2}$$

Similarly

$$n = N_C e^{-(E_C-E_F)/KT}$$

$$n \times p = N_V \times N_C e^{-E_G/KT}$$

Substituting the values of  $N_C$  and  $N_V$ ,

$$n \times p = n_i^2 = AT^3 e^{-E_G/KT}$$

## 1.6.3 CARRIER CONCENTRATIONS IN A SEMICONDUCTOR

$$dn = N(E) \times f(E) \times dE$$

$dn$  = number of conduction electrons per cubic meter whose energy lies between  $E$  and  $E + dE$

$f(E)$  = The probability that a quantum state with energy  $E$  is occupied by the electron.

$N(E)$  = Density of States.

$$f(E) = \frac{1}{1 + e^{(E-E_F)/KT}}$$

The concentration of electrons in the conduction band is

$$n = \int_{E_C}^{\infty} N(E) \times f(E) \times dE$$

for

$$E \geq E_C$$

$(E - E_C)$  is  $\gg KT$ .

$$\therefore f(E) = e^{-(E-E_F)/KT}$$

$$\therefore e^{(E-E_F)/KT} \gg 1$$

$$\therefore n = \int_{E_C}^{\infty} \gamma(E-E_C)^{1/2} \times e^{-(E-E_F)/KT} \cdot dE$$

Simplifying this integral, we get,

$$n = N_C \times e^{-(E_C-E_F)/KT}$$

where 
$$N_C = 2 \left( \frac{2\pi m_n \bar{K} T}{h^2} \right)^{3/2}$$

$\bar{K}$  = Boltzman's Constant in J/°K

' $N_i$ ' is constant.

where  $m_n$  = effective mass of the electron.

Similarly the number of holes /  $m^3$  in the Valence Band

$$p = N_V e^{-(E_F-E_V)/KT}$$

where 
$$N_V = 2 \left( \frac{2\pi m_p \bar{K} T}{h^2} \right)^{3/2}$$

**Fermi Level** is the maximum energy level that can be occupied by the electrons at 0 °K. **Fermi Level** or characteristic energy represents the energy state with 50% probability of being filled if no forbidden bond exists. If  $E = E_F$ , then  $f(E) = 1/2$  for any value of temperature.  $f(E)$  is the probability that a quantum state with energy  $E$  is occupied by the electron.

**1.6.4 FERMI LEVEL IN INTRINSIC SEMICONDUCTOR**

$$n = p = n_i$$

$$n = N_C e^{-(E_C-E_F)/KT}$$

$$p = N_V e^{-(E_F-E_V)/KT}$$

$$n = p$$

or 
$$N_C e^{-(E_C-E_F)/KT} = N_V e^{-(E_F-E_V)/KT}$$

Electrons in the valence band occupy energy levels up to ' $E_F$ '. ' $E_F$ ' is defined that way. Then the additional energy that has to be supplied so that free electron will move from valence band to the conduction band is  $E_C$

$$\begin{aligned} \frac{N_C}{N_V} &= e^{\frac{-(E_F-E_V)}{KT} + \frac{(E_C+E_F)}{KT}} \\ &= e^{\frac{-2E_F+E_C+E_V}{KT}} \end{aligned}$$

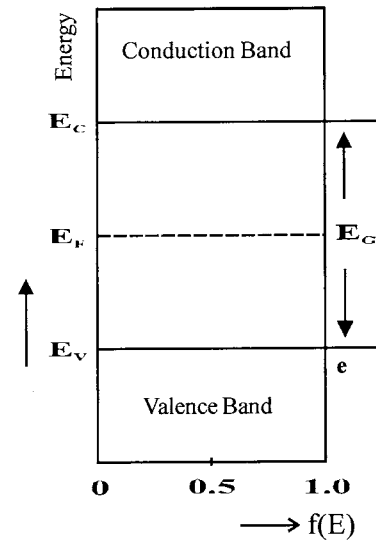


Fig. 1.5 Energy band diagram.

Taking logarithms on both sides,

$$\ln \frac{N_C}{N_V} = \frac{E_C + E_V - 2E_F}{KT}$$

$$\therefore E_F = \frac{E_C + E_V}{2} - \frac{KT}{2} \ln \frac{N_C}{N_V}$$

$$N_C = 2 \left( \frac{2\pi m_n \bar{K}T}{h^2} \right)^{\frac{3}{2}} \quad \dots\dots\dots (1.17)$$

$$N_V = 2 \left( \frac{2\pi m_p \bar{K}T}{h^2} \right)^{\frac{3}{2}} \quad \dots\dots\dots (1.18)$$

where  $m_n$  and  $m_p$  are effective masses of holes and electrons. If we assume that  $m_n = m_p$ , (though not valid),

$$N_C = N_V$$

$$\therefore \ln \frac{N_C}{N_V} = 0$$

$$\therefore E_F = \frac{E_C + E_V}{2} \quad \dots\dots\dots (1.19)$$

The graphical representation is as shown in Fig. 1.5. **Fermi Level in Intrinsic Semiconductor lies in the middle of Energy gap  $E_G$ .**

**Problem 1.15**

In p-type Ge at room temperature of 300 °K, for what doping concentration will the fermi level coincide with the edge of the valence bond ? Assume  $\mu_p = 0.4$  m.

**Solution**

$$E_F = E_V$$

when

$$N_A = N_V$$

$$\therefore E_F = E_V + kT \ln \frac{N_V}{N_A}$$

$$\therefore N_V = 4.82 \times 10^{15} \left( \frac{mp}{m} \right)^{\frac{3}{2}} \times T^{3/2} = 4.82 \times 10^{15} (0.4)^{3/2} (300)^{3/2} \\ = 6.33 \times 10^{18}.$$

$$\therefore \text{Doping concentration } N_A = 6.33 \times 10^{18} \text{ atoms/cm}^3.$$

**Problem 1.16**

If the effective mass of an electron is equal to twice the effective mass of a hole, find the distance in electron volts (ev) of fermi level in as intrinsic semiconductor from the centre of the forbidden bond at room temperature.



**Solution**

For Intrinsic Semiconductor,

$$E_F = \left[ \left( \frac{E_C + E_V}{2} \right) - \frac{KT}{2} \ln \left( \frac{N_C}{N_V} \right) \right]$$

If  $m_p = m_n$   
then  $N_C = N_V$ .

Hence  $E_F$  will be at the centre of the forbidden band. But if  $m_p \neq m_n$ .  $E_F$  will be away from the centre of the forbidden band by

$$\frac{KT}{2} \ln \cdot \frac{N_C}{N_V} = \frac{3}{4} \frac{kT}{2} \cdot \ln \frac{m_n}{m_p}$$

$$\therefore N_C = 2 \left( \frac{2\pi m_n \bar{K}T}{n^2} \right)^{3/2}$$

$$N_V = 2 \left( \frac{2\pi m_p \bar{k}T}{n^2} \right)^{3/2}$$

$$= \frac{3}{4} \times 0.026 \ln(2)$$

$$= 13.5 \text{ m. eV}$$

**1.7 p-n DIODE EQUATION**

The hole current in the *n-side*  $I_{pn}(x)$  is given as

$$I_{pn}(x) = \frac{Ae \times D_p}{L_p} p_n(0) e^{-x/L_p}$$

But  $p_n(0) = p_{no} (e^{V/V_T} - 1)$

$$I_{pn}(0) = \frac{Ae \times D_p}{L_p} \times p_{no} (e^{V/V_T} - 1)$$

$D_p$  = Diffusion coefficient of holes

$D_n$  = Diffusion coefficient of electrons.

$\therefore e^{-x/L_p}$  at  $x = 0$  is 1.

Similarly the electron current due to the diffusion of electrons from *n-side* to *p-side* is obtained from the above equation itself, by interchanging n and p.

$$\therefore I_{np}(0) = \frac{Ae \times D_n}{L_n} \times n_{po} (e^{V/V_T} - 1)$$

The total diode current is the sum of  $I_{pn}(0)$  and  $I_{np}(0)$

$$\text{or} \quad I = I_0 \left( e^{V/V_T} - 1 \right) \quad \dots\dots\dots (1.20)$$

where 
$$I_0 = \frac{AeD_p}{L_p} \times p_{no} + \frac{AeD_n}{L_n} \times n_{po}$$

In this analysis we have neglected charge generation and recombination. Only the current that results as a result of the diffusion of the carriers owing to the applied voltage is considered.

**Reverse Saturation Current**

$$I = I_0 \times \left( e^{V/V_T} - 1 \right)$$

This is the expression for current  $I$  when the diode is forward biased. If the diode is reverse biased,  $V$  is replaced by  $-V$ .  $V_T$  value at room temperature is  $\sim 26$  mV. If the reverse

bias voltage is very large,  $e^{-V/V_T}$  is very small. So it can be neglected.

$$\therefore I = -I_0$$

$I_0$  will have a small value and  $I_0$  is called the **Reverse Saturation Current**.

$$I_0 = \frac{AeD_p p_{no}}{L_p} + \frac{AeD_n n_{po}}{L_n}$$

In *n-type* semiconductor,

$$n_n = N_D$$

But  $n_n \times p_n = n_i^2 \quad \therefore p_n = \frac{n_i^2}{N_D}$

In *p-type* semiconductor,

$$p_p = N_A \quad \therefore n_p = \frac{n_i^2}{N_A}$$

Substituting these values in the expression for  $I_0$ ,

$$I_0 = Ae \left( \frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right) \times n_i^2$$

where  $n_i^2 = A_0 T^3 e^{-E_G/KT}$

$E_G$  is in electron volts =  $e \cdot V_G$ , where  $V_G$  is in Volts.

$$\therefore n_i^2 = A_0 T^3 e^{-\frac{E_{G0}}{KT}}$$

$$E_{G0} = V_G \cdot e$$

But  $\frac{KT}{e} = \text{Volt equivalent of Temperature } V_T$ .

For Germanium,  $D_p$  and  $D_n$  decrease with temperature and  $n_i^2$  increases with  $T$ . Therefore, temperature dependance of  $I_0$  can be written as,

$$I_0 = K_1 T^2 e^{-V_G/V_T}$$

For Germanium, the current due to thermal generation of carriers and recombination can be neglected. But for Silicon it cannot be neglected. So the expression for current is modified as

$$I = I_0 \left( e^{\frac{V}{\eta V_T}} - 1 \right)$$

where  $n = 2$  for small currents and  $n = 1$  for M large currents.

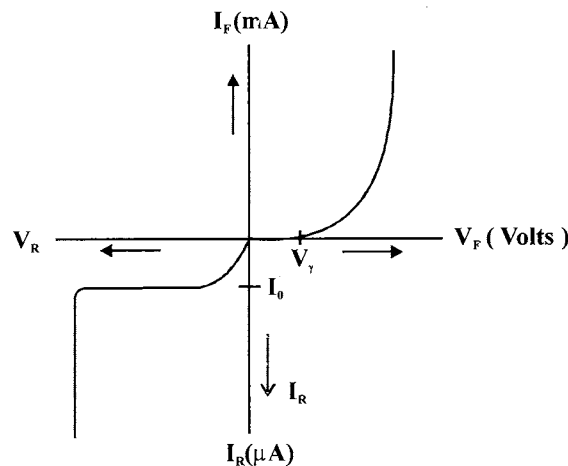
**1.8 VOLT-AMPERE (V-I) CHARACTERISTICS**

The general expression for current in the *p-n junction* diode is given by

$$I = I_0 \left( e^{\frac{V}{\eta V_T}} - 1 \right)$$

$\eta = 1$  for Germanium and 2 for Silicon. For Silicon, ' $\eta$ ' will be less than that for Germanium.  $V_T = 26 \text{ mV}$ .

If ' $V$ ' is much larger than  $V_T$ , 1 can be neglected. So ' $I$ ' increases exponentially with forward bias voltage ' $V$ '. In the case of reverse bias, if the reverse voltage  $-V \gg V_T$ , then  $e^{-V/V_T}$  can be neglected and so reverse current is  $-I_0$  and remains constant independent of ' $V$ '. So the characteristics are as shown in Fig. 1.6 and not like theoretical characteristics. The difference is that the practical characteristics are plotted at different scales. If plotted to the same scale, (reverse and forward) they may be similar to the theoretical curves. Another point is, in deriving the equations the breakdown mechanism is not considered. As ' $V$ ' increases **Avalanche multiplication** sets in. So the actual current is more than the theoretical current.



**Fig 1.6 V-I Characteristics of p-n junction diode.**

### CUT IN VOLTAGE $V_\gamma$

In the case of Silicon and Germanium, diodes there is a *Cut In* or *Threshold* or *Off Set* or *Break Point Voltage*, below which the current is negligible. It's magnitude is 0.2V for Germanium and 0.6V for Silicon ( Fig. 1.7 ).

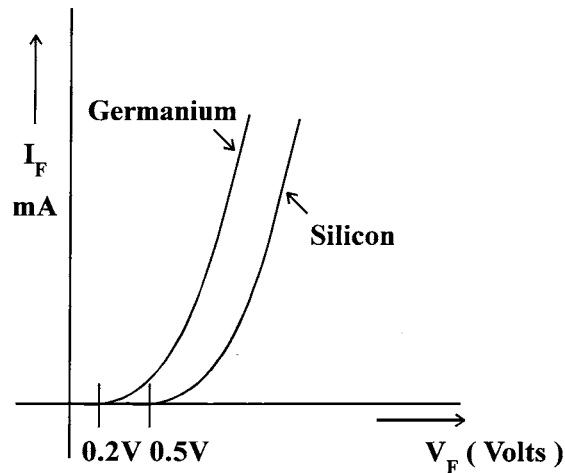


Fig 1.7 Forward characteristics of a diode.

### 1.9 TEMPERATURE DEPENDENCE OF V-I CHARACTERISTICS

$$\sigma = (\mu_n n + \mu_p p)e,$$

So the electrical characteristics of a semiconductor depends upon ' $n$ ' and ' $p$ ', the concentration of holes and electrons.

The expression for  $n = N_C e^{-(E_C - E_F)/KT}$  and the expression for  $p = N_V e^{-(E_F - E_V)/KT}$

These are valid for both intrinsic and extrinsic materials.

The electrons and holes, respond to an external field as if their mass is  $m^*$  ( $m^* = 0.6m$ ) and not ' $m$ '. So this  $m^*$  is known as *Effective Mass*.

With impurity concentration, only  $E_F$  will change. In the case of intrinsic semiconductors,  $E_F$  is in the middle of the energy gap, indicating equal concentration of holes and electrons.

If donor type impurity is added to the intrinsic semiconductor it becomes n-type. So assuming that all the atoms are ionized, each impurity atom contributes at least one free electron. So the first  $N_D$  states in the conduction band will be filled. Then it will be more difficult for the electrons to reach Conduction Band, bridging the gap between Covalent Bond and Valence Bond. So the number of electron hole pairs, thermally generated at that temperature will be decreased. *Fermi level is an indication of the probability of occupancy of the energy states*. Since Because of doping, more energy states in the ConductionBand are filled, the fermi level will move towards the Conduction Band.

EXPRESSION FOR  $E_G$

$$n = N_C \times e^{\frac{-(E_C - E_F)}{KT}}$$

$$p = N_V \cdot e^{\frac{-(E_F - E_V)}{KT}}$$

$$n \times p = N_C \times N_V \times e^{\frac{-(E_C - E_V)}{KT}}$$

But  $(E_C - E_V) = E_G$

and  $n \times p = n_i^2$

$$\therefore n_i^2 = N_C \times N_V \times e^{-\frac{E_G}{KT}}$$

$$\frac{n_i^2}{N_C \times N_V} = e^{\frac{E_G}{KT}}$$

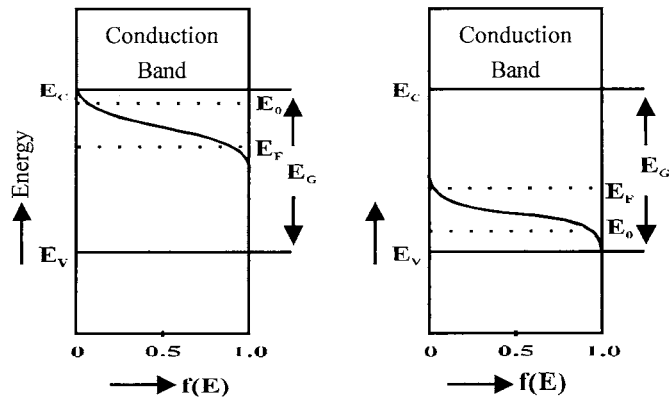
Taking logarithms,

$$\ln\left(\frac{n_i^2}{N_C N_V}\right) = -\frac{E_G}{KT}$$

or  $-\ln\left(\frac{N_C N_V}{n_i^2}\right) = -\frac{E_G}{KT}$

or  $E_G = KT \ln\left(\frac{N_C N_V}{n_i^2}\right)$  ..... ( 1.21 )

The position of Fermi Level is as shown in Fig. 1.8.



For n-type semiconductor      For p-type semiconductor

Fig. 1.8

Similarly, in the case of *p-type* materials, the Fermi level moves towards the valence band since the number of holes has increased. In the case of *n-type* semiconductor, the number of free electrons has increased. So energy in Covalent Bond has increased. Fermi Level moves towards conduction band. **Similarly in *p-type* semiconductors, fermi Level moves towards Valence Band.** So it is as shown in Fig. 1.8.

