CHAPTER 1

BASIC MATHEMATICAL TECHNIQUES

1.1 Introduction

To understand automata theory, one must have a strong foundation about discrete mathematics. Discrete mathematics is a branch of mathematics dealing with sets, functions, relations, graphs and trees. It plays a vital role in the concepts of theoretical computer science.

In this chapter we discuss from set theory by covering basic notations, types of sets, various operations on sets. Also we will show how languages, grammar are generated from strings. Functions, relations and graphs are needed to prove the theorems, mathematical induction is more important as it is applied to many topics of theoretical computer science. Discrete mathematics is the study of mathematical structures and objects which are not continuous. It excludes continuous mathematics like calculus and analysis. Discrete mathematics is a branch of mathematics dealing with countable sets. The set of objects studies in discrete mathematics can be finite, hence it is also termed as finite mathematics. Concepts and notations of discrete mathematics are useful in studying computer algorithms, cryptography, automated theorem proving and software development.

Theoretical computer science is one of the areas that closely relates to discrete mathematics. It highly depends on graph theory, logic automatic theory and formal language which are related to computability set theory, information theory, game theory etc., that are also useful in computational mathematics.

1.2 Sets

(a) **Set:** A set is a collection of elements without any order. A set is described as a list of all its elements. If an element x belongs to a set x then it is written as $x \in X$.

There are three ways to represent a set.

1. ***Roster or tabulation method*:** In this method the elements of the set are listed within brackets separated by commas. This is also called as list notation. This is only suitable for the sets having finite elements.

*Ex:*1.1 $X_1 = \{1, 2, 3...\}$ set of integers

$X_2 = \{1, 3, 5...\}$ set of odd numbers

$X_3 = \{1, 2, 3, 5, 7...\}$ set of prime numbers

2. ***Set builder method***: In this method the property that characterizes the elements is defined

   *Ex:*1.2 $X_1 = \{x|x \text{ is an odd integer}\}$ set of odd numbers

   $X_2 = \{x \in E|x \text{ is a vowel from English alphabet } E\}$ set of vowels

   The notation "x|" is read as "the set of all x such that". This notation is also known as predicate notation. General form of this notation is $\{x|P(x)$ where P is some predicate$\}$.

3. ***Recursive method***: It defines set of rules to generate its members

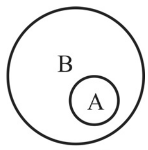
   *Ex:*1.3 The set E of even numbers greater than 2 is defined by a set of rules
   (a) $4 \in E$ (b) if $x \in E$, $x + 2 \in E$ (c) nothing else belongs to E

(b) Cardinality: The number of distinct elements in a set is called the cardinality

If set $A = \{1, 2, 3\}$ its cardinality is 3.

1.2.1 Types of Sets

(a) Sub set*:* If every element in a set A is also an element of set B, then A is called a subset of B. then it is written as $A \subseteq B$

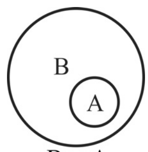


$A \subseteq B$

(b) Empty set: A set without elements is called empty. It is denoted by $A = \{\}$.

(c) Singleton set: A set containing only one element is called Singleton set. $X_1 = \{0\}$, $X_2 = \{x\}$ are single tons.

(d) Super set: if A is subset of B. Then B is superset of A. Which is written as $B \supset A$



$B \supset A$

(e) Universal set: A set U is called universal set if U is the super set of all the sets which are under our consideration.

(f) Finite set: If a set contains exactly n distinct elements i.e., if cardinality of a set is defined then it is a finite set.

(g) Power set: the power set of a set S is denoted by 2^S. If $S = \{x_1, x_2\}$, then its power set has $2^S = \{\varnothing, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}$

(h) Proper subsets: If two sets A and B are such that $A \subseteq B$ and $A \neq B$ then A is a proper subset. We write as $A \subseteq B$

(i) Equal sets: if two sets A and B have same elements, then they are equal $A = B$.

(j) Disjoint sets: If there are no common elements in two sets A and B, then they are disjoint. $A \cap B = \{\varnothing\}$

(k) Equivalent sets: If two sets have same cardinality then they are said to be equivalent A = {a, b, c} = N = {1, 2, 3} then A and N are equivalent sets.