*To calculate the exact position of the Fermi Level in n-type Semiconductor:*

In *n-type* semiconductor,

$$n \simeq N_D$$

But  $n = N_C \times e^{-(E_C - E_F)/KT}$

$$\therefore N_D = N_C \times e^{-(E_C - E_F)/KT}$$

or  $\frac{N_D}{N_C} = e^{-(E_C - E_F)/KT}$

Taking logarithms,

$$\ln \frac{N_D}{N_C} = \frac{-(E_C - E_F)}{KT}$$

or  $KT \times \left\{ \ln \frac{N_D}{N_C} \right\} = - (E_C - E_F)$

or  $E_F = E_C - KT \times \left\{ \ln \frac{N_C}{N_D} \right\}$  ..... ( 1.22 )

***So Fermi Level  $E_F$  is close to Conduction Band  $E_C$  in n-type semiconductor.***

Similarly for *p-type* material,

$$p = N_A$$

But  $p = N_V \times e^{-(E_F - E_V)/KT}$

$$\therefore \frac{N_A}{N_V} = e^{-(E_F - E_V)/KT}$$

Taking Logarithms,

$$\ln \frac{N_A}{N_V} = \frac{-(E_F - E_V)}{KT}$$

$$KT \times \ln \frac{N_A}{N_V} = E_V - E_F$$

or  $E_F = E_V + KT \times \ln \frac{N_V}{N_A}$  ..... ( 1.23)

$$\therefore N_A = N_V$$

***Fermi Level is close to Valance Band  $E_V$  in p-type semiconductor.***

**Problem 1.17**

In n type silicon, the donor concentration is 1 atom per  $2 \times 10^8$  silicon atoms. Assuming that the effective mass of the electron equals true mass, find the value of temperature at which, the fermi level will coincide with the edge of the conduction band. Concentration of Silicon =  $5 \times 10^{22}$  atom/cm<sup>3</sup>.

**Solution**

Donor atom concentration = 1 atom per  $2 \times 10^8$  Si atom.

Silicon atom concentration =  $5 \times 10^{22}$  atoms/cm<sup>3</sup>

$$\therefore N_D = \frac{5 \times 10^{22}}{2 \times 10^8} = 2.5 \times 10^{14} / \text{cm}^3.$$

For n-type, Semiconductor,

$$E_F = E_C - KT \ln \left( \frac{N_C}{N_D} \right)$$

If  $E_F$  were to coincide with  $E_C$ , then

$$N_C = N_D$$

$$N_D = 2.5 \times 10^{14} / \text{cm}^3.$$

$$N_C = 2 \left\{ \frac{2\pi m_n \bar{K} T}{h^2} \right\}^{\frac{3}{2}}$$

$h$  = Plank's Constant;  $\bar{K}$  = Boltzman Constant

$m_n$  the effective mass of electrons to be taken as =  $m_E$

$$\therefore N_C = 2 \left\{ \frac{2 \times 3.14 \times 9.1 \times 10^{-31} \times \bar{K} \times T}{h^2} \right\}^{\frac{3}{2}}$$

$$= 4.28 \times 10^{15} T^{\frac{3}{2}}$$

$$\therefore N_C = N_D,$$

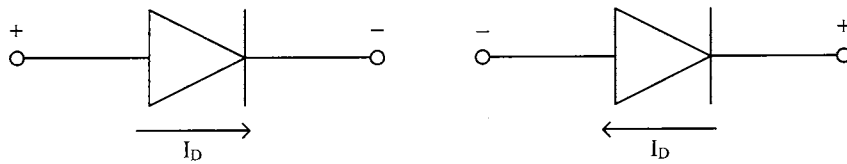
$$4.28 \times 10^{15} T^{\frac{3}{2}} = 2.5 \times 10^{14}$$

$$\therefore T = 0.14 \text{ } ^\circ\text{K}$$

that *p-side* of the p-n junction is heavily doped. If *n-side* is heavily doped, it would be  $N_D$ .

### 1.10 IDEAL VS PRACTICAL

**Ideal:** The p-n junction diode allows flow of heavy current during forward bias condition and allows a small level of current when it is reverse biased. The direction of the arrow in the following Fig.1.9(a) depicts the direction of flow of current when the diode is forward biased. Similarly the direction of the arrow in Fig.1.9 (b) depicts the flow of current direction when the diode is reversed biased.



*Fig.1.9(a) Illustration of flow of current when the diode is forward biased*

*Fig. 1.9(b) Illustration of flow of current when the diode is Reverse biased*

#### ANALOGY

A common analogy that can be employed to describe the behaviour of a p-n junction diode is Mechanical switch.

During the forward bias the p-n junction diode allows flow of heavy currents through the junction. And hence the amount of resistance offered by the p-n junction diode during forward bias condition is very small and assumed to be zero. Thus the pn junction diode acts as a closed switch when the diode is forward biased. On the other hand when the diode is reverse biased it offers a flow very small level of current across the junction which is assumed to be zero. In otherwords it offers very high amount of resistance during its reverse bias (prevents flow of current across the junction i.e., offer  $\infty$  resistance). Thus the p-n junction diode acts on a open switch during the reverse bias.

The following Fig.1.10(a) shows the analogy of closed switch of p-n junction diode and Fig. 1.10(b) shows the analogy of open switch of p-n junction diode.



*Fig. 1.10(a) Analogy of p-n junction diode as closed switch*      *Fig.1.10(b) Analogy of p-n junction diode as open switch*

#### PRACTICAL

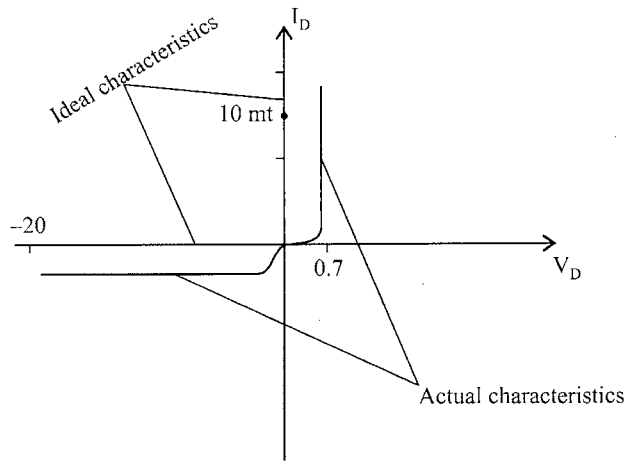
Even though the p-n junction diode offers a flow of heavy currents during forward bias, a small amount of current (which is due to presence of minority charge carriers across the junction) opposes the flow of forward biased current ( $I_D$ ). In otherwords the p-n junction diode offers a small amount of forward resistance ( $r_f$ ) which opposes the flow forward biased current ( $I_D$ ).



Similarly when the diode is reverse biased the switch is assumed to be open but in practical the p-n junction diode allows the flow small amounts of currents through the junction which can be referred as leakage current (or) minority charge carrier current.

**IDEAL VS PRACTICAL**

The following figure shows super imposition of practical characteristics in ideal characteristics. From the figure we can observe that the ideal diode does not offer any forward resistance and hence it does not require any additional voltage to make  $I_D$  to flow through the diode. In otherwords the forward bias current  $I_D$  starts flowing through the diode even when applied forward bias voltage is at 0V.



Where as in case of practical diode as it offers a small amount of forward resistance ( $r_f$ ) it requires a knee voltage (or) threshold voltage of 0.6 in case of silicon (0.3 of germanium) to make forward current  $I_D$  to flow through the junction.

**ANALOGY: CLOSED SWITCH**

**Ideal:** When the switch is closed, the resistance between the contacts is assumed to be zero. In the plot a point chosen in vertical axis where the diode current 10 mA and voltage across diode is 0V. According to Ohm's law.

$$R_F = \frac{V_D}{I_D} = \frac{0V}{10 \text{ mA}} = 0 \Omega$$

**Practical:** In practical diode the minimum voltage required to obtain  $I_D$  is 0.6 (for Si). Hence

$$R_F = \frac{V_D}{I_D} = \frac{0.6}{10 \text{ mA}} = 60 \Omega$$

**ANALOGY: OPEN SWITCH**

**Ideal:** For any value of voltage on horizontal axis (any point on X-axis has zero coordinate) the current is zero. Applying ohm's law. In plot  $V_D$  is taken as 20.

$$R_r = \frac{V_D}{I_D} = \frac{20}{0 \text{ mA}} = \infty$$

**Practical:** For practical diode a small level of current (usually in order of  $\eta\text{A}$  for si and  $\mu\text{A}$  for Ge) flows when the diode is reverse biased. Hence

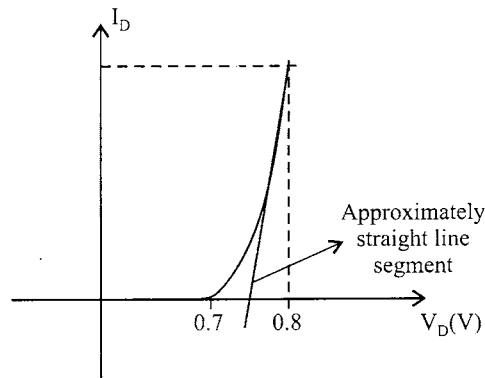
$$R_r = \frac{20}{10 \text{ mA (assume)}} = \frac{20}{10 \times 10^{-9}} = 2000 \text{ M}\Omega \text{ for si}$$

	Ideal	Practical
When switch is closed	$R_f = 0$	$R_f = R_f$ (in order of ohms)
When switch is open	$R_r = \infty$	$R_r = R_r$ (in order of $\text{M}\Omega$ )

**1.11 DIODE EQUIVALENT CIRCUITS**

An equivalent circuit is a combination of elements that are chosen to represent actual characteristics of device for particular operating region.

**Piecewise Linear Equivalent Circuit:** The best method of obtaining equivalent circuit is to approximate its characteristics with straight line segments. The forward bias characteristics of p-n junction diode is shown in following Fig. 1.11.



**Fig. 1.11** Approximation of diode characteristics

The above Fig. 1.11 represent approximation of exact characteristics of p-n junction diode. From the figure it is clear that this approximation does not yield the exact duplication of actual characteristics of p-n junction diode. However these approximation yields sufficiently close to actual curve to represent the behaviour of the diode.

The actual characteristics have knee voltage of 0.7 V but the approximate model has knee voltage greater than 0.7 V. To achieve this 0.7 V of knee voltage connect a battery of 0.7 V to the p-n junction diode assuring its polarity is in opposition with the applying voltage as show in following Fig.1.12 and assuming the diode to be ideal, the ideal diode does not conduct any current when it is in reverse bias condition hence approximation of reverse bias characteristics of diode can be neglected.

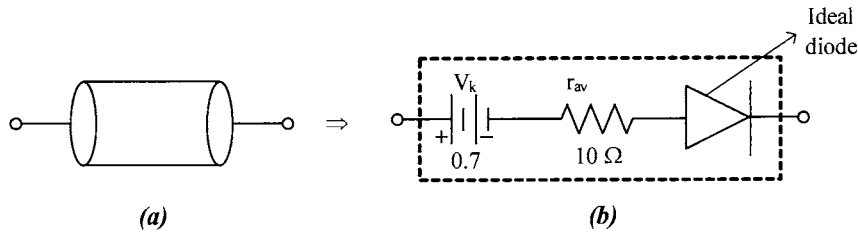


Fig. 1.12 Equivalent circuit of diode

The above Fig. 1.12 assures that the diode is not conducted until voltage across it reaches greater than its knee voltage i.e., 0.7. And once the diode starts conducting it offers a specified value of resistance  $r_{av}$ .

The approximate value of  $r_{av}$  can be obtained from the operating point of the diode. Assuming its operating point to be (10, 0.8) i.e., the current through the diode is 10 mA when applied voltage is 0.8 (show in Fig. 1.12(a)). We get

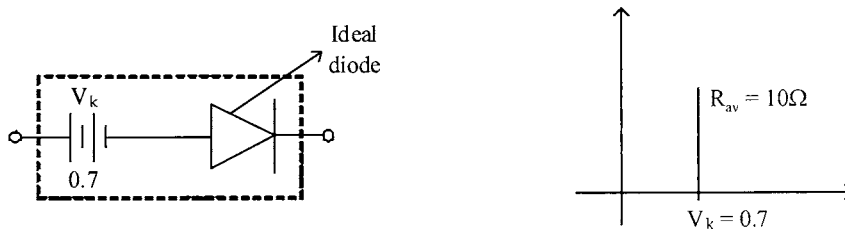
$$r_{av} = \left. \frac{\Delta V_d}{\Delta I_d} \right|_{pt \text{ to } pt} = \frac{0.8 - 0.7}{10 \text{ mA} - 0 \text{ mA}} = \frac{0.1}{10 \text{ mA}} = 10 \Omega$$

$$\therefore r_{av} = 10 \Omega$$

The 0.7 V is knee voltage i.e., minimum voltage required to make diode ready for conducting and at exact knee point the diode will be ready to conduct, but it will not be conducting hence  $I_D$  at knee voltage is 0 mA. When the voltage across the diode just crosses its knee voltage the diode starts conducting and hence  $I_D$  starts increasing. But at point when applied voltage is equal to knee voltage the diode will be in equilibrium resulting  $I_D = 0$ .

**SIMPLIFIED EQUIVALENT CIRCUIT**

In general the value of  $r_{av}$  is very small in comparison with the other elements of the circuit and hence it can be neglected. Removing  $r_{av}$  from diode equivalent circuits give simplified equivalent diode circuit. The following shows the simplified equivalent circuit of the diode and its characteristics.



## 1.12 BREAK DOWN MECHANISMS IN SEMICONDUCTOR DIODES

There are three types of breakdown mechanisms in semiconductor devices.

1. Avalanche Breakdown
2. Zener Breakdown
3. Thermal Breakdown

### 1.12.1 AVALANCHE BREAKDOWN

When there is no bias applied to the diode, there are certain number of thermally generated carriers. When bias is applied, electrons and holes acquire sufficient energy from the applied potential to produce new carriers by removing valence electrons from their bonds. These thermally generated carriers acquire additional energy from the applied bias. They strike the lattice and impart some energy to the valence electrons. So the valence electrons will break away from their parent atom and become free carriers. These newly generated additional carriers acquire more energy from the potential (since bias is applied). So they again strike the lattice and create more number of free electrons and holes. This process goes on as long as bias is increased and the number of free carriers gets multiplied. This is known as *avalanche multiplication*. Since the number of carriers is large, the current flowing through the diode which is proportional to free carriers also increases and when this current is large, avalanche breakdown will occur.

### 1.12.2 ZENER BREAKDOWN

Now if the electric field is very strong to disrupt or break the covalent bonds, there will be sudden increase in the number of free carriers and hence large current and consequent breakdown. Even if thermally generated carriers do not have sufficient energy to break the covalent bonds, the electric field is very high, then covalent bonds are directly broken. This is *Zener Breakdown*. A junction having narrow depletion layer and hence high field intensity will have zener breakdown effect. ( $\cong 10^6$  V/m). If the doping concentration is high, the depletion region is narrow and will have high field intensity, to cause Zener breakdown.

### 1.12.3 THERMAL BREAKDOWN

If a diode is biased and the bias voltage is well within the breakdown voltage at room temperature, there will be certain amount of current which is less than the breakdown current. Now keeping the bias voltage as it is, if the temperature is increased, due to the thermal energy, more number of carriers will be produced and finally breakdown will occur. This is *Thermal Breakdown*.

In zener breakdown, the covalent bonds are ruptured. But the covalent bonds of all the atoms will not be ruptured. Only those atoms, which have weak covalent bonds such as an atom at the surface which is not surrounded on all sides by atoms will be broken. But if the field strength is not greater than the critical field, when the applied voltage is removed, normal covalent bond structure will be more or less restored. This is Avalanche Breakdown. But if the field strength is very high, so that the covalent bonds of all the atoms are broken, then normal structure will not be achieved, and there will be large number of free electrons. This is *Zener Breakdown*.

In Avalanche Breakdown, only the excess electron, loosely bound to the parent atom will become free electron because of the transfer of energy from the electrons possessing higher energy.

### 1.13 ZENER DIODE CHARACTERISTICS

This is a *p-n junction* device, in which zener breakdown mechanism dominates. *Zener diode is always used in Reverse Bias.*

Its constructional features are:

1. *Doping concentration is heavy on p and n regions of the diode, compared to normal p-n junction diode.*
2. *Due to heavy doping, depletion region width is narrow.*
3. *Due to narrow depletion region width, electric field intensity  $E = \frac{V}{d} = \frac{V_z}{W}$  will be high, near the junction, of the order of  $10^6 \text{V/m}$ . So Zener Breakdown mechanism occurs.*

In normal *p-n junction* diode, avalanche breakdown occurs if the applied voltage is very high. The reverse characteristic of a p-n junction diode is shown in Fig. 1.13.

When the Zener diode is reverse biased, the current flowing is only the reverse saturation current  $I_0$  which is constant like in a reverse biased diode. At  $V = V_z$ , due to high electric field  $\left(\frac{V_z}{W}\right)$ , Zener breakdown occurs. Covalent bonds are broken and suddenly the number of free electrons increases. So  $I_z$  increases sharply and  $V_z$  remains constant, since,  $I_z$  increases through Zener resistance  $R_z$  decreases. So the product  $V_z = R_z \cdot I_z$  almost remains constant. If the input voltage is decreased, the Zener diode regains its original structure. (But if  $V_i$  is increased much beyond  $V_z$ , electrical breakdown of the device will occur. The device loses its semiconducting properties and may become a short circuit or open circuit. ***This is what is meant by device breakdown.***)

#### Applications

1. *In Voltage Regulator Circuits*
2. *In Clipping and Clamping Circuits*
3. *In Wave Shaping Circuits.*

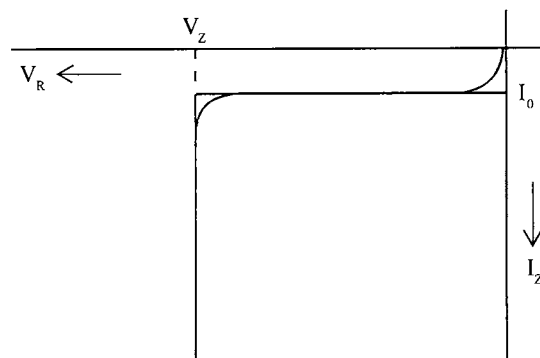


Fig 1.13 Reverse characteristic of a Zener diode.

### 1.14 p-n JUNCTION DIODE AS RECTIFIER

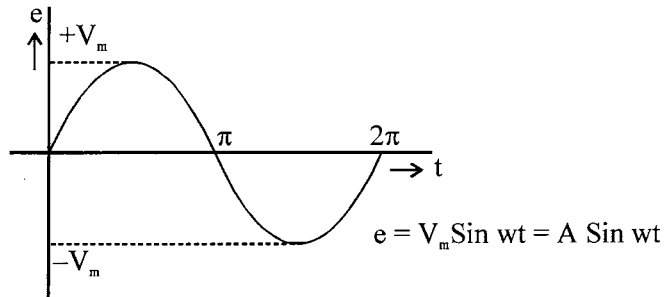
The electronic circuits require a D.C. source of power. For transistor A.C. amplifier circuit for biasing, D.C. supply is required. The input signal can be A.C. and so the output signal will be amplified A.C. signal. But without biasing with D.C. supply, the circuit will not work. So more or less all electronic A.C. instruments require D.C. power. To get this, D.C. batteries can be used. But they will get dried quickly and replacing them every time is a costly affair. Hence it is economical to convert A.C. power into D.C.

**Rectifier** is a circuit which offers low resistance to the current in one direction and high resistance in the opposite direction.

**Rectifier** converts sinusoidal signal to unidirectional flow and not pure D.C.

**Filter** converts unidirectional flow into pure D.C.

If the input to the rectifier is a pure sinusoidal wave, the average value of such a wave is zero, since the positive half cycle and negative half cycle are exactly equal.

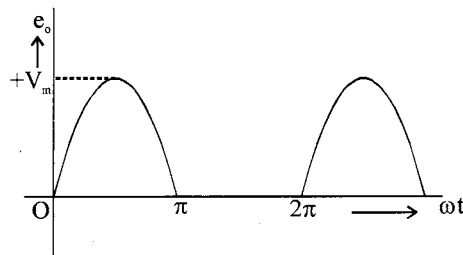


**Fig 1.14 AC input wave form.**

$$T_{av} = \frac{1}{2\pi} \int_0^{2\pi} A \sin(\omega t) dt$$

### 1.15 HALF-WAVE RECTIFIER

If this signal is given to the rectifier circuit, say Half Wave Rectifier Circuit, the output will be as shown in Fig. 1.15.



**Fig. 1.15 Half wave rectified output (unidirectional flow).**

Now the average value of this waveform is *not zero*, since there is no negative half. Hence a rectifier circuit converts A.C. Signal with zero average value to a unidirectional wave form with non zero average value.

The rectifying devices are semiconductor diodes for low voltage signals and vacuum diodes for high voltage circuit. A basic circuit for rectification is as shown in Fig. 1.16.

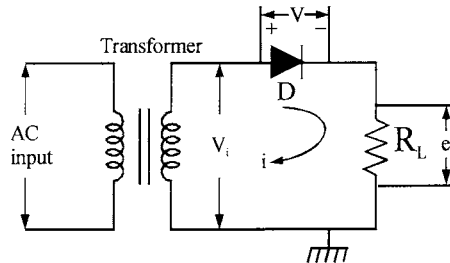


Fig 1.16 Halfwave Rectifier (HWR) circuit.

A.C input is normally the A.C. main supply. Since the voltage is 230V, and such a high voltage cannot be applied to the semiconductor diode, step down transformer should be used. If large D.C. voltage is required vacuum tubes should be used. Output voltage is taken across the load resistor  $R_L$ . Since the peak value of A.C. signal is much larger than  $V_\gamma$ , we neglect  $V_\gamma$  for analysis.

1.15.1 MAXIMUM OR PEAK CURRENT

The output current waveform for half wave rectification is shown in Fig. 1.17.

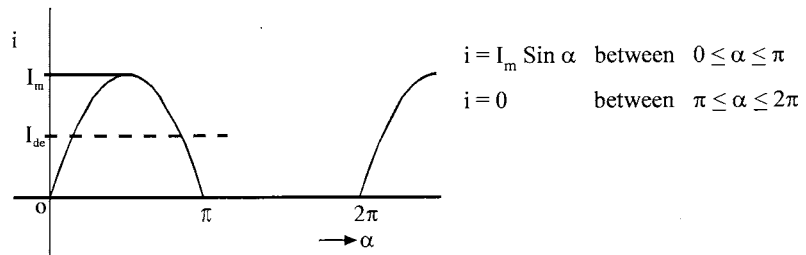


Fig 1.17 Half wave rectified output.

$$\therefore \begin{aligned} i &= I_m \sin \alpha & \alpha = \omega t & \quad 0 \leq \alpha \leq \pi \\ i &= 0 & & \quad \pi \leq \alpha \leq 2\pi \end{aligned}$$

$$I_m = \frac{V_m}{R_f + R_L}$$

$R_f$  is the forward resistance of the diode.  $R_L$  is the load resistance

1.15.2 READING OF D.C. AMMETER

If a D.C. Ammeter is connected in the rectifier output circuit, what reading will it indicate? Is it the peak value or will the needle oscillate from zero to maximum and then to zero and so on, or will it indicate average value? The meter is so constructed that it reads the average value.

By definition, average value =  $\frac{\text{Area of the curve}}{\text{Base}}$

$\therefore$  For Half wave rectified output, base value is  $2\pi$  for one cycle,

$$I_{DC} = \frac{1}{2\pi} \int_0^{\pi} I_m \sin \alpha \, d\alpha = \frac{I_m}{2\pi} \cdot [-\cos \alpha]_0^{\pi}$$

$$I_{DC} = \frac{I_m}{2\pi} [1 + 1] = \frac{I_m}{\pi}$$

Upper limit is only  $\pi$ , because the signal is zero from  $\pi$  to  $2\pi$ . The complete cycle is from 0 to  $2\pi$ .

### 1.15.3 READING OF A.C. AMMETER

An A.C. ammeter is constructed such that the needle deflection indicates the effective or RMS current passing through it.

Effective or RMS value of an A.C. quantity is the equivalent D.C. value which produces the same heating effect as the alternating component. If some A.C. current is passed through a resistor, during positive and negative half cycles, also because of the current, the resistor gets heated, or there is some equivalent power dissipation. What is the value of D.C. which produces the same heating effects as the A.C. quantity? The magnitude of this equivalent D.C. is called the RMS or Effective Value of A.C.

$$\begin{aligned} \text{By definition } I_{rms} &= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (I_m \sin \alpha)^2 \, d\alpha} = \sqrt{\frac{I_m^2}{2\pi} \int_0^{2\pi} \sin^2 \alpha \, d\alpha} \\ &= \frac{I_m}{\sqrt{2\pi}} \sqrt{\int_0^{2\pi} \left\{ \frac{1 - \cos 2\alpha}{2} \right\} \, d\alpha} = \frac{I_m}{\sqrt{2\pi}} \sqrt{\left[ \frac{\alpha}{2} \right]_0^{2\pi} + \left[ \frac{\sin 2\alpha}{4} \right]_0^{2\pi}} \end{aligned}$$

$$\therefore \text{RMS value of a sine wave} = \frac{I_m}{\sqrt{2\pi}} \times \sqrt{\frac{2\pi}{2} + 0}$$

$$= \frac{I_m \sqrt{\pi}}{\sqrt{2} \sqrt{\pi}} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

$$\text{Form Factor} = \frac{\text{RMS Value}}{\text{Average Value}}$$

$$\text{For a sine wave, } I_{average} = 0.636 I_m = 2 \times \frac{1}{2\pi} \int_0^{\pi} (I_m \sin \alpha) \, d\alpha$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

$$\text{Form factor} = \frac{\left( \frac{I_m}{\sqrt{2}} \right)}{\left( \frac{I_m}{\pi} \right)} = \frac{0.707 I_m}{0.636 I_m} = 1.11$$

### 1.15.4 PEAK INVERSE VOLTAGE

For the circuit shown, the input is A.C. signal. Now during the positive half cycle, the diode conducts. The forward resistance of the diode  $R_f$  will be small.

$$\therefore \text{The voltage across the diode } v = i \times R_f$$

$$i = I_m \sin \alpha \quad 0 \leq \alpha \leq \pi.$$

$$\therefore v = I_m R_f \sin \alpha \quad 0 \leq \alpha \leq \pi.$$



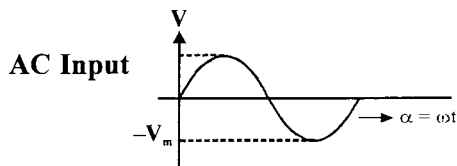


Fig. 1.18

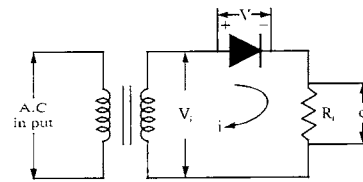


Fig. 1.19 Half wave rectifier circuit

Since  $R_f$  is small, the voltage across the diode ‘V’ during positive half cycle will be small, and the waveform is as shown. But during the negative half cycle, the diode will not conduct. Therefore, the current ‘i’ through the circuit is zero. So the voltage across the diode is not zero but the voltage of the secondary of the transformer  $V_i$  will appear across the diode ( $\because$  effectively the diode is across the secondary of the transformer.)

$$\therefore v = V_m \sin \alpha \quad \pi \leq \alpha \leq 2\pi.$$

The waveform across the diode is Fig. 1.20.

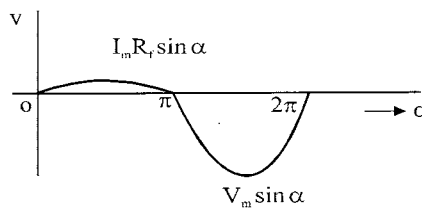


Fig 1.20 Voltage waveform across the diode

So the D.C. Voltage that is read by a D.C. Voltmeter is the average value.

$$V_{DC} = \frac{1}{2\pi} \left[ \int_0^{\pi} I_m R_f \cdot \sin \alpha \, d\alpha + \int_{\pi}^{2\pi} V_m \sin \alpha \, d\alpha \right]$$

$$= \frac{1}{2\pi} [(2)I_m R_f - (2)I_m (R_f + R_L)]$$

$$\therefore V_m = I_m (R_f + R_L)$$

$$\therefore \boxed{V_{DC} = \frac{-I_m R_L}{\pi}} \quad \dots\dots\dots ( 1.24 )$$

If we connect a CRO across the diode, this is the waveform that we see is as shown in Fig. 1.20. So in the above circuits the diode is being subjected to a maximum voltage of  $V_m$ . It occurs when the diode is not conducting. Hence it is called the **Peak Inverse Voltage (PIV)**

### 1.15.5 REGULATION

The variation of D.C output voltage as a function of D.C load current is called 'regulation'.

$$\% \text{ Regulation} = \frac{V_{\text{No Load Voltage}} - V_{\text{Full Load}}}{V_{\text{Full Load}}} \times 100\%$$

For an ideal power supply output voltage is independent of the load or the output in voltage remains constant even if the load current varies, like in Zener diode near breakdown. Therefore, **regulation is zero or it should be low for a given circuit.**

#### EXPRESSION FOR $V_{\text{DC}}$ THE OUTPUT DC VOLTAGE

For half wave rectifier circuit (Fig. 1.21),  $I_{\text{DC}}$  the average value is :

$$\begin{aligned} I_{\text{DC}} &= \frac{1}{2\pi} \int_0^{2\pi} i \cdot d\alpha \\ &= \frac{1}{2\pi} \int_0^{\pi} I_m \sin \alpha \cdot d\alpha = \frac{I_m}{\pi} \end{aligned}$$

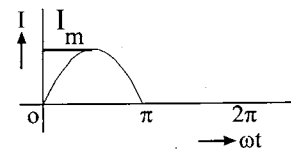


Fig 1.21 HWR current output

$$I_{\text{DC}} = \frac{I_m}{\pi} \quad \dots\dots\dots ( 1.25 )$$

But  $I_m = \frac{V_m}{R_f + R_L}$   $R_L = \text{Load Resistance}$

$R_f = \text{Forward Resistance of the Diode.}$

$$V_{\text{DC}} = I_{\text{DC}} R_L$$

$$\therefore I_{\text{DC}} = \frac{V_m / \pi}{R_f + R_L}$$

But  $I_{\text{DC}} = \frac{V_m}{\pi(R_f + R_L)}$

$$\therefore V_{\text{DC}} = \frac{V_m \cdot R_L}{\pi(R_f + R_L)}$$

Adding and Subtracting  $R_f$ ,

$$V_{DC} = \frac{V_m \cdot (R_L + R_f - R_f)}{\pi(R_f + R_L)} = \frac{V_m}{\pi} - \frac{V_m \cdot R_f}{\pi(R_f + R_L)}$$

$$I_{DC} = \frac{V_m}{\pi(R_f + R_L)}$$

$I_{DC}$  is determined by  $R_L$ . Hence  $V_{DC}$  depends upon  $R_L$ .

$$\boxed{V_{DC} = \frac{V_m}{\pi} - I_{DC} \times R_f} \quad \dots\dots\dots ( 1.26 )$$

This expression indicates that  $V_{DC}$  is  $\frac{V_m}{\pi}$  at *no load or when the load current is zero*, and it decreases with increase in  $I_{DC}$  linearly since  $R_f$  is more or less constant for a given diode. The larger the value of  $R_f$ , the greater is the decrease in  $V_{DC}$  with  $I_{DC}$ . But, the series resistance of the secondary winding of the transformer  $R_s$  should also be considered.

For a given circuit of half wave rectifier, if a graph is plotted between  $V_{DC}$  and  $I_{DC}$  the slope of the curve gives  $(R_f + R_s)$  where  $R_f$  is the forward resistance of diode and  $R_s$  the series resistance of secondary of transformer.

**REGULATION FOR HWR**

The regulation indicates how the DC voltage varies as a function of DC load current. In general, the percentage of regulation for ideal power supply is zero. The percentage of regulation is defined as

$$\% \text{ Regulation} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100$$

As we know that,

$$V_{DC} = \frac{V_m}{\pi} - I_{DC} \cdot R_f$$

$$V_{NL} = \frac{V_m}{\pi} \text{ and } V_{FL} = \frac{V_m}{\pi} - I_{DC} \cdot R_f$$

$$\therefore \% \text{ Regulation} = \frac{\frac{V_m}{\pi} - \frac{V_m}{\pi} + I_{DC} \cdot R_f}{\frac{V_m}{\pi} - I_{DC} \cdot R_f} \times 100$$

$$= \frac{I_{DC} \cdot R_f}{\frac{V_m}{\pi} - I_{DC} \cdot R_f} = \frac{1}{\frac{V_m}{\pi (I_{DC} \cdot R_f)} - 1} \times 100$$

$$= \frac{1}{\frac{V_m}{\pi} \times \frac{\pi}{V_m} \frac{(R_f + R_L)}{R_f} - 1} \times 100 \left[ \because I_{DC} = \frac{V_m}{\pi(R_f + R_L)} \right]$$

$$\boxed{\% \text{ Regulation} = \frac{R_f}{R_L} \times 100}$$

Suppose for a given rectifier circuit, the specifications are 15V and 100 mA, i.e., the no load voltage is 15V and max load current that can be drawn is 100 mA. If the value of  $(R_f + R_s) = 25 \Omega$  then the percentage regulation of the circuit is :

$$\text{No Load Voltage} = 15\text{V}$$

$$\text{Drop across Diode} = I_m (R_f + R_s)$$

$$R_s = \text{Transformer Secondary Resistance}$$

$$\begin{aligned} \text{Max. Voltage with load} &= 15\text{V} - (I_m \times (R_f + R_s)) \\ &= 15 - 100 \text{ mA} \times 25\Omega \\ &= 15 - 2.5 \text{ volts} = 12.5\text{V} \end{aligned}$$

$$\therefore \text{Percentage Regulation} = \frac{15 - 12.5}{12.5} \times 100 = \frac{2.5}{12.5} \times 100 \simeq 20\%.$$

### 1.15.6 RIPPLE FACTOR

The purpose of a rectifier circuit is to convert A.C. to D.C. But the simple circuit shown before will not achieve this. Rectifier converts A.C. to unidirectional flow and not D.C. So filters are used to get pure D.C. Filters convert unidirectional flow into D.C. Ripple factor is a measure of the fluctuating components present in rectifier circuits.

$$\text{Ripple factor, } \gamma = \frac{\text{RMS Value of alternating components of the waveform}}{\text{Average Value of the waveform}}$$

$$\gamma = \frac{I'_{\text{rms}}}{I_{\text{DC}}} = \frac{V'_{\text{rms}}}{V_{\text{DC}}}$$

$I'_{\text{rms}}$  and  $V'_{\text{rms}}$  denote the value of the A.C components of the current and voltage in the output respectively. While determining the ripple factor of a given rectifier system experimentally, a voltmeter or ammeter which can respond to high frequencies ( greater than power supply frequency 50 Hz ) should be used and a capacitor should be connected in series with the input meter in order to block the D.C. component. Ripple factor should be small. ( Total current  $i = I_m \sin \omega t$  according to Fourier Series, only A.C. is sum of D.C. and harmonics ).

We shall now derive the expression for the ripple factor. The instantaneous current is given by  $i' = i - I_{\text{DC}}$ .