1.2.2 Operations on Sets

(a) Union: It is the set of elements which belong to two sets A or B. It is denoted by $A \cup B$

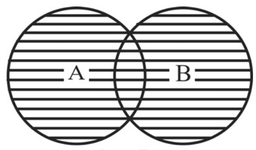
*Ex:*1.4 If $A = \{1, 3, 5\}$ $B = \{2, 4, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

$A \cup B = \{x|x \in A \text{ or } x \in B\}$

*Ex:*1.5 $A \cup A = A$

$A \cup \varnothing = A$



$A \cup B$

(b) Intersection: It is the set of elements which belong to both A and B. It is denoted by $A \cap B$

*Ex:*1.6 If $A = \{1, 3, 5\}$ $B = \{1, 2, 4\}$

$A \cap B = \{1\}$

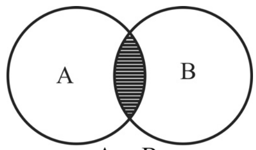
*Ex:*1.7 if $A = \{x|x \in \text{ even numbers}\}$

$B = \{x|x \in \text{ odd number}\}$

$A \cap B = \{\ \}$

$A \cap A = A$

$A \cap \varnothing = \varnothing$



$A \cap B$

(c) Complement: It is the set of elements which belong to U but do not belong to set A. It is denoted by A'. $A' = \{x|x \in U \text{ and } x \notin A\}$

*Ex:*1.8 If $U = \{x|x \in \text{ integers}\}$

$A = \{x|x \in \text{ even numbers}\}$

$A' = \{x|x \in \text{odd numbers}\}$

$A' = U - A$

(d) Set difference: It is the set of all elements in A and not in B. It is denoted by A – B.

$A - B = \{x|x \in A \text{ and } x \notin B\}$

*Ex:*1.9 If $A = \{a, c, d, g, h, i\}$

$B = \{a, b, c, e, f, g, i\}$

$A - B = \{d, h\}$

$B - A = \{b, e, f\}$

*Ex:*1.10 $A - \varnothing = A$, $\varnothing - A = \varnothing$

$A - A = \varnothing$

(e) Symmetric difference: The symmetric difference of two sets A and B is given by

$$A \,\Delta B = (A - B) \cup (B - A)$$

*Ex:*1.11 If $A = \{1, 3, 5, 7, 10\}$ $B = \{1, 2, 4, 6, 8, 9, 10\}$

$A - B = \{3, 7\}$ $B - A = \{2, 4, 6, 8, 9\}$

1.3 Set Identities

There are some general laws about sets which follow from the definitions of set theoretic generations, subsets etc. these equations below hold for any sets A, B and universal set U.

(a) Identity law

$$A \cup \varnothing = A,\ A \cap U = A$$

(b) Idempotent law

$$A \cup A = A,\ A \cap A = A$$

(c) Domination Law

$$A \cup U = U,\ A \cap \varnothing = \varnothing$$

(d) Complementation law

$$(A')' = A$$

(e) Commutative law

$$\mathbf{A \cup B = B \cup A, A \cap B = B \cap A}$$

(f) Distributive law

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(g) **De Morgon's law**

$$(A \cup B)' = A' \cap B'$$

$$(A \cap B)' = A' \cup B'$$

(h) Absorption laws

$$(A \cup B) \cap A = A$$

$$(A \cap B) \cup A = A$$

(i) Difference laws

$$A - B = A \cap B'$$

$$B - A = B \cap A'$$

(j) Symmetric difference laws

$$A \Delta B = (A - B) \cup (B - A)$$

$$B \Delta A = (B - A) \cup (A - B)$$

(k) Associative laws

$$(A \cup B) \cup C = A \cup (B \cup C), (A \cap B) \cap C = A \cap (B \cap C)$$

1.4 Relations between Sets

(a) Partitions of a set: If A is a non empty set, then that family of set $\{A_1, A_2 \ldots A_n\}$ is a partition of the set A if

(i) $A = \bigcup_{i=1}^{m} A_i$

(ii) $A_i \cap A_j = \varnothing$ if $i \neq j$

*Ex:*1.12 If $A = \{a, b, c, d, e, f, g, h\}$

$A_1 = \{a, b, c, f\}$, $A_2 = \{d, e, g, h\}$

Then $\{A_1, A_2\}$ is a partition of A

(b) Ordered pairs: The ordered pairs are the sets resulting from a set whose elements are ordered in particular fashion.