'i' is the total current. In a half wave rectifier, some D.C components are also present. Hence A.C component is,

$$i' = (i - I_{\text{DC}}) [ ( \text{Total current} - I_{\text{DC}} ) ]$$

$$\text{RMS value is } I'_{\text{rms}} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (i - I_{\text{DC}})^2 d\alpha}$$

$$\therefore I'_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (i^2 - 2I_{DC}i + I_{DC}^2) d\alpha}$$

Now,  $\frac{1}{2\pi} \int_0^{2\pi} i^2 d\alpha =$  Square of the rms value of a sine wave by definition.  
 $= (I_{rms})^2$

$$\frac{1}{2\pi} \int_0^{2\pi} i d\alpha = \text{The average value or D.C value } I_{DC}$$

$I_{DC}$  is constant. So taking this term outside,

$$\frac{I_{DC}^2}{2\pi} \int_0^{2\pi} d\alpha = \frac{I_{dc}}{2\pi} [2\pi]$$

$$\therefore I'_{rms} = \sqrt{(I_{rms})^2 - 2I_{DC}^2 + I_{DC}^2}$$

The rms ripple current is

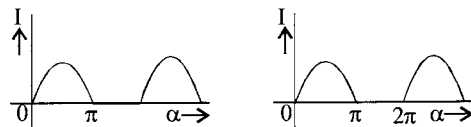
$$I'_{rms} = \sqrt{(I_{rms})^2 - I_{DC}^2}$$

Ripple factor,  $\gamma = \frac{I'_{rms}}{I_{dc}} = \frac{\sqrt{(I_{rms})^2 - I_{DC}^2}}{I_{DC}}$

$$\gamma = \sqrt{\left(\frac{I_{rms}}{I_{DC}}\right)^2 - 1} \dots\dots\dots ( 1.27 )$$

This is independent of the current waveshape and is not restricted to a half wave configuration. If a capacitor is used to block D.C. and then  $I_{rms}$  or  $V_{rms}$  is measured,

Then,  $\gamma = \frac{I_{rms}}{I_{DC}}$



$$\therefore I'_{rms} = \sqrt{I_{rms}^2 - I_{DC}^2}$$

and  $I_{DC} = 0$  (blocked by Capacitor) output waveform

For Half Wave Rectifier Circuit ( HWR ),

$$I = I_m \sin \alpha$$

$$I_{av} = I_{DC} = \frac{1}{2\pi} \int_0^{\pi} I_m \sin \alpha d\alpha = \frac{I_m}{\pi}$$

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi} \int_0^{\pi} I_m^2 \sin^2(\alpha) d\alpha} = \frac{I_m}{2}$$

$$I_{\text{DC}} = \frac{I_m}{\pi}; \quad V_{\text{DC}} = \frac{V_m}{\pi} \quad \text{Peak Inverse Voltage (PIV)} = V_m$$

$$\text{Ripple Factor} \quad \gamma = \frac{\text{RMS value of ripple current}}{\text{Average value of the current}} = \frac{I'_{\text{rms}}}{I_{\text{DC}}}$$

$$\text{Total current} \quad I = I_{\text{DC}} + I'(\text{ripple})$$

$$I'(\text{ripple}) = (I - I_{\text{DC}})$$

$$\begin{aligned} I'_{\text{rms}} &= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (I - I_{\text{DC}})^2 d\alpha} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (I^2 - 2II_{\text{DC}} + I_{\text{DC}}^2) d\alpha} \\ &= \sqrt{(I_{\text{rms}})^2 - 2II_{\text{DC}} + I_{\text{DC}}^2} \end{aligned}$$

$$\therefore I'_{\text{rms}} = \sqrt{(I_{\text{rms}})^2 - I_{\text{DC}}^2}$$

$$\gamma = \frac{I'_{\text{rms}}}{I_{\text{DC}}} = \sqrt{\left(\frac{I_{\text{rms}}}{I_{\text{DC}}}\right)^2 - 1}$$

$$\text{for Half Wave Rectifier, } \gamma = \sqrt{\left(\frac{I_m \times \pi}{2 \times I_m}\right)^2 - 1} = \sqrt{\left(\frac{\pi}{2}\right)^2 - 1} = 1.21$$

### 1.15.7 RATIO OF RECTIFICATION ( $\eta$ )

$$\text{Ratio of Rectification} = \frac{\text{DC power delivered to the load}}{\text{AC input power from transfer secondary}}$$

$$P_{\text{DC}} = I_{\text{DC}}^2 \times R_L$$

$$\text{But} \quad I_{\text{DC}} \text{ for HWR} = \frac{I_m}{\pi}$$

$$\therefore P_{\text{DC}} = \left(\frac{I_m}{\pi}\right)^2 \times R_L$$

$P_{\text{AC}}$  is what a Wattmeter would indicate if placed in the HWR circuit with its voltage terminal connected across the secondary of the transformer.

From  $\pi - 2\pi$  the diode is not conducting. Hence  $I_{\text{ac}} = 0$ .

$$P_{\text{ac}} = (I_{\text{rms}})^2 (R_f + R_L)$$

Q from  $\pi - 2\pi$  the AC is not being converted to D.C. So this power is wasted, as heat across diode and transformer.

$$\therefore I_{\text{rms}}(\pi - 2\pi) \text{ portion of AC is } = \sqrt{\frac{1}{2\pi} \int_{\pi}^{2\pi} I_m^2 \sin^2 \alpha d\alpha} = \frac{I_m}{2}$$

$I_{\text{rms}}$  for HWR is  $I_m/2$ . During negative half cycle, diode is not conducting and current  $I$  in the loop is zero even though Voltage  $V$  is present.

$$\therefore P_{\text{AC}} = \left(\frac{I_m}{2}\right)^2 (R_f + R_L) = \left(\frac{I_m}{2}\right)^2 \times R_L$$

$$\therefore \frac{P_{\text{DC}}}{P_{\text{AC}}} = \frac{I_m^2 \cdot R_L \times 4}{\pi^2 I_m^2 \times R_L} = \frac{4}{\pi^2} = 0.406$$

$$\therefore \text{Ratio of rectification for HWR} = 0.406$$

Considering ideal diode. If we consider  $R_f$  also, the expression =  $\frac{0.406}{1 + \left(\frac{R_f}{R_L}\right)}$

The AC input power is not converted to D.C. Only part of it is converted to D.C and is dissipated in the load. The balance of power is also dissipated in the load itself as AC power. We have to consider the rating of the secondary of the transformer. In ratio of rectification we have considered only the A.C output as the secondary of the transformer.

**1.15.8 TRANSFORMER UTILIZATION FACTOR**

$$\begin{aligned} \text{Transformer Utilization Factor (TUF)} &= \frac{\text{D.C power delivered to the load}}{\text{AC rating of transformer secondary}} \\ &= \frac{P_{\text{DC}}}{P_{\text{AC rated}}} \end{aligned}$$

This term TUF is not ratio of rectification, because all the rated current of the secondary is not being drawn by the circuit.

$$P_{\text{ac}} = I_{\text{DC}}^2 \cdot R_L - \left(\frac{I_m}{\pi}\right)^2 \cdot R_L$$

$$P_{\text{ac}} = V_{\text{rms}} \times I_{\text{rms}}$$

$$V_{\text{rms}} = \frac{V_m}{\sqrt{2}} = \text{Rated voltage of (Secondary Transformer)}$$

$$I_{\text{rms}} = I_m/2$$

$$P_{\text{ac}} = V_{\text{rms}} \times I_{\text{rms}}$$

But  $V_m = I_m (R_f + R_L) \simeq I_m R_L$  ( $R_f$  is small )

$$\text{TUF} = \frac{I_m^2}{\pi^2} \times R_L / \left( \frac{I_m R_L}{\sqrt{2}} \times \frac{I_m}{2} \right) = \frac{2\sqrt{2}}{\pi^2} = 0.287$$

Transformer Utilization Factor for Half Wave Rectifier is 0.287

### 1.15.9 DISADVANTAGES OF HALF WAVE RECTIFIER

1. High Ripple Factor (1.21)
2. Low ratio of rectification (0.406)
3. Low TUF (0.287)
4. D.C saturation of transformer.

For half wave configuration,

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi} \int_0^{\pi} I_m^2 \sin^2 \alpha \, d\alpha} = \frac{I_m}{2}$$

Because during negative half cycle the diode will not conduct hence  $i = 0$  in the loop from  $\pi - 2\pi$ . So integration is from  $0 - \pi$  only. Even though  $V$  is present,  $i = 0$  for one half cycle. Therefore, Power is zero ( $V \times I = P = 0$ , since,  $I = 0$ )

$$I_{\text{DC}} = \frac{1}{2\pi} \int_0^{\pi} I_m \sin \alpha \, d\alpha = \frac{I_m}{2\pi} [-\cos \alpha]_0^{\pi} = \frac{I_m}{2\pi} |1 + 1| = \frac{I_m}{\pi}$$

$$\therefore \text{Ripple Factor for Half Wave Rectification } \frac{I_{\text{rms}}}{I_{\text{DC}}} = \frac{I_m/2}{I_m/\pi} = \frac{\pi}{2}$$

$$\gamma = \sqrt{(1.57)^2 - 1} = 1.21$$

$$\therefore \frac{\pi}{2} = 1.57$$

$$\therefore \gamma > 1$$

So the ripple voltage exceeds D.C voltage. Hence HWR is a poor circuit for converting AC to DC.

### 1.15.10 POWER SUPPLY SPECIFICATIONS

The input characteristics which must be specified for a power supply are :

1. The required output D.C voltage
2. Regulation
3. Average and peak currents in each diode
4. Peak inverse voltage (PIV)
5. Ripple factor.

### 1.16 RIPPLE FACTOR

The purpose of a rectifier circuit is to convert A.C. to D.C. But the simple circuit shown before will not achieve this. Rectifier converts A.C. to unidirectional flow and not D.C. So filters are used to get pure D.C. Filters convert unidirectional flow into D.C. Ripple factor is a measure of the fluctuating components present in rectifier circuits.

$$\text{Ripple factor, } \gamma = \frac{\text{RMS Value of alternating components of the waveform}}{\text{Average Value of the waveform}}$$

$$\gamma = \frac{I'_{\text{rms}}}{I_{\text{DC}}} = \frac{V'_{\text{rms}}}{V_{\text{DC}}}$$

$I'_{\text{rms}}$  and  $V'_{\text{rms}}$  denote the value of the A.C components of the current and voltage in the output respectively. While determining the ripple factor of a given rectifier system experimentally, a voltmeter or ammeter which can respond to high frequencies ( greater than power supply frequency



50 Hz ) should be used and a capacitor should be connected in series with the input meter in order to block the D.C. component. Ripple factor should be small. ( Total current  $i = I_m \sin \omega t$  according to Fourier Series, only A.C. is sum of D.C. and harmonics ).

We shall now derive the expression for the ripple factor. The instantaneous current is given by  $i' = i - I_{DC}$ .

$i$  is the total current. In a half wave rectifier, some D.C components are also present. Hence A.C component is,

$$i' = (i - I_{DC}) [ ( \text{Total current} - I_{DC} ) ]$$

$$\text{RMS value is } I'_{\text{rms}} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (i - I_{DC})^2 d\alpha}$$

$$\therefore I'_{\text{rms}} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (i^2 - 2I_{DC} \cdot i + I_{DC}^2) d\alpha}$$

Now,  $\frac{1}{2\pi} \int_0^{2\pi} i^2 d\alpha = \text{Square of the rms value of a sine wave by definition.}$   
 $= (I_{\text{rms}})^2$

$$\frac{1}{2\pi} \int_0^{2\pi} i \cdot d\alpha = \text{The average value or D.C value } I_{DC}$$

$I_{DC}$  is constant. So taking this term outside,

$$\frac{I_{DC}^2}{2\pi} \int_0^{2\pi} d\alpha = \frac{I_{DC}}{2\pi} [2\pi]$$

$$\therefore I'_{\text{rms}} = \sqrt{(I_{\text{rms}})^2 - 2I_{DC}^2 + I_{DC}^2}$$

The rms ripple current is

$$I'_{\text{rms}} = \sqrt{(I_{\text{rms}})^2 - I_{DC}^2}$$

Ripple factor,  $\gamma = \frac{I'_{\text{rms}}}{I_{dc}} = \frac{\sqrt{(I_{\text{rms}})^2 - I_{DC}^2}}{I_{DC}}$

$$\gamma = \sqrt{\left(\frac{I_{\text{rms}}}{I_{DC}}\right)^2 - 1} \dots\dots\dots ( 1.28 )$$

This is independent of the current waveshape and is not restricted to a half wave configuration. If a capacitor is used to block D.C. and then  $I_{\text{rms}}$  or  $V_{\text{rms}}$  is measured,

Then,  $\gamma = \frac{I_{rms}}{I_{DC}}$

$\therefore I'_{rms} = \sqrt{I_{rms}^2 - I_{DC}^2}$

and  $I_{DC} = 0$  (blocked by Capacitor) output waveform

For Half Wave Rectifier Circuit ( HWR ),

$$I = I_m \sin \alpha$$

$$I_{av} = I_{DC} = \frac{1}{2\pi} \int_0^{\pi} I_m \sin \alpha \cdot d\alpha = \frac{I_m}{\pi}$$

$$I_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{\pi} I_m^2 \sin^2(\alpha) d\alpha} = \frac{I_m}{2}$$

$$I_{DC} = \frac{I_m}{\pi}; V_{DC} = \frac{V_m}{\pi} \quad \text{Peak Inverse Voltage ( PIV )} = V_m$$

Ripple Factor  $\gamma = \frac{\text{RMS value of ripple current}}{\text{Average value of the current}} = \frac{I'_{rms}}{I_{DC}}$

Total current  $I = I_{DC} + I'(\text{ripple})$   
 $I'(\text{ripple}) = (I - I_{DC})$

$$I'_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{\pi} (I - I_{DC})^2 d\alpha} = \sqrt{\frac{1}{2\pi} \int_0^{\pi} (I^2 - 2I \cdot I_{DC} + I_{DC}^2) \cdot d\alpha}$$

$$= \sqrt{(I_{rms})^2 - 2I_{DC}^2 + I_{DC}^2}$$

$$\therefore I'_{rms} = \sqrt{(I_{rms})^2 - I_{DC}^2}$$

$$\gamma = \frac{I'_{rms}}{I_{DC}} = \sqrt{\left(\frac{I_{rms}}{I_{DC}}\right)^2 - 1}$$

for Half Wave Rectifier,  $\gamma = \sqrt{\left(\frac{I_m \times \pi}{2 \times I_m}\right)^2 - 1} = \sqrt{\left(\frac{\pi}{2}\right)^2 - 1} = 1.21$

### 1.17 FULL WAVE RECTIFIER ( FWR )

Since half wave rectifier circuit has poor ripple factor, for ripple voltage is greater than DC voltage, it cannot be used. So now analyze a full wave rectifier circuit.

The circuit is as shown in Fig. 1.22.

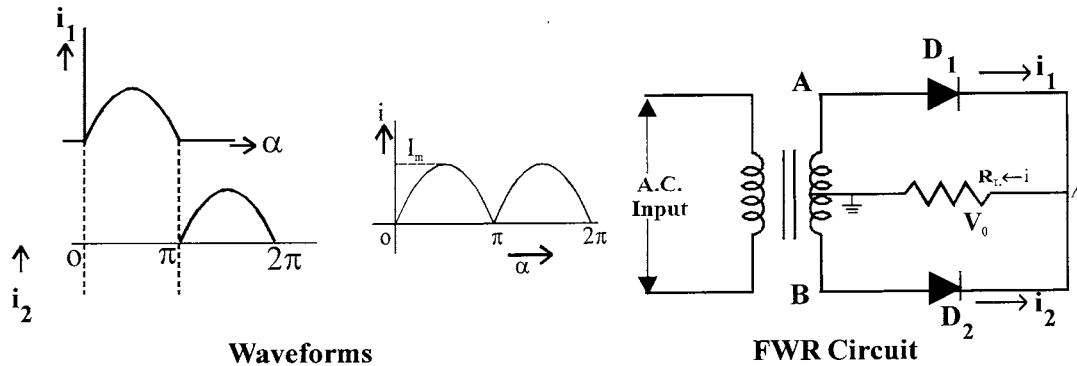


Fig 1.22

During the +ve half cycle,  $D_1$  conducts and the current through  $D_2$  is zero  
 During the -ve half cycle,  $D_2$  conducts and the current through  $D_1$  is zero  
 A centre tapped transformer is used.

The total current  $i$  flows through the load resistor  $R_L$  and the output voltage  $V_0$  is taken across  $R_L$ .

For half wave rectifier, circuit, 
$$I_{DC} = \frac{1}{2\pi} \int_0^{2\pi} I_m \sin \alpha d\alpha = \frac{I_m}{\pi}$$

For full wave rectifier, circuit,  $I_{DC}$  = Twice that of half wave rectifier circuit

$$\therefore I_{DC} = 2 I_m / \pi$$

For half wave rectifier circuit, 
$$I_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{\pi} I_m^2 \sin^2 \alpha d\alpha} = \frac{I_m}{2}$$

For full wave rectifier circuit, 
$$I_{rms} = \sqrt{2 \times \frac{1}{2\pi} \int_0^{\pi} I_m^2 \sin^2 \alpha d\alpha}$$

$$I_{rms} = \sqrt{2} \times \frac{I_m}{2} = \frac{I_m}{\sqrt{2}}$$

$$I_m = \frac{V_m}{R_f + R_L}$$

$R_f$  is the forward resistance of each diode.

A centre tapped transformer is essential to get full wave rectification. So there is a phase shift of  $180^\circ$ , because of centre tapping. So  $D_1$  is forward biased during the input cycle of  $0$  to  $\pi$ ,  $D_2$  is forward biased during the period  $\pi$  to  $2\pi$  since the input to  $D_2$  has a phase shift of  $180^\circ$  compared to the input to  $D_1$ . So positive half cycle for  $D_2$  starts at a full wave rectifier circuit, while the D.C. current starts through the load resistance  $R_L$  is twice that of the Half Wave Rectifier Circuit.

Therefore, for Half Wave Rectifier Circuit,

$$I_{DC} = \frac{I_m}{\pi}$$

For Full Wave Rectifier Circuit,

$$I_{DC} = \frac{2I_m}{\pi}$$

Hence, Ripple Factor is improved.

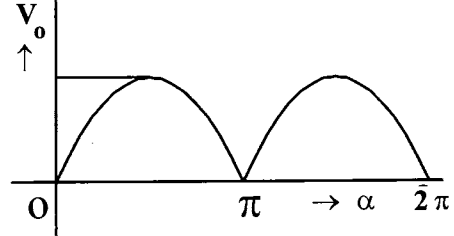


Fig 1.23 FWR voltage output.

### 1.17.1 RIPPLE FACTOR

$$\frac{I'_{rms}}{I_{DC}} = \frac{I_m/\sqrt{2}}{2I_m/\pi} = \frac{\pi}{2\sqrt{2}} = 1.11$$

$$I'_{rms} = \sqrt{I_{rms}^2 - I_{DC}^2}; \quad \gamma = \frac{\sqrt{I_{rms}^2 - I_{DC}^2}}{I_{DC}}$$

Therefore,  $\gamma = 1.21$  for HWR, and it is 0.482 for FWR.

### 1.17.2 REGULATION FOR FWR

The regulation indicates how D.C voltage varies as a function of load current. The percentage of regulation is defined as

$$\% \text{ Regulation} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100$$

The variation of DC voltage as a function of load current is as follows

$$\begin{aligned} V_{DC} &= I_{DC} R_L \\ &= \frac{2I_m}{\pi} \cdot R_L \left[ \because I_m = \frac{V_m}{R_f + R_L} \right] \\ &= \frac{2V_m}{\pi(R_f + R_L)} \cdot R_L \left[ \because I_m = \frac{V_m}{R_f + R_L} \right] \\ &= \frac{2V_m}{\pi} \left[ 1 - \frac{R_f}{R_f + R_L} \right] \\ &= \frac{2V_m}{\pi} - \frac{2V_m}{\pi(R_f + R_L)} \times R_f \end{aligned}$$

$$V_{DC} = \frac{2V_m}{\pi} - I_{DC} \cdot R_f$$

$$V_{NL} = \frac{V_m}{\pi}, \quad V_{FL} = \frac{V_m}{\pi} - I_{DC} \cdot R_f$$

$$\begin{aligned} \% \text{ Regulation} &= \frac{\frac{2V_m}{\pi} - \frac{2V_m}{\pi} + I_{DC} \cdot R_f}{\frac{2V_m}{\pi} - I_{DC} \cdot R_f} \times 100 \\ &= \frac{I_{DC} R_f}{\frac{2V_m}{\pi} - I_{DC} R_f} \times 100 \\ &= \frac{I_{DC} R_f}{I_{DC} R_f \left( \frac{2V_m}{\pi I_{DC} R_f} \right) - 1} \times 100 \\ &= \frac{1}{\left( \frac{2V_m}{\pi} \times \frac{(R_f + R_L)}{2V_m R_f} - 1 \right)} \times 100 \end{aligned}$$

$$\% \text{ Regulation} = \frac{R_f}{R_L} \times 100$$

**1.17.3 RATIO OF RECTIFICATION**

D.C. power delivered to Load  
 -----  
 A.C. input power from transformer secondary

$$P_{DC} = I_{DC}^2 \times R_L \text{ for FWR,}$$

$$I_{DC} = \frac{2I_m}{\pi} = \frac{4I_m^2}{\pi^2} R_L.$$

$P_{AC}$  is what a Wattmeter would indicate if placed in the FWR Circuit with its voltage terminals connected across the transformer secondary.

$$\begin{aligned} P_{ac} &= (I_{rms})^2 \cdot (R_f + R_L); I_{rms} \text{ for FWR} = \frac{I_m}{\sqrt{2}} \\ &= \left( \frac{I_m}{\sqrt{2}} \right)^2 \cdot (R_f + R_L). \\ &= \frac{I_m^2}{2} (R_f + R_L). \end{aligned}$$

If we assume that  $R_f$  is the forward resistance of the diode is very small, compared to  $R_L$ .

$$R_f + R_L \cong R_L.$$

$$\therefore P_{AC} = \frac{I_m^2}{2} (R_L).$$

$$\text{Ratio of Rectification} = \frac{P_{DC}}{P_{AC}} = \frac{4I_m^2 R_L \times 2}{\pi^2 I_m^2 \times R_L} = \frac{8}{\pi^2} = 0.812$$

For HWR it is 0.406. Therefore, for FWR the ratio of rectification is twice that of HWR.

#### 1.17.4 TRANSFORMER UTILIZATION FACTOR

In fullwave rectifier using center-tapped transformer, the secondary current flows through each half separately in every half cycle. While the primary of transformer carries current continuously. Hence TUF is calculated for primary and secondary windings separately and then the average TUF is determined.

$$\begin{aligned} \text{Secondary TUF} &= \frac{\text{DC power delivered to load}}{\text{AC rating of transformer secondary}} \\ &= \frac{(I_{DC})^2 R_L}{V_{rms} I_{rms}} = \frac{\left(\frac{2 I_m}{\pi}\right)^2 \cdot R_L}{\frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}} \\ &= \frac{4 I_m^2}{\pi^2} \times \frac{2 R_L}{I_m^2 (R_f + R_L)} = \frac{8}{\pi^2} \times \frac{1}{1 + \frac{R_f}{R_L}} \end{aligned}$$

if  $R_f \ll R_L$ , then secondary TUF = 0.812.

The primary of the transformer in feeding two HWR's separately. These two HWR's work independently of each other but feed a common load. Therefore,

$$\text{TUF for primary} = 2 \times \text{TUF of HWR} = 2 \times \frac{0.287}{1 + \frac{R_f}{R_L}}$$

if  $R_f \ll R_L$ , then TUF for primary = 0.574

The average TUF for FWR using center tapped transformer

$$= \frac{\text{TUF of primary} + \text{TUF of secondary}}{2} = \frac{0.812 + 0.574}{2} = 0.693$$

Therefore, the transformer is utilized 69.3% in FWR using center tapped transformer.

#### 1.17.5 PEAK INVERSE VOLTAGE FOR FULL WAVE RECTIFIER

With reference to the FWR circuit, during the -ve half cycle,  $D_1$  is not conducting and  $D_2$  is conducting. Hence maximum voltage across  $R_L$  is  $V_m$ , since voltage is also present between A and O of the transformer, the total voltage across  $D_1 = V_m + V_m = 2V_m$  ( Fig. 1.22 ).

#### 1.17.6 D.C. SATURATION

In a FWR, the D.C. currents  $I_1$  and  $I_2$  flowing through the diodes  $D_1$  and  $D_2$  are in opposite direction and hence cancel each other. So there is no problem of D.C. current flowing through the core of the transformer and causing saturation of the magnetic flux in the core.

PIV is  $2 V_m$  for each diode in FWR circuit because, when  $D_1$  is conducting, the drop across it is zero. Voltage delivered to  $R_L$  is  $V_m$ .  $D_2$  is across  $R_L$ . During the same half cycle,  $D_2$  is not conducting. Therefore, peak voltage across it is  $V_m$  from the second half of the transformer. The total voltage across  $D_2$  is  $V_m + V_m = 2V_m$ .

### 1.18 BRIDGE RECTIFIERS

The circuit is shown in Fig. 1.24. During the positive half cycle,  $D_1$  and  $D_2$  are forward biased.  $D_3$  and  $D_4$  are open. So current will flow through  $D_1$  first and then through  $R_L$  and then through  $D_2$  back to the ground. During the -ve half cycle  $D_4$  and  $D_3$  are forward biased and they conduct. The current flows from  $D_3$  through  $R_L$  to  $D_4$ . Hence the direction of current is the same. So we get full wave rectified output.

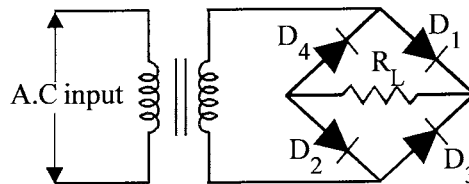


Fig 1.24 Bridge rectifier circuit.

In Bridge rectifier circuit, there is no need for centre tapped transformer. So the transformer secondary line to line voltage should be one half of that, used for the FWR circuit, employing two diodes.

$$I_{DC} = \frac{2I_m}{\pi}$$

where

$$I_m = \frac{V_m}{R_S + 2R_f + R_L}$$

$$V_{DC} = \frac{2V_m}{\pi} - I_{DC}(R_S + 2R_f)$$

$R_S$  = Resistance of Transformer Secondary.

$R_f$  = Forward Resistance of the Diode.

$R_L$  = Load Resistance.

$2R_f$  should be used since two diodes in series are conducting at the same time. The ripple factor and ratio of rectification are the same as for Full Wave Rectifier.

#### 1.18.1 ADVANTAGES OF BRIDGE RECTIFIER

1. The peak inverse voltage (PIV) across each diode is  $V_m$  and not  $2V_m$  as in the case of FWR. Hence the Voltage rating of the diodes can be less.
2. Centre tapped transformer is not required.
3. There is no D.C. Current flowing through the transformer since there is no centre tapping and the return path is to the ground. So the transformer utilization factor is high.

#### 1.18.2 DISADVANTAGES

1. Four diodes are to be used.
2. There is some voltage drop across each diode and so output voltage will be slightly less compared to FWR. But these factors are minor compared to the advantages.

Bridge rectifiers are available in a package with all the 4 diodes incorporated in one unit. It will have two terminals for A.C. Input and two terminals for DC output. Selenium rectifiers are also available as a package.

### 1.19 THE HARMONIC COMPONENTS IN RECTIFIER CIRCUIT

The analytical representation of output current wave of halfwave rectifier using fourier series is given by

$$i = I_m \left[ \frac{1}{\pi} + \frac{1}{2} \sin \omega t - \frac{2}{\pi} \sum_{k=\text{even}} \frac{\cos k\omega t}{(k+1)(k-1)} \right] \quad \dots(1.29)$$

The lowest angular frequency present in eq. (1.29) is of a.c source. And all other terms present in above equation are of even harmonic of power source.

Similarly in case of full wave rectifier in which one diode conducts for positive half cycle and the other diode conducting for negative half cycle and the net output current is difference of the currents conducted in positive half cycle ( $i_1(\alpha)$ ) and the current conducted in negative half cycle ( $i_2(\alpha)$ ). i.e.,

$$\text{Net current } i = i_1(\alpha) + i_2(\alpha) \quad (i_2(\alpha) = i_2(\alpha + \pi)) \quad \dots(1.30)$$

Here both  $i_1(\alpha)$  and  $i_2(\alpha)$  are same in magnitude but opposite in polarity. The difference  $i_1(\alpha) + i_2(\alpha)$  results in cancellation of lowest angular frequency of power source present in eq. (1.29). And hence the total current.

$i = i_1 + i_2$  attains of form

$$i = I_m \left[ \frac{2}{\pi} - \frac{4}{\pi} \sum_{\substack{k=\text{even} \\ k \neq 0}} \frac{\cos k\omega t}{(k+1)(k-1)} \right] \quad \dots(1.31)$$

Here in eq. (1.31) the fundamental frequency ' $\omega$ ' has been eliminated from the equation. And hence lowest frequency in the output being  $2\omega$ , i.e., the second harmonic term. This offers an definite advantage in the effectiveness of filtering the output.

The other advantage of fullwave rectifier is that the direction of the current during two halves of cycle is of opposite in nature, thus the magnetic cycle through which iron core is take is essentially that of the altering current and this eliminates d.c. saturation of the transformer, which would yield additional harmonics in the output.



1.20 INDUCTOR FILTER

FILTERS

A power supply must provide ripple free source of power from an A.C. line. But the output of a rectifier circuit contains ripple components in addition to a D.C. term. It is necessary to include a filter between the rectifier and the load in order to eliminate these ripple components. Ripple components are high frequency A.C. signals in the D.C. output of the rectifier. These are not desirable, so they must be filtered. So filter circuits are used.

Flux linkages per ampere of current  $L = \frac{N\phi}{I}$ . The ability of a component to develop induced voltage when alternating current is flowing through the element is the property of the inductor. Types of Inductors are :

- 1. Iron Cored
- 2. Air Cored

An inductor opposes any change of current in the circuit. So any sudden change that might occur in a circuit without an inductor are smoothed out with the presence of an inductor. In the case of AC, there is change in the magnitude of current with time.

Inductor is a short circuit for DC and offers some impedance for A.C. ( $X_L = j\omega L$ ). So it can be used as a filter. AC voltage is dropped across the inductors, whereas D.C. passes through it. Therefore, A.C is minimized in the output.

Inductor filter is used with FWR circuit. Therefore, HWR gives 121% ripple and using filter circuit for such high ripple factor has no meaning. FWR gives 48% ripple and by using filter circuit we can improve it. According to Fourier Analysis, current, 'i' in HWR Circuit is

$$i = \frac{I_m}{\pi} + \frac{I_m}{2} \sin \omega t - \frac{2I_m}{\pi} \frac{\cos 4\omega t}{3}$$

Current in the case of FWR is  $i \cong \frac{2I_m}{\pi} - \frac{4I_m}{3\pi} \cos 2\omega t - \frac{4I_m \cos 4\omega t}{15\pi}$

The reactance offered by inductor to higher order frequencies like  $4\omega t$  can be neglected. So the output current is

$$i \cong \frac{2I_m}{\pi} - \frac{4I_m}{3\pi} \cos 2\omega t$$

the fundamental harmonic  $\omega$  is eliminated.

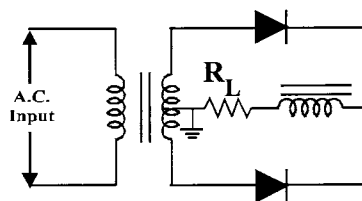


Fig 1.25 Inductor filter circuit.

The circuit for FWR with inductor filter is as shown in Fig. 1.25.

One winding of transformer can be used as 'L'. For simplification, diode and choke (inductor) resistances can be neglected, compared with  $R_L$ . The D.C. component of the current is

$$I_m = \frac{V_m}{R_L}$$

Impedance due to L and  $R_L$  in Series  $|Z| = \sqrt{R_L^2 + (2\omega L)^2}$

For second Harmonic, the frequency is  $2\omega$ .

Therefore, A.C. Component of current  $I_m = V_m / \sqrt{R_L^2 + 4\omega^2 L^2}$ . So substituting these values in the expression, for current,

$$I = \frac{2I_m}{\pi} - \frac{4I_m}{3\pi} \cos 2\omega t$$

by inductor to higher order frequencies like  $4\omega t$  etc., we get,

$$i = \frac{2V_m}{\pi R_L} - \frac{4V_m}{3\pi \sqrt{R_L^2 + 4\omega^2 L^2}} \cos(2\omega t - \theta)$$

where  $\theta$  is the angle by which the load current lags behind the voltage and is given by

$$\theta = \tan^{-1} \frac{2\omega L}{R_L}$$

### 1.20.1 RIPPLE FACTOR

$$\gamma = I_{r,rms} / I_{D.C.} = I_{r,rms} / I_{DC} = 2V_m / \pi R_L$$

$$I_{r,rms} = \frac{I_m}{\sqrt{2}} = \frac{4V_m}{3\sqrt{2}\pi \sqrt{R_L^2 + 4\omega^2 L^2}}$$

$$\begin{aligned} \therefore \text{Ripple factor} &= \frac{4V_m \times \pi R_L}{3\sqrt{2}\pi \sqrt{R_L^2 + 4\omega^2 L^2} \times 2V_m} \\ &= \frac{2}{3\sqrt{2}} \times \frac{i}{\sqrt{1 + \frac{4\omega^2 L^2}{R_L^2}}} \end{aligned}$$

$$\text{If } \frac{4\omega^2 L^2}{R_L^2} \gg 1, \text{ then}$$

$$\text{Ripple Factor } r = \frac{R_L \times 2}{3\sqrt{2} \times 2\omega L}$$

$$\therefore \text{Ripple factor} = \frac{R_L}{3\sqrt{2}\omega L} \quad \dots\dots\dots (1.32)$$

Therefore, for higher values of L, the ripple factor is low. If  $R_L$  is large, then also  $\gamma$  is high. Hence inductor filter should be used where the value of  $R_L$  is low. Suppose the output wave form from FWR supply is as shown in Fig. 1.26, then,

$V_{\gamma p-p}$  is the peak to peak value of the ripple voltage. Suppose

$$V_{DC} = 300V, \text{ and } V_{\gamma p-p} \text{ is } 10V.$$

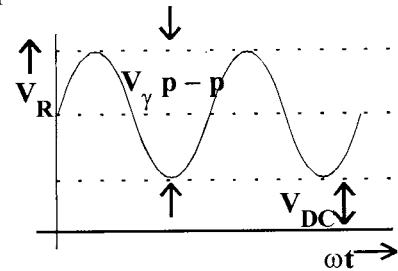


Fig 1.26 Ripple voltage  $V_{\gamma}$

then  $V_R$  maximum =  $\frac{10}{2} = 5V$ . Therefore,  $V_{\gamma rms} = \frac{V_{\gamma max}}{\sqrt{2}} = \frac{5}{\sqrt{2}} = 3.54V$

$$\therefore \text{Ripple Factor } \gamma = \frac{V_{r rms}}{V_x} = \frac{3.54}{300} = 0.0118.$$

$$\% \text{ ripple} = \text{Ripple factor} \times 100 \% = 1.18$$

**1.20.2 REGULATION**

$$V_{DC} = I_{DC} \cdot R_L; \quad I_{DC} \text{ for F W R is } \frac{2I_m}{\pi}$$

$$V_{DC} = \frac{2I_m R_L}{\pi} = \frac{2V_m}{\pi}$$

$$I_m \cdot R_L = V_m$$

Therefore,  $V_{DC}$  is constant irrespective of  $R_L$ . But this is true if  $L$  is ideal. In practice

$$V_{DC} = \frac{2V_m}{\pi} - I_{DC} \cdot R_f$$

where  $R_f$  is the resistance of diode.

An inductor stores magnetic energy when the current flowing through it is greater than the average value and releases this energy when the current is less than the average value.

Another formula that is used for Inductor Filter for

$$\text{Ripple Factor} = 0.236 \frac{R_C + R_L}{\omega_L} \dots\dots\dots ( 1.33 )$$

where  $R_L$  is load resistance  $R_C$  is the series resistance of the inductors. This is the same as the

above formula  $\frac{R_L}{3\sqrt{2}\omega_L}$

$$\therefore \frac{1}{3\sqrt{2}} = 0.236.$$

**Problem 1.18**

A FWR is used to supply power to a 2000Ω Load. Choke Inductors of 20 H inductance and capacitors of 16μf are available. Compute the ripple factor using 1. One Inductor filter 2. One capacitor filter 3. Single L type filter.