*Ex:*1.13 If $A = \{a, b\}$

$\langle a, b\rangle$ and $\langle b, a\rangle$ are ordered pairs $\langle a, b\rangle \neq \langle b, a\rangle$

(c) Duality principle: If A is any identity involving set, if B is a set obtained by replacing $\cap$ by $\cup$ and $\varnothing$ by U, and B is also true then B is called dual of A.

Duality of $A \cup (B \cap A) = A$ is $A \cap (B \cup A) = A$

(d) Cartesian product: The set of all ordered pairs of elements (x, y) if $x \in A$ and $y \in B$ is called cartesian product of sets A and B. It is denoted by $A \times B$.

$A \times B = \{(x, y) \mid x \in A, y \in B\}$

*Ex:*1.14 $A = \{1, 2\}$ $B = \{3, 4, 5\}$

$A \times B = \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}$

$B \times A = \{(3, 1), (3, 2), (4, 1), (4, 2), (5, 1) (5, 2)\}$

1.5 Closure Property of Sets

A set is closed with respect to that operation if the operation can always be completed with elements in the set. "Closure" is a property of a set on a given operation with elements in the set. "Closure" is a property of a set on a given operation

*Ex:*1.15 If A = {2, 4, 6, 8,….} it is closed with respect to addition because the sum of any two of them is another even number belong to A.

1.6 De Morgan's Laws

Let A and B be two sets, then

$$\left(\overline{A \cup B}\right) = \bar{A} \cap \bar{B}$$

$$\left(\overline{A \cap B}\right) = \bar{A} \cup \bar{B}$$

1.7 Relations

A relation from A to B is the subset of A × B. If R a relation from A to B, then R is a set of ordered pair in which first element belongs to A and second one to B. it is represented by a R b.

*Ex:*1.16 If N_1 and N_2 two sets of relations we can define R as a relation between N_1 and N_2 as N_1 is cube of N_2.

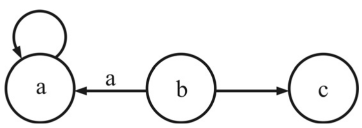
$$R = \{(1,1), (2, 8), (3, 27)..\}$$
$$= \{(a, b) \mid a \in N_1, b \in N_2 \text{ and } b = a^3\}$$

A relation may be represented by using matrices or graphs.

Ex: Consider two sets $X = \{1, 2, 3\}$ and $Y = \{a, b\}$, then the relation RXY is given by

$$R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \left|\begin{array}{cc} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{array}\right| \end{array}$$

Ex: Let $A = \{a, b, c\}$ is a set and a relation $R = \{(a, a), (b, a), (b, c)\}$ then the relation graph is



1.7.1 Types of Relations

(a) **Binary relation:** If A and B are two sets and $R \subseteq A \times B$ then we call R as binary relation from A to B. A relation $R \subseteq A \times A$ is called a relation on A.

(b) **Reflexive relation:** A relation on A is said to be reflexive if for each a $\in$ A, a is related to a.

*Ex:*1.17 If A = {a, b} R = {(a, a) (b, b)}

(c) **Symmetric relation:** A relation R on a set A is said to be symmetric if whenever a R b implies b R a, where (a, b) $\in$ A

Ex: If A = {1, 2, 3}

R = {(1, 3), (3, 1), (1, 2), (2, 1)}

(d) **Transitive relation:** A relation R on a set A is said to be transit if there are relations a R b and b R c, then we can say there is a relation a R c or

If (a, b) (b, c) $\in$ R

Then (a, c) $\in$ R

(e) **Equivalence relation:** A binary relation R on a set A will be known as equivalence relation if and only if

(a) R is reflexive relation

(b) R is symmetric relation

(c) R is transitive relation

1.7.2 Domain and Range of a Relation

If there is a relation a R b, $<a,b>\in R$, then dom R = $\{a \mid <a,b> \in R \text{ for some } b\}$ is called as domain and range R = $\{b \mid <a,b>\in R, \text{ for some } a\}$ is called range of relation.