**Solution**

**1. One Inductor Filter :**

$$I = \frac{2I_m}{\pi} - \frac{4I_m}{3\pi} \cos (2\omega t - \phi ) \text{ where } \phi = \text{Tan}^{-1} \left( \frac{2\omega L}{R} \right)$$

$$I_{DC} = \frac{2V_m}{\pi R_L}$$

$$V = \frac{I \text{ ripple}}{I_{DC}} = \frac{R_L}{3\sqrt{2}\omega_L} = \frac{100}{3 \times 1.414 \times 2} = 0.074$$

### 2. Capacitor filter :

The ripple voltage for a capacitor filter is of Triangular waveform approximately. The rms

value for Triangular wave is  $\frac{E_d}{2\sqrt{3}}$  or  $\frac{V_{rms}}{2\sqrt{3}}$ .

$$\therefore E_{rms} = \frac{E_d}{2\sqrt{3}}$$

Suppose the discharging of the capacitor will cut one for one full half cycle .then the change last by the capacitor

$$Q = \omega t = I_{DC} \times \frac{1}{2f}$$

$$\therefore \text{voltage} = \frac{Q}{c}$$

$$\therefore E_d = \frac{I_{DC}}{2fc}$$

$$\gamma = \frac{I_{DC}}{2fc \times 2\sqrt{3} \times I_{DC}} = \frac{1}{4fc\sqrt{3}R_L} = \frac{1}{4 \times \sqrt{3} \times 50 \times 16 \times 10^{-6} \times 2000} = 0.09$$

### 3. L Type filter :

$$\gamma = \frac{\sqrt{2}}{3} \times \frac{1}{4\omega^2 LC} = 0.0037$$

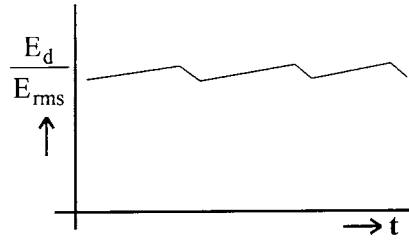


Fig 1.27  $V_o$  of capacitor filter.

### Problem 1.19

A diode whose internal resistance is  $20 \Omega$  is to supply power to a  $1000\Omega$  load from a  $110V$  ((rms) source of supply. Calculate (a) The peak load current. (b) The DC load current (c) AC Load Current (d) The DC diode voltage. (e) The total input power to the circuit. (f) % regulation from no load to the given load.

### Solution

Since only one diode is being used, it is for HWR Circuit.

$$(a) \quad I_m = \frac{V_m}{R_f + R_L} = \frac{110\sqrt{2}}{1020} = 152.5\text{mA.}$$

$$(b) \quad I_{DC} = \frac{I_m}{\pi} = \frac{152.5}{\pi} = 48.5\text{mA.}$$

$$(c) \quad I_{rms} = \frac{I_m}{2} = \frac{152.5}{2} = 76.2\text{mA.}$$

$$(d) \quad V_{dc} = \frac{-I_m \cdot R_L}{\pi} = \frac{-152.5 \times 1}{\pi} = -48.5\text{V.}$$

$$(e) \quad P_i = I_{rms}^2 \times (R_f + R_L) = 5.92\text{W.}$$

$$(f) \quad \% \text{ regulation} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\% = \frac{V_m - I_{DC} \cdot R_L}{I_{DC} \cdot R_L} = 2.06\%$$

**Problem 1.20**

Show that the maximum DC output power  $P_{DC} = V_{DC} I_{DC}$  in a half wave single phase circuit occurs when the load resistance equals the diode resistance  $R_F$ .

**Solution**

$$P_{DC} = I_{DC}^2 \cdot R_L = \frac{V_m^2 \cdot R_L}{\pi^2 (R_F + R_L)}$$

when  $R_L$  is very large,  $V_{DC} \cong \frac{V_m}{\pi}$ ;  $I_{DC} = \frac{V_{DC}}{R_L} = \frac{V_m}{\pi \cdot R_L}$

for  $P_{DC}$  to be maximum,  $\frac{dP_{DC}}{dR_L} = 0$ .

or  $\frac{V_m^2}{\pi^2} \left( \frac{(R_F + R_L)^2 - (R_F + R_L) \cdot R_L}{(R_F + R_L)^4} \right) = 0$ .

$$R_F + R_L = 2R_L \text{ or } R_L = R_F.$$

**Therefore,  $P_{DC}$  is maximum when  $R_L$  is equal to  $R_F$ .**

**Problem 1.21**

A 1mA meter whose resistance is  $10\Omega$  is calibrated to read rms volts when used in a bridge circuit with semiconductor diodes. The effective resistance of each element may be considered to be zero in the forward direction and  $\infty$  in the reverse direction. The sinusoidal input voltage is applied in series with a  $5 - K\Omega$  resistance. What is the full scale reading of this meter?

**Solution**

$$I_{DC} = \frac{2I_m}{\pi} \text{ for bridge rectifier circuit.}$$

$$I_m = \frac{V_m}{R_L};$$

$$V_m = \sqrt{2} V_{rms}$$

$$\therefore I_{DC} = \frac{2\sqrt{2} \times V_{rms}}{\pi R_L};$$

$$R_L = 5K\Omega + 10 \Omega = (5000 + 10) \Omega = 5010 \Omega$$

$$1mA = \frac{2\sqrt{2} \times V_{rms}}{\pi \times 5010}$$

$$\therefore V_{rms} = \frac{1 \times 10^{-3} \times \pi \times 5010}{2\sqrt{2}} = 5.56V.$$

**1.21 L-SECTION FILTER**

In an inductor filter, ripple decreases with increase in  $R_L$ . In a capacitor filter, ripple increases with increase in  $R_L$ . A combination of these two into a choke **input or L-Section Filter** should then make the ripple independent of load resistance.

If  $L$  is small, the capacitor will be charged to  $V_m$ , the diodes will be cut off allowing a short pulse of current. As  $L$  is increases, the pulses of current are smoothed and made to flow for a larger period, but at reduced amplitude. But for a critical value of inductance  $L_c$ , either one diode

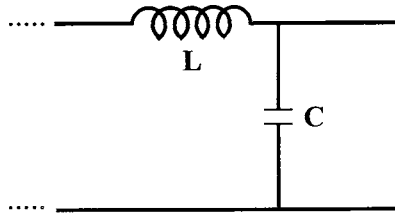


Fig 1.28 (a) Circuit.

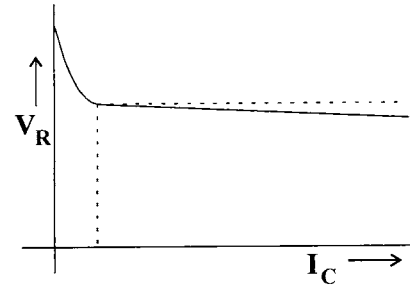


Fig 1.28 (b) LC filter characteristic.

or the other will be always conducting, with the result that the input voltage  $V$  and  $I$  to the filter are full wave rectified sine waves.

The circuit and graph of DC output voltage for LC Filter is as shown in Fig. 1.28(a) and (b) respectively.

$$V_{DC} = \frac{2V_m}{\pi}$$

for Full Wave Rectifier Circuit. Considering ideal elements, conduction angle is increased when inductor is placed because there is some drop across 'L'. So C cannot charge to  $V_m$ . Therefore, the diode will be forward biased for a longer period.

The ripple circuit which passes through 'L', is not allowed to develop much ripple voltage across  $R_L$ , if  $X_C$  at ripple frequency is small compared to  $R_L$ . Because the current will pass through C only. Since  $X_C$  is small and Capacitor will get charged to a constant voltage. So  $V_o$  across  $R_L$  will not vary or ripple will not be there. Since for a properly designed LC Filter,

$$X_C \ll R_L$$

and

$$X_L \gg X_C \quad \text{at } \omega = 2\pi f$$

$X_L$  should be greater than  $X_C$  because, all the AC should be dropped across  $X_L$  itself so that AC Voltage across C is nil and hence ripple is low.

### 1.21.1 RIPPLE FACTOR IN LC FILTER

AC Component of current through L is determined by  $X_L$ .

$$X_L = 2\omega L$$

RMS value of ripple current for Full Wave Rectifier with L Filter is,

$$I'_{rms} = \frac{4V_m}{3\pi\sqrt{2}X_L} = \text{RMS Value of Ripple Current for L Filter}$$

$$I'_{rms} = \frac{2}{3\sqrt{2}X_L} = \frac{2V_m}{\pi}$$

$$\therefore I'_{rms} = \frac{2}{3\sqrt{2}X_L} \cdot V_{DC} = \frac{\sqrt{2}V_{DC}}{3X_L}$$

The ripple voltage in the output is developed by the ripple current flowing through  $X_C$ .

$$\therefore V'_{rms} = I'_{rms} \cdot X_C = \frac{\sqrt{2}}{3} \cdot \frac{X_C}{X_L} \cdot V_{DC}$$

$$\text{Ripple factor} = \frac{V'_{\text{rms}}}{V_{\text{DC}}}$$

$$\gamma = \frac{\sqrt{2} \cdot X_C}{3X_L}$$

$$X_C = \frac{1}{j2\omega C} \cdot X_L = jL\omega L = 2\pi fL$$

$$\therefore \gamma = \frac{\sqrt{2}}{3} \cdot \frac{1}{2\omega C} \cdot \frac{1}{2\omega L}$$

$X_L = 2\omega L$  since ripple is being considered at a frequency, twice the line frequency.

If  $f = 60 \text{ Hz}$ ,  $\gamma = \frac{0.83}{LC}$

**1.21.2 BLEEDER RESISTANCE**

For the LC filter, the graph between  $I_{\text{DC}}$  and  $V_{\text{DC}}$  is as shown in Fig. 1.29. For light loads, i.e., when the load current is small, the capacitor gets charged to the peak value and so the no load voltage is  $2V_m/\pi$ . As the load resistance is decreased,  $I_{\text{DC}}$  increases so the drop across other elements  $V_{\text{iz}}$  diodes and choke increases and so the average voltage across the capacitor will be less than the peak value of  $2V_m/\pi$ . The output voltage remains constant beyond a certain point  $I_B$  and so the regulation will be good. The voltage  $V_{\text{DC}}$  remains constant even if  $I_{\text{DC}}$  increases, because, the capacitor gets charged every time to a value just below the peak voltage, even though the drop across diode and choke increases. So for currents above  $I_B$ , the filter acts more like an inductor filter than C filter and so the regulation is good. Therefore, for LC filters, the load is chosen such that, the  $I_{\text{DC}} \geq I_B$ . The corresponding resistance is known as **Bleeder Resistance** ( $R_B$ ). So Bleeder Resistance is the value for which  $I_{\text{DC}} \geq I_B$  and good regulation is obtained, and conduction angle is  $180^\circ$ . When  $R_L = R_B$ , the conduction angle =  $180^\circ$ , and the current is continuous. So just as we have determined the critical inductance  $L_C$ , when  $R_L = R_B$ , the Bleeder Resistance,  $I_{\text{DC}}$  = the negative peak of the second harmonic term.

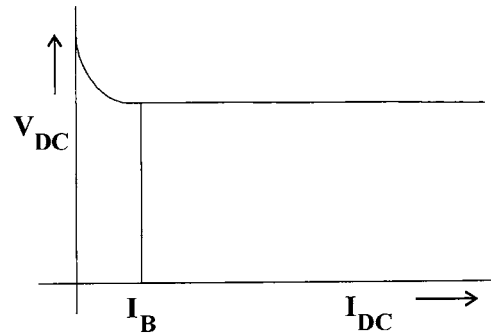


Fig 1.29 LC filter characteristic.

$$\therefore I_{\text{DC}} = \frac{2V_m}{\pi R_B}$$

$$I' \text{ peak} = \frac{4V_m}{3\pi} \cdot \frac{1}{X_L}$$

$$\therefore \frac{2V_m}{\pi R_B} = \frac{4V_m}{3\pi} \cdot \frac{1}{X_L}$$

$$R_B = \frac{3 \cdot X_L}{2}; \quad R_B \text{ is Bleeder Resistance.}$$

$I_{DC}$  should be equal to or greater than 'I' peak, since, ' $X_L$ ' determines the peak value of the ripple component. If ' $X_L$ ' is large 'I' peak can be  $\leq I_{DC}$  and so the ripple is negligible or pure D.C. is obtained, where  $X_L = 2\omega L$  corresponding to the second harmonic. Therefore, R should be at least equal to this value of  $R_B$ , the Bleeder Resistor.

### 1.21.3 SWINGING CHOKE

The value of the inductance in the LC circuits should be  $> L_C$  the minimum value so that conduction angle of the diodes is  $180^\circ$  and ripple is reduced. But if the current  $I_{DC}$  is large, then the inductance for air cored inductors as  $L = N\phi/I$  (flux linkages per ampere of current), as 'I' increases, 'L' decreases and may become below the critical inductance value  $L_C$ . Therefore, Iron cored inductors or chokes are chosen for filters such that the value of L varies within certain limits and when 'I' is large, the core saturates, and the inductance value will not be  $< L_C$ . Such chokes, whose inductance varies with current within permissible limits, are called **Swinging Chokes**. (In general for inductors  $\phi/I$  remains constant so that L is constant for any value of current I) This can be avoided by choosing very large value of L so that, even if current is large, Inductance is large enough  $> L_C$  But this increases the cost of the choke. Therefore, swinging choke are used.

$$L_C \geq R_L / 3\omega$$

$$R_L = \text{Load Resistance}$$

$$L_C = \text{Critical Inductance}$$

$$\omega = 2\pi f$$

At no load  $R_L = \infty$ . Therefore, L should be  $\infty$  which is not possible. Therefore, Bleeder Resistance of value  $= 3X_L / 2$  is connected in parallel with  $R_L$ , so that even when  $R_L$  is  $\infty$ , the conduction angle is  $180^\circ$  for each diode.

The inductance of an iron - cored inductor depends up on the D.C. current flowing through it, L is high at low currents and low at high currents. Thus its L varies or swings within certain limits. This is known as **Swinging Choke**.

Typical values are

$$L = 30 \text{ Henrys at } I = 20 \text{ mA and}$$

$$L = 4 \text{ Henrys at } I = 100 \text{ mA.}$$

### Problem 1.22

Design a full wave rectifier with an LC filter (single section) to provide 9V DC at 100mA with a maximum ripple of 2%. Line frequency  $f = 60 \text{ HZ}$ .

### Solution

$$\text{Ripple factor } \gamma = \frac{\sqrt{3}}{2} \times \frac{1}{2\omega C} \times \frac{1}{2\omega L} = \frac{0.83}{LC} \mu$$

$$\therefore 0.02 = \frac{0.83}{LC} \text{ or } LC = \frac{0.83}{0.02} = 42 \mu$$

$$R_L = \frac{V_{DC}}{I_{DC}} = \frac{9V}{0.1} = 90\Omega$$

$$LC \geq \frac{R_L}{3\omega} \geq \frac{R_L}{1130}$$



But LC should be 25% larger.  $\therefore$  for  $f = 60$  Hz, the value of LC should be  $\geq \frac{R_L}{900}$ .

$$LC \geq \frac{R_L}{900} \geq \frac{90}{900} = 0.1 \text{ Henry.}$$

If  $L = 0.1$ H, then  $C = \frac{42}{0.1} = 420\mu\text{f}$ . This is high value

If  $L = 1$ H, then  $C = 42 \mu\text{f}$ .

If the series resistance of L is assumed to be  $50 \Omega$ , the drop across L is

$$I_{DC} \times R = 0.1 \times 50 = 5\text{V.}$$

**Transformer Rating :**

$$V_{DC} = 9\text{V} + 5\text{V} = 14\text{V}$$

$$\therefore V_m = \frac{\pi}{2} (9 + 5) = 22\text{V}$$

$$\therefore \text{rms value is } \frac{22}{\sqrt{2}} = 15.5\text{V}$$

Therefore, a  $15.5 - 0 - 15.5$  V, 100mA transformer is required. PIV of the diodes is  $2V_m$  Because it is FWR Circuit.

$$\therefore \text{PIV} = 44\text{V}$$

So, diodes with 44V, 100mA ratings are required.

**Problem 1.23**

In a FWR with C filter circuit,  $V_r$ p-p = 0.8v and the maximum voltage is 8.8 volts (Fig. 1.30 ).  $R_L = 100\Omega$  and  $C = 1050 \mu\text{f}$ . Power line frequency = 60 Hz. Determine the Ripple Factor and DC output voltage from the graph and compare with calculated values.

**Solution**

$$V_r' \text{ p-p} = 0.8\text{V.}$$

$$\therefore V_{rms}' = \frac{0.8}{2\sqrt{3}} = 0.231\text{V}$$

$$V_{DC} = V_m - \frac{V_r' \text{ p-p}}{2} = 8.8 - \frac{0.8}{2} = 8.4\text{V.}$$

$$\therefore \gamma = \frac{V'_{rms}}{V_{DC}} = \frac{0.2}{8.4} = 0.0238 \text{ or } 2.38\%$$

$$\begin{aligned} \text{Theoretical values, } \gamma &= \frac{1}{4\sqrt{3fR_L \cdot C}} \\ &= \frac{1}{4\sqrt{3 \times 60 \times 100 \times 1050\mu\text{f}}} \\ &= 0.057 \text{ or } 5.75\% \end{aligned}$$

$$V_{DC} = \frac{4fR_L C}{1 + 4fR_L C} \times V_m = 8.46\text{V}$$

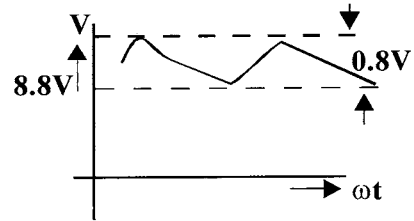


Fig 1.30 For problem 23

**1.22 π-SECTION FILTER**

In the LC filter or L section filter, there is some voltage drop across L. If this cannot be tolerated and more D.C. output voltage  $V_0$  is desired, pi or  $\pi$  filter or CLC filter is to be used. The ripple factor will be the same as that of L section filter, but the regulation will be poor. It can be regarded as a L section filter with  $L_1$  and  $C_1$ , before which there is a capacitor C, i.e. the capacitor filter. The input to the L section filter is the output of the capacitor filter C. The output of capacitor filter will be a triangular wave superimposed over D.C.

Now the output  $V_0$  is the voltage across the input capacitor C less by the drop across  $L_1$ . The ripple contained in the output of 'C' filter is reduced by the L section filter  $L_1 C_1$ .

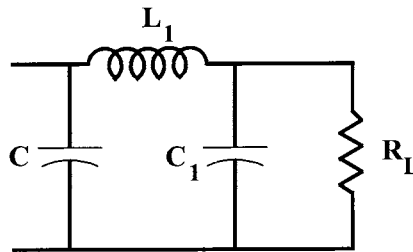


Fig 1.31 CLC or  $\pi$  section filter

**1.22.1 π-SECTION FILTER WITH A RESISTOR REPLACING THE INDUCTOR**

Consider the circuit shown here. It is a  $\pi$  filter with L replaced by R. If the resistor R is chosen equal to the reactance of L (the impedance in  $\Omega$  should be the same), the ripple remains unchanged.

$V_{DC}$  for a single capacitor filter =  $(V_m - I_{DC}/4fC)$  If you consider  $C_1$  and R, the output across  $C_1$  is  $(V_m - I_{DC}/4fC)$ . There is some drop across R. Therefore, the net output is

$$V_m - \frac{I_{DC}}{4fC} - I_{DC} \times R$$

The Ripple Factor for  $\pi$  sector is

$$\gamma = \frac{\sqrt{2}X_c X_{c1}}{R_L \cdot X_{L1}} \dots\dots\dots (1.34)$$

So by this, saving in the cost, weight and space of the choke are made . But this is practical only for low current power supplies.

Suppose a FWR output current is 100 mA and a 20 H choke is being used in  $\pi$  section filter. If this curve is to be replaced by a resistor,  $X_L = R$ . Taking the ripple frequency as  $2f = 100$  Hz,

$$\begin{aligned} X_L &= \omega L = 2\pi(2f).L = 4\pi fL \\ &= 4 \times 3.14 \times 50 \times 20 = 12560\Omega \\ &\approx 12 \text{ K}\Omega \end{aligned}$$

Voltage drop across R =  $12,000 \times 0.1 = 1200\text{V}$ , which is very large.

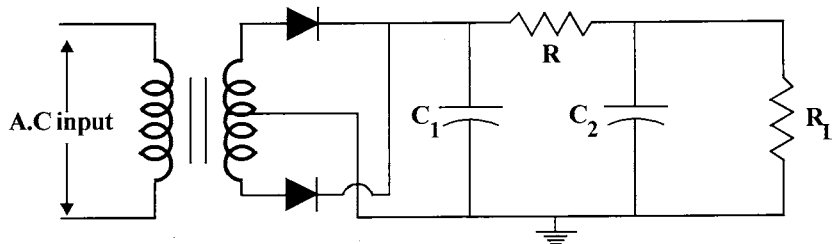


Fig 1.32 FWR circuit  $\pi$  section / CRC filter.

### 1.22.2 EXPRESSION FOR RIPPLE FACTOR OF $\pi$ -SECTION FILTER

The output of the capacitor in the case of a capacitor filter is a Triangular wave. So assuming the output  $V_o$  across C to be a Triangular wave, it can be represented by Fourier series as

$$V = V_{DC} - \frac{V'P - P}{\pi} \left( \sin 2\omega t - \frac{\sin 4\omega t}{2} + \frac{\sin 6\omega t}{3} \dots \right)$$

But the ripple voltage peak to peak  $V' p-p = \frac{I_{DC}}{2fc}$ . ( for  $\Delta$  wave )

Neglecting 4<sup>th</sup> and higher harmonic, the rms voltage of the Second Harmonic Ripple is

$$\begin{aligned} \frac{v' p-p}{\pi\sqrt{2}} &= \frac{I_{DC}}{2\sqrt{2}\pi fc} = \frac{I_{DC} \times 2}{4\sqrt{2}\pi fc} = \frac{\sqrt{2}I_{DC}}{4\pi fc} \\ \frac{1}{4\pi fc} &= X_c \end{aligned}$$

It is the capacitive reactance corresponding to the second harmonic.

$$V'_{rms} = \sqrt{2}I_{DC}X_c \quad (\text{across 'C' filter only})$$

The output of C filter is the input for the L section filter of  $L_1C_1$  . Therefore, as we have done in the case of L section filter,

$$\text{Current through the inductor ( harmonic component )} = \frac{\sqrt{2}I_{DC}X_c}{X_{L_1}}$$

$$\text{Output Voltage} = \text{Current} ( X_C ) \quad ( V_o = I.X_C )$$

$$V'_{rms} = \frac{\sqrt{2}I_{DC}X_c}{X_{L_1}}.X_{C_1} \quad (\text{after L section})$$

$$\text{Ripple factor, } \gamma = \frac{v'_{\text{rms}}}{V_{\text{DC}}} = \frac{(\sqrt{2} \cdot I_{\text{DC}} \cdot X_C) X_{C1}}{V_{\text{DC}} X_{L1}}$$

$$V_{\text{DC}} = I_{\text{DC}} \cdot R_L$$

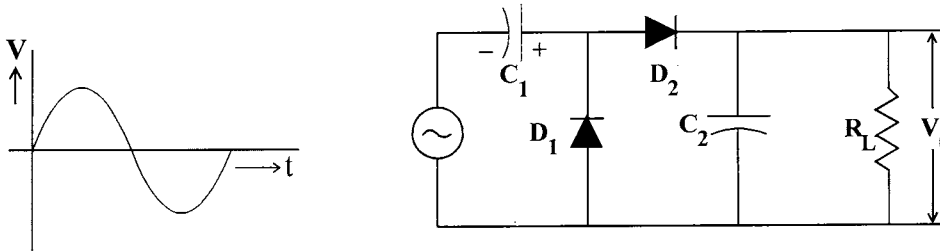
$$\therefore \gamma = \frac{(\sqrt{2} \cdot X_C)}{R_L} \left( \frac{X_{C1}}{X_{L1}} \right)$$

where all reactances are calculated at the 2nd harmonic frequency  $\omega = 2\pi f$ .

DC output voltage = ( The DC voltage for a capacitor filter ) - ( The drop across  $L_1$  ).

**Problem 1.24**

Design a power supply using a  $\pi$ -filter to give DC output of 25V at 100 mA with a ripple factor not to exceed 0.01%. Design of the circuit means, we have to determine L, C, diodes and transformers.



**Fig 1.33 Peak to peak detector.**

**Solution**

Design of the circuit means, we have to determine L, C, Diodes and Transformer

$$R_L = \frac{V_{\text{DC}}}{I_{\text{DC}}} = \frac{25\text{V}}{100\text{ mA}} = \frac{25\text{V}}{0.1\text{ A}} = 250\Omega$$

$$\text{Ripple factor } \gamma = \sqrt{2} \cdot \frac{X_C}{R_L} \cdot \frac{X_{C1}}{X_{L1}}$$

$X_C$  can be chosen to be =  $X_{C1}$

$$\therefore \gamma = \sqrt{2} \cdot \frac{X_C^2}{R_L \cdot X_{L1}}$$

This gives a relation between C and L.

$$C^2 L = y$$

There is no unique solution to this.

Assume a reasonable value of L which is commercially available and determine the corresponding value of capacitor. Suppose L is chosen as 20 H at 100 mA with a D.C. Resistance of  $375\Omega$  ( of Inductor ).

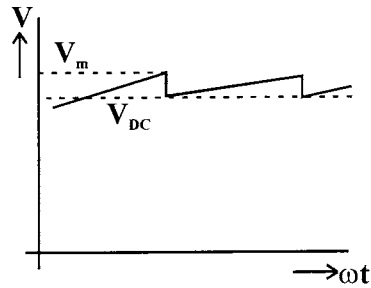


Fig 1.34 For problem 1.7.

$$\therefore c^2 = \frac{y}{L}$$

or 
$$c = \sqrt{\frac{y}{L}}$$

$$V_{DC} = V_m - \frac{V_\gamma}{2}$$

$$V_\gamma = \frac{I_{DC}}{2fc}$$

Now the transformer voltage ratings are to be chosen.

The voltage drop across the choke = choke resistance  $\times I_{DC} = 375 \times 100 \times 10^{-3} = 37.5V$ .

$$V_{DC} = 25V.$$

Therefore, voltage across the first capacitor C in the  $\pi$ -filter is

$$V_c = 25 + 37.5 = 62.5V.$$

The peak transformer voltage, to centre tap is

$$V_m = (V_c) + \frac{V_\gamma}{2} \text{ (for C filter)}$$

$$V_\gamma = \frac{I_{DC}}{2fc}$$

$$\therefore V_m = 62.5 V + \frac{0.1}{4 \times 50 \times c}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} \cong 60v$$

Therefore, a transformer with 60-0-60V is chosen . The ratings of the diode should be, current of 125 mA and voltage = PIV =  $2V_m = 2 \times 84.6V = 169.2V$ .

**Problem 1.25**

A full wave rectifier with LC filter is to supply 250 V at 100 mA. D.C. Determine the ratings of the needed diodes and transformer, the value of the bleeder resistor and the ripple, if  $R_C$  of the choke =  $400\Omega$ .  $L = 10\text{H}$  and  $C = 20\ \mu\text{F}$ .

**Solution**

$$R_L = \frac{V_{DC}}{I_{DC}}$$

$$R_L = \frac{250\text{V}}{0.1} = 2,500\Omega.$$

For the choke input resistor,

$$E_{DC} = \frac{2E_m}{\pi(1 + \frac{R_C}{R_L})}$$

and

$$I_{rms} \cong I_{DC}$$

$$E_m = \frac{\pi E_{DC}}{2} (1 + \frac{R_C}{R_L}) = \frac{\pi \times 250}{2} (1 + \frac{400}{2500}) = 455\text{V}$$

$$\therefore E_{rms} = \frac{455}{\sqrt{2}} = 322\text{V}$$

Therefore, the transformer should supply 322V rms on each side of the centre tap. This includes no allowance for transformer impedance, so that the transformer should be rated at about 340 volts 100 mA, D.C

$$\text{The Bleeder Resistance } R_B = \frac{3X_L}{2}$$

$$L = 10000\Omega.$$

$$I_B = \frac{2E_m}{3\pi\omega L} = \frac{2 \times 455}{3\pi \times 377 \times 10} = 0.0256\text{A}$$

$$\text{Ripple factor } \gamma = \frac{0.47}{4\omega^2 LC - 1} = \frac{0.47}{4 \times 377^2 \times 20 \times 10^{-6}}$$

The current ratings of each diode = 0.00413. Should be 50mA.

**1.23 USE OF ZENER DIODE AS REGULATOR**

Voltage Regulator Circuits are electronic circuits which give constant DC output voltage, irrespective of variations in *Input Voltage*  $V_i$ , *current drawn by the load*  $I_L$  from output terminals, and *Temperature*  $T$ . Voltage Regulator circuits are available in discrete form using BJTs, Diodes etc and in IC (Integrated Circuit) form. The term voltage regulator is used when the output delivered is DC voltage. The input can be DC which is not constant and fluctuating. If the input is AC, it is converted to DC by Rectifier and Filter Circuits and given to I.C. Voltage Regulator circuit, to get constant DC output voltage. If the input is A.C 230 V from mains, and the output desired is constant DC, a stepdown transformer is used and then Rectifier and filter circuits are used, before the electronic regulator circuit. The block diagrams are shown in Fig. 1.35 and 1.36.

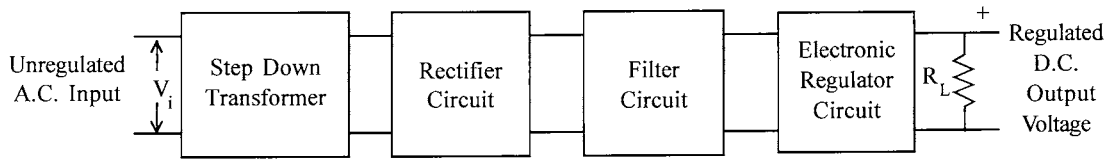


Fig. 1.35 Block diagram of voltage regulator with A.C input.

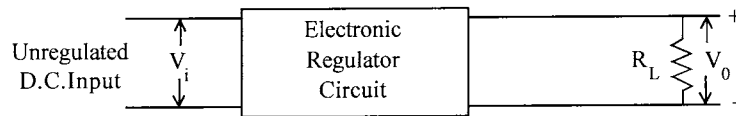


Fig. 1.36 Block diagram of voltage regulator with D.C input.

The term *Voltage Stabilizer* is used, if the output voltage is AC and not DC. The circuits used for voltage stabilizers are different. The voltage regulator circuits are available in IC form also. Some of the commonly used ICs are,  $\mu A$  723, LM 309, LM 105, CA 3085 A.

7805, 7806, 7808, 7812, 7815 : Three terminal positive Voltage Regulators.

7905, 7906, 7908, 7912, 7915 : Three terminal negative Voltage Regulators.

The Voltage Regulator Circuits are used for electronic systems, electronic circuits, IC circuits, etc.

The specifications and Ideal Values of Voltage Regulators are :

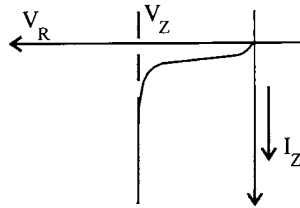
Specifications	Ideal Values
1. Regulation ( $S_V$ )	: 0 %
2. Input Resistance ( $R_i$ )	: $\infty$ ohms
3. Output Resistance ( $R_o$ )	: 0 ohms
4. Temperature Coefficient ( $S_T$ )	: 0 mv/oc.
5. Output Voltage $V_o$	: -
6. Output current range ( $I_L$ )	: -
7. Ripple Rejection	: 0 %

Different types of Voltage Regulators are

1. Zener regulator
2. Shunt regulator
3. Series regulator
4. Negative voltage regulator
5. Voltage regulator with foldback current limiting
6. Switching regulators
7. High Current regulator

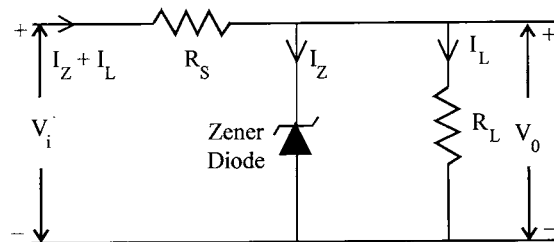
### 1.23.1 ZENER VOLTAGE REGULATOR CIRCUIT

A simple circuit without using any transistor is with a zener diode Voltage Regulator Circuit. In the reverse characteristic voltage remains constant irrespective of the current that is flowing through Zener diode. The voltage in the break down region remains constant (Fig. 1.37).



**Fig. 1.37 Zener diode reverse characteristic.**

Therefore in this region the zener diode can be used as a voltage regulator. If the output voltage is taken across the zener, even if the input voltage increases, the output voltage remains constant. The circuit as shown in Fig. 1.38.



**Fig. 1.38 Zener regulator circuit.**

The input  $V_i$  is DC. Zener diode is reverse biased.

If the input voltage  $V_i$  increases, the current through  $R_S$  increases. This extra current flows through the zener diode and not through  $R_L$ . Therefore zener diode resistance is much smaller than  $R_L$  when it is conducting. Therefore  $I_L$  remains constant and so  $V_0$  remains constant.

#### **The limitations of this circuits are**

1. The output voltage remains constant only when the input voltage is sufficiently large so that the voltage across the zener is  $V_Z$ .
2. There is limit to the maximum current that we can pass through the zener. If  $V_i$  is increased enormously,  $I_Z$  increases and hence breakdown will occur.
3. Voltage regulation is maintained only between these limits, the minimum current and the maximum permissible current through the zener diode. Typical values are from 10 mA to 1 ampere.

### 1.23.2 SHUNT REGULATOR

The shunt regulator uses a transistor to amplify the zener diode current and thus extending the Zener's current range by a factor equal to transistor  $h_{FE}$ . (Fig. 1.39)



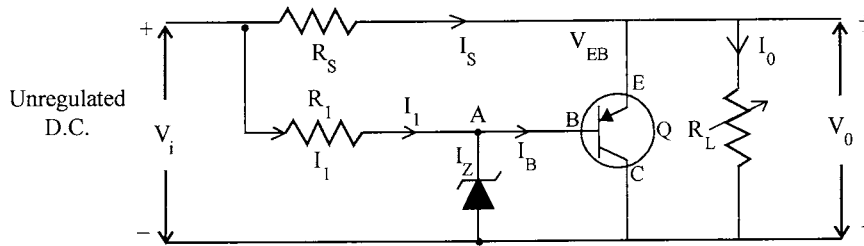


Fig. 1.39 Shunt regulator circuit

Zener current, passes through  $R_1$

Nominal output voltage

$$= V_Z + V_{EB}$$

The current that gets branched as  $I_B$  is amplified by the transistor. Therefore the total current  $I_0 = (\beta + 1) I_B$ , flows through the load resistance  $R_L$ . Therefore for a small current through the zener, large current flows through  $R_L$  and voltage remains constant. In other words, for large current through  $R_L$ ,  $V_0$  remains constant. Voltage  $V_0$  does not change with current.

**Problem 1.26**

For the shunt regulator shown, determine

1. The nominal voltage
2. Value of  $R_1$ ,
3. Load current range
4. Maximum transistor power dissipation.
5. The value of  $R_S$  and its power dissipation.

$V_i = \text{Constant}$ . Zener diode 6.3V, 200mW, requires 5mA minimum current.