1.8 Operation on Relations

(a) **Complement:** Complement of a relation $R \subseteq A \times B$ is defined as $R^1 = (A \times B) - R$

*Ex:*1.18 If $R = \{<a,d>, <a,e>, <b,c>\}$ and U = {a, b} × {c, d}

$R^1 = \{<a,c>, <b,d>, <b,e>\}$

We have U = {a, b} × {c, d, e}

$= \{<a,c>, <a,d>, <a,e>, <b,c>, <b,d>, <b,e>\}$

$\therefore$ $R^1 = \{<a,c>, <b,d>, <b,e>\}$ R complement

and $(R^{-1}) = \{<d,a>, <e,a>, <c,b>\}$ R inverse

$(R^{-1})^{-1} = R$

*Ex:*1.19 Let x = {0, 1, 2, 3….} set of natural numbers if R be "is less than" then what is R^{-1} ?

Since R is less than, R^{-1} is greater than, i.e., in $< a, b >$ a is greater than b.

$$R^{-1} = \{ < a, b > | < b, a > \in R \} \text{ where } a > b$$

(b) Inverse: The inverse of a relation $R \subseteq A \times B$ is defined as $R^{-1} = \{< b, a >| < a, b > \in R\}$

The relation from B to A which consists of those ordered pairs belong to R^{-1}.

*Ex:*1.20 Let $R = \{(a, 1), (b, 2), (c, 3)\}$

Then $R^{-1} = \{(1, a), (2, b), (3, c)\}$

1.8.1 Closures of Relations

(a) Reflexive closure: If R is relation on Set A, then $R \cup \Delta_A$ is reflexive closure of R where

$$\Delta_A = \{(a, a) \mid a \in A\}$$

To obtain the reflexive closure of a relation R we just add the diagonal relation elements to R (i.e., $\{(a, a) \mid a \in A\}$

(b) Symmetric closure: If R is relation on set A, then $R \cup R^{-1}$ is the symmetric closure of R where R^{-1} is the inverse of R.

*Ex:*1.21 If $R = \{ (a, a), (b, b), (a, b)\}$

Then $S = R \cup R^{-1}$ is symmetric closure

Hence $S = \{(a, a), (b, b), (a, b), (b, a)\}$

(c) Transitive closure: If R is a relation on set A, then R* is Transitive Closure, if (a, b) $\in$ R*, (b, c) $\in$ R*

Then (a, c) $\in$ R*. If there is a path between a one in relation R, we can say that

$$R^* \bigcup_{n=1}^{\infty} R^n$$

1.9 Function

A function relates exactly one output for each of its admissible inputs. A function is a process of correspondence between input and output.

A function f from set A to set B is a relation from A to B that satisfies following conditions

1. For each element in the domain of f (Set A) is paired with one element in range of f (set B) i.e., from $< a, b > \in f$ and $< a, c > \in f$ follows that b = c.

2. Dom f = A

   If f is a relation between a and b given by ordered pair (a, b), then f(a) = b, b is called the image of a under f. The image of the domain under f is called the range of f.

   Ex: Let A = {a,b} and B = {1, 2, 3}, then the following relations from A to B are functions from A to B

   $$P = \{<a,1>,<b,1>\}$$

   $$Q = \{<a,2>,<b,3>\}$$

   The notation F: $A \rightarrow B$ is used for "F is a function from A to B". A function maps each argument (domain of a function) into their corresponding value (range of a function)

1.9.1 Types of Functions

(a) Onto function: A function f from a set A to a set B is said to be onto, if and only if for every element in B, there is an element in A such that f(A) = B. Every element in B is mapped to at least one element of A.

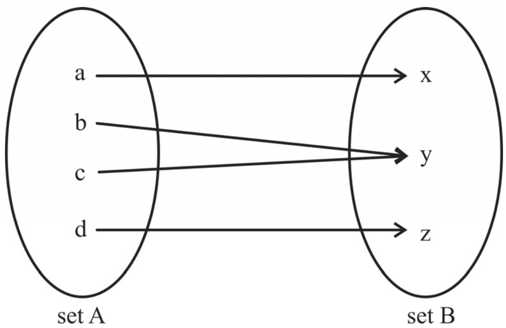


Fig. 1.1 Onto mapping

(b) Into function: A function f is said to be into function if there is at least one element of set B which is not mapped by any element of set A.

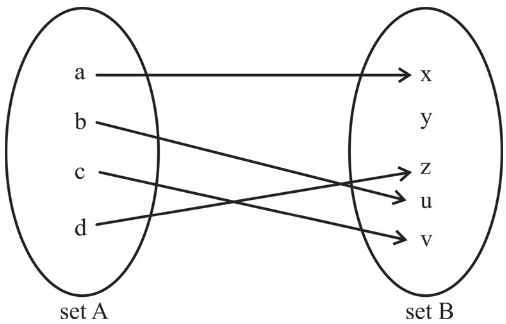


Fig. 1.2 Into mapping

(c) One to one function: A function f is said to be one to one if all the elements of set. A map to all distinct elements of set B is without mapping.

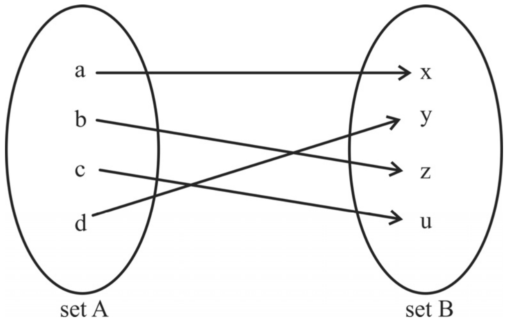


Fig. 1.3 One to one mapping

(d) One to One into function: A function f is said to be one to one function if it is both one to one and into function.

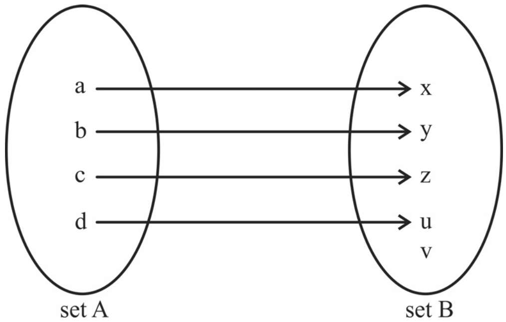


Fig. 1.4 One to one into mapping

(e) Many to one function: A function f is said to be many to one function if at least one element of co-domain B is mapped by two or more elements of domain A.

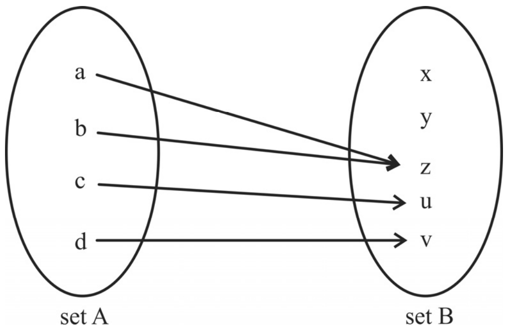


Fig. 1.5 Many to one mapping

(f) Many to one into function: A function f is said to be many to one into function if it is both many to one and into function. One or more elements of A map to some elements of B and some elements of B are not mapped by any elements of set A.

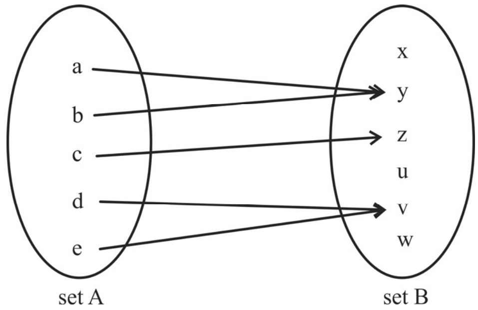


Fig. 1.6 Many to one into mapping

(g) Many to one onto function: A function f is said to be many to one onto function if it is both many to one and onto function. One element of set N is mapped by atleast one element of set A and two or more elements of A map to some elements of B.

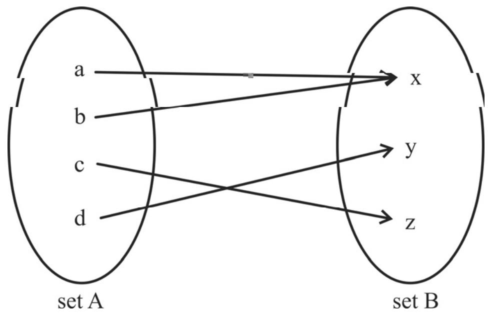


Fig. 1.7 Many to one onto mapping

(h) Constant function: A function f is said to be a constant function of every element of A maps