**Transistor Specifications :**

$$V_{EB} = 0.2V, h_{FE} = 49, I_{CBO} = 0.$$

1. The nominal output voltage is the sum of the transistor  $V_{EB}$  and zener voltage.

$$V_0 = 0.2 + 6.3 = 6.5V = V_{EB} + V_Z$$

2.  $R_1$  must supply 5mA to the zener diode :

$$\therefore R_1 = \frac{8V - 6.3}{5 \times 10^{-3}} = \frac{1.7}{5 \times 10^{-3}} = 340 \Omega$$

3. The maximum allowable zener current is

$$\frac{\text{Power rating}}{\text{Voltage rating}} = \frac{0.2}{6.3} = 31.8 \text{ mA}$$

The load current range is the difference between minimum and maximum current through the shunt path provided by the transistor. At junction A, we can write,

$$I_B = I_Z - I_1$$

$I_1$  is constant at 5 mA

$$\therefore I_B = I_Z - I_1$$

$I_1$  is constant at 5 mA

$$\begin{aligned} \therefore I_B &= 5 \times 10^{-3} - 5 \times 10^{-3} = 0 \\ I_B (\text{min}) &= I_{Z \text{ Max}} - I_2 \\ &= 31.8 \times 10^{-3} - 5 \times 10^{-3} = 26.8 \text{ mA} \end{aligned}$$

The transistor emitter current  $I_E = I_B + I_C$

$$\begin{aligned} I_C &= \beta I_B = h_{FE} I_B \\ \therefore I_E &= (\beta + 1) I_B = (h_{FE} + 1) I_B \end{aligned}$$

$I_B$  ranges from a minimum of 0 to maximum of 26.8 mA

$$\begin{aligned} \therefore \text{Total load current range is } (h_{FE} + 1) I_B \\ = 50 (26.8 \times 10^{-3}) = \mathbf{1.34 \text{ A}} \end{aligned}$$

4. The maximum transistor power dissipation occurs when the current is maximum  $I_E \simeq I_C$

$$P_D = V_o I_E = 6.5 (1.34) = \mathbf{8.7 \text{ W}}$$

5.  $R_S$  must pass 1.34 A to supply current to the transistor and  $R_L$ .

$$R_S = \frac{V_i - V_o}{1.34} = \frac{8 - 6.5}{1.34} = 1.12 \Omega$$

The power dissipated by  $R_S$ ,

$$\begin{aligned} &= I_S^2 R_S \\ &= (1.34)^2 \cdot (1.12) = 3 \text{ W} \end{aligned}$$

### REGULATED POWER SUPPLY

An unregulated power supply consists of a transformer, a rectifier, and a filter. For such a circuit regulation will be very poor i.e. as the load varies (*load means load current*) [No load means no load current or 0 current. Full load means full load current or short circuit], we want the output voltage to remain constant. But this will not be so for unregulated power supply. The short comings of the circuits are :

1. Poor regulation
2. DC output voltage varies directly as the a.c. input voltage varies
3. In simple rectifiers and filter circuits, the d.c. output voltage varies with temperature also, if semiconductor devices are used.

An electronic feedback control circuit is used in conjunction with an unregulated power supply to overcome the above three short comings. Such a system is called a “*regulated power supply*”.

#### Stabilization

The output voltage depends upon the following factors in a power supply.

1. Input voltage  $V_i$
2. Load current  $I_L$
3. Temperature

$\therefore$  Change in the output voltage  $\Delta V_o$  can be expressed as

$$\Delta V_0 = \frac{\partial V_0}{\partial V_i} \cdot \Delta V_i + \frac{\partial V_0}{\partial I_L} \cdot \Delta I_L + \frac{\partial V_0}{\partial T} \cdot \Delta T$$

$$\Delta V_0 = S_V \Delta V_i + R_0 \Delta I_L + S_T \Delta T$$

Where the three coefficients are defined as,

- (i) **Stability factors.**

$$S_V = \left. \frac{\Delta V_0}{\Delta V_i} \right|_{\Delta I_L=0, \Delta T=0}$$

This should be as small as possible. Ideally 0 since  $V_0$  should not change even if  $V_i$  changes.

- (ii) **Output Resistance**

$$R_0 = \left. \frac{\Delta V_0}{\Delta I_L} \right|_{\Delta V_i=0, \Delta T=0}$$

- (iii) **Temperature Coefficient**

$$S_T = \left. \frac{\Delta V_0}{\Delta T} \right|_{\Delta V_i=0, \Delta I_L=0}$$

The smaller the values of the three coefficients, the better the circuit.

## SUMMARY

- ◆ Energy possessed by an electron rotating in an orbit with radius  $r$ ;

$$E = -\frac{e^2}{8\pi\epsilon_0 r}$$

- ◆ Expression for the radius of orbit,  $r = \frac{n^2 h^2 \epsilon_0}{\pi m e^2}$

- ◆ Expression for wavelength of emitted radiation,  $\lambda = \frac{12,400}{(E_2 - E_1)}$

- ◆ Types of Electronic Emissions from the surface.

1. Thermionic Emission
2. Secondary Emission
3. Photo electric Emission
4. High Field Emission

- ◆ Expression for Threshold Frequency  $f_T$  in photoelectric emission,

$$f_T = \frac{e\phi}{h}$$

- ◆ Fermi Level lies close to  $E_C$  in *n-type* semiconductor

- ◆ Fermi Level lies close to  $E_V$  in *p-type* semiconductor

- ◆  $\sigma = ne\mu_n + pe\mu_p$

- ◆ Hall Voltage,  $V_H = \frac{BI}{\rho\omega}$

- ◆ Expression for current through a *p-n junction*.

$$\text{Diode is } I = I_0 \left( e^{\frac{V}{\eta V_T}} - 1 \right)$$

- ◆ The three types of breakdown mechanisms in semiconductor diodes are

1. Avalanche Breakdown
2. Zener Breakdown
3. Thermal Breakdown.

- ◆ Rectifier circuit converts AC to Unidirectional Flow.

- ◆ Filter Circuit converts Unidirectional flow to DC. It minimises Ripple.

- ◆ The different types of filter circuits are

1. Capacitor Filter
2. Inductor Filter
3. L-Section ( LC ) Filter.
4.  $\pi$ -Section Filter
5. CRC and CLC Filters

- ◆ Ripple Factor =  $\frac{I'_{rms}}{I_{DC}}$ . For Half Wave Rectifier,  $\gamma = 1.21$ , for Full Wave Rectifier  $\gamma = 0.482$

- ◆ Expression for Ripple Factor for C Filter,

$$\gamma = \frac{1}{4\sqrt{3}fCR_L}$$

- ◆ Expression for Ripple Factor for L Filter,

$$\gamma = \frac{R_L}{4\sqrt{3}\omega L}$$

- ◆ Expression for Ripple Factor for LC Filter,

$$\gamma = \frac{\sqrt{2}}{3} \times \frac{1}{2\omega C} \times \frac{1}{2\omega L}$$

- ◆ Expression for Ripple Factor for  $\pi$  Filter,

$$\gamma = \frac{\sqrt{2}X_C}{R_L} \left( \frac{X_{C1}}{X_{L1}} \right)$$

- ◆ Critical Inductance  $L_C \geq \frac{R_L}{3\omega}$

- ◆ Bleeder Resistance  $R_B = \frac{3X_L}{2}$

**OBJECTIVE TYPE QUESTIONS**

1. Free electron concentration in semiconductors is of the order of .....
2. Insulators will have resistivity of the order of .....
3. Expression for J in the case of a semiconductor with concentrations n and p is .....
4. According to Law of Mass Action in semiconductors, .....
5. Intrinsic concentration depends on temperature T as .....
6. Depletion region width varies with reverse bias voltages as .....
7. In *p-type* semiconductor, Fermi Level lies close to .....
8. In *n-type* semiconductor, Fermi Level lies close to .....
9. The rate at which  $I_0$  changes with temperature in Silicon diode is .....
10. Value of Volt equivalent of Temperature at 25°C is .....
11. Einstein's relationship in semiconductors is .....
12. A very poor conductor of electricity is called .....

13. When donor impurities are added allowable energy levels are introduced a little ..... the ..... band.
14. Under thermal equilibrium, the product of the free negative and positive concentration is a constant independent of the amount of ..... doping and this relationship, called the mass action law and is given by  $np = \dots\dots\dots$
15. The unneutralized ions in the neighbourhood of the junction are referred to as ..... charges and this region depleted of mobile charges is called ..... charge region.
16. General expression for  $E_o$ , the contact difference of potential in an open circuited *p-n junction* in terms of  $N_C$ ,  $N_A$  and  $n_i$  is .....
17. Typical value of  $E_o = \dots\dots\dots$ eV
18. The equation governing the law of the junction is .....
19. The expression for the current in a forward biased diode is .....
20. The value of cut in voltage in the case of Germanium diode ..... and Silicon Diode ..... at room temperature.
21. Expression for  $V_B$  the barrier potential in terms of depletion region width  $W$  is  $V_B = \dots\dots\dots$
22. Expression for  $I_o$  in terms of temperature  $T$  and  $V_T$  is .....
23. Zener breakdown mechanism needs relatively ..... electric field compared to Avalanche Breakdown.
24. Rectifier Converts ..... to .....
25. Filter Circuit Converts ..... to .....
26. Ripple Factor in the case of Full Wave Rectifier Circuit is .....
27. The characteristics of a swinging choke is .....
28. In the case of LC Filter Circuits, Bleeder Resistance ensures
  - (1) .....
  - (2) .....

### ESSAY TYPE QUESTIONS

1. Explain the concept of 'hole'. How *n-type* and *p-type* semiconductors are formed? Explain.
2. Derive the expression for  $E_G$  in the case of intrinsic semiconductor.
3. Derive the expression for  $E_G$  in the case of *p-type* and *n-type* semiconductors.
4. With the help of necessary equations, Explain the terms Drift Current and Diffusion Current.
5. Explain about Hall Effect. Derive the expression for Hall Voltage. What are the applications of Hall Effect?
6. Distinguish between Thermistors and Sensistors.

7. Derive the expression for contact difference of potential  $V_0$  in an open circuited p-n junction.
8. Draw the forward and reverse characteristics of a p-n junction diode and explain them qualitatively.
9. Derive the expression for Transistor Capacitance  $C_T$  in the case of an abrupt p-n junction.
10. Compare Avalanche, Zener and Thermal Breakdown Mechanisms.
11. Derive the expression for  $E_0$  in the case of open circuited p-n junction diode.
12. Qualitatively explain the forward and reverse characteristic of p-n junction diode.
13. How junction capacitances come into existence in p-n junction diode.
14. Distinguish between Avalanche, Zener and Thermal Breakdown Mechanisms
15. Obtain the expression for ripple factor in the case of Full Wave Rectifier Circuit with Capacitor Filter.
16. Explain the terms Swinging Choke and Bleeder Resistor.
17. Compare C, L, L-Section,  $\pi$ -Section ( CLC and CRC ) Filters in all respects.

#### MULTIPLE CHOICE QUESTIONS

1. The force of electron 'F' between nucleus and electron with charge 'e' and radius 'r' is, proportional to,  $F \propto$ 
  - (a)  $\frac{e^2}{r^2}$
  - (b)  $\frac{r^2}{e^2}$
  - (c)  $e^2 r^2$
  - (d)  $\frac{e}{r}$
2. The energy possessed by the electron, W orbiting round the nucleus is,
  - (a)  $\frac{-e}{8\pi \epsilon_0 r}$
  - (b)  $\frac{-e^2}{8\pi \epsilon_0 r}$
  - (c)  $\frac{-e.r}{8\pi \epsilon_0}$
  - (d)  $\frac{-e^2}{8\pi \epsilon_0 r^2}$
3. The expression for the Kinetic Energy E of free electron in terms of momentum of the electron 'p' and mass of the electron m is,
  - (a)  $\frac{p}{2m}$
  - (b)  $\frac{p}{2m^2}$
  - (c)  $\frac{p^2}{2m^2}$
  - (d)  $\frac{p^2}{2m}$
4. The expression for radius of stable state 'e' r, is
  - (a)  $\frac{n^2 h^2 \epsilon_0}{\pi m e^2}$
  - (b)  $\frac{n h \epsilon_0}{\pi^2 m^2 e^2}$
  - (c)  $\frac{n^2 h \epsilon_0^2}{\pi m e^2}$
  - (d)  $\frac{n^2 h^2 \epsilon_0^2}{\pi m e^2}$
5. The value of the radius of the lowest state or ground state is, given,  $h = 6.626 \times 10^{-34}$  J - Secs  $\epsilon_0 = 10^{-9}/36\pi$ 
  - (a)  $0.28 \text{ \AA}$
  - (b)  $0.98 \text{ \AA}$
  - (c)  $0.18 \text{ \AA}$
  - (d)  $0.58 \text{ \AA}$
6. The energy required to detach an electron from its parent atom is called,
  - (a) Ionization potential
  - (b) Electric potential
  - (c) Kinetic energy
  - (d) Threshold potential

7. Electron collision without transfer of energy in collision is called,
- (a) Null collision (b) Impact collision  
(c) Elastic collision (d) Stiff collision
8. Wave mechanics in electron theory is also known as
- (a) Eienstein theory (b) Quantum mechanics  
(c) Bohr mechanics (d) Classical theory
9. The expression for threshold frequency  $f_T$  to cause photoelectric emission is, with usual notation is,
- (a)  $\frac{e\phi^2}{h}$  (b)  $\frac{e\phi}{h}$  (c)  $\frac{eh}{\phi}$  (d)  $\frac{e^2\phi^2}{h^2}$
10. The conductivity of a good conductor is typically
- (a)  $10^{-3} \Omega/\text{cm}$  (b)  $10^3 \Omega/\text{cm}$  (c)  $10^3 \text{ } \Omega/\text{cm}$  (d)  $10 \text{ } \Omega/\text{m}$
11. The free electron concentration of a good conductor is of the order of
- (a)  $10^{10}$  electrons/ $\text{m}^3$  (b)  $10^{28}$  electron/ $\text{m}^3$   
(c)  $10^2$  electrons/ $\text{cm}^3$  (d)  $10^{10}$  electrons/ $\text{m}^3$
12. Fermi level in Intrinsic semiconductor lies
- (a) close to conduction band (b) close to valence band  
(c) In the middle (d) None of the these
13. The potential which exists in a p-n junction to cause drift of charge carriers is called
- (a) contact potential (b) diffusion potential  
(c) ionisation potential (d) threshold potential
14. The rate of increase of reverse saturation current for Germanium diode is,
- (a) 5% (b) 4% (c) 1% (d) 7%
15. Special types of diodes in which transition time and storage time are made small are called ...
- (a) Snap diodes (b) Rectifier diodes (c) Storage diodes (d) Memory diodes
16. The Circuit which converts unidirectional flow to D.C. is called
- (a) Rectifier circuit (b) Converter circuit  
(c) filter circuit (d) Eliminator
17. For ideal Rectifier and filter circuits, % regulations must be ...
- (a) 1% (b) 0.1 % (c) 5% (d) 0%
18. The value of current that flows through  $R_L$  in a ' $\pi$ ' section filter circuit at no load is
- (a)  $\infty$  (b) 0.1 mA (c) 0 (d) few mA



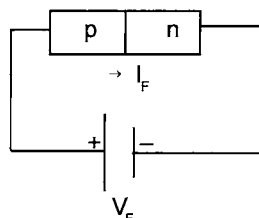
CONCEPTUAL QUESTIONS (Interview Questions)

- When it is said that an electron moves from valence band to conduction band, will there be physical movement of the electron ?

Ans : No, The energy band diagram represents the energy levels of the electrons. When an electron “moves” from valence band to conduction band, it only means that the energy possessed by the electron has increased from  $E_v$  electron volts to  $E_c$  electron volts. The energy diagram is only Pictorial representation of the energy possessed by the electrons. The energy levels of different electrons will be different.

- What does the arrow in the symbol of a P-n junction diode ( $\rightarrow$ ) indicate?

Ans : The arrow mark  $\begin{matrix} A & K \\ (p) & (n) \end{matrix}$  indicates the direction of current flow when the P - n junction is forward biased.



Direction of  $I_F$  is the same as that of the direction of arrow.

- When a Semiconductor device ‘breaks down’; what does it mean?

Ans : It does not mean physical breakdown. The shape of the device remains the same. But the device loses its Semiconductor properties. The device may become a short or open circuited.

- What is the difference between Avalanche and zener breakdown mechanisms?

Ans : In Zener breakdown mechanism, due to sudden increase in the electric field strength  $\epsilon$ , when it reaches a critical value, covalent bonds break and due to this there will be sudden increase in the number of free electrons and hence, the reverse current which was small till that time increases suddenly. But in Avalanche breakdown the field strength is not that high. A free electron with lot of energy collides with another electron, transfers its energy and makes it a free electron. This in turn generates some more free electrons. Thus there is multiplication of carriers. This will occur usually in a forward biased device. Avalanche breakdown precedes Zener breakdown, for comparison, though both cannot occur in the same device at a time.

- How for a low voltage like 6.8 V, in the case of Fz 6.8 Zener doide zener breakdown occurs, which requires high electric field strength?

Ans : Electric field strength  $\epsilon = \frac{V}{d}$ . Even if V is small, if ‘d’ is very small,  $\epsilon$  can be very

high. This is what happens in a Zener diode. Due to heavy doping depletion, region width near the junction becomes very small. So, when the voltage  $V_z$  reaches the required value, electric field ‘ $\epsilon$ ’ reaches the critical value, covalent bonds will be broken and Zener breakdown occurs.

*Specifications for Silicon Diode*

Sl.No.	Parameter	Symbol	Typical Value	Units
1.	Reverse breakdown voltage	$V_{br}$	75	V
2.	Static Reverse Current	$I_R$	5	$\mu A$
3.	Static Forward voltage (for silicon)	$V_F$	0.5	V
4.	Total Capacitance	$C_T$	2	pf
5.	Reverse Recovery Time	$t_{rr}$	4	ns
6.	Continuous Power Dissipation	P	500	mw
7.	Max. Forward Current	$I_F$	50	mA

*Some type numbers of Diodes*

1.	0A79	:	0	→	Semiconductor Device.
			A	→	Denotes Germanium device.
2.	BY127	:	B	→	Denotes Silicon Device.
			Y	→	Rectifying Diode.
			Number 127		is owing a type no and has significance.
3.	1N4153	:	N	→	Bipolar Device.
			1	→	Single Polar function.
4.	BY100:800V, 1A				Silicon Diode.

*Specifications of a junction diode*

Sl.No.	Parameter	Symbol	Typical Value	Units
1.	Working Inverse voltage	WIV	80	V
2.	Average Rectified current	$I_U$	100	A
3.	Continuous forward current	$I_F$	300	mA
4.	Peak repetitive forward current	$i_f$	400	mA
5.	Forward Voltage	$V_f$	0.6	V
6.	Reverse current	$I_R$	500	nA
7.	Breakdown voltage	BV	100	V

*Specifications of a Germanium Tunnel Diode IN 2939*

Sl.No.	Parameter	Symbol	Typical Value	Units
1.	Forward current	$I_F$	5	mA
2.	Reverse current	$I_R$	10	mA
3.	Peak current	$I_p$	1	mA
4.	Valley current	$I_v$	0.1	mA
5.	Peak voltage	$V_p$	50	V
6.	Valley voltage	$V_v$	30	V
7.	Ratio of $I_p$ to $I_v$	$I_p/I_v$	10	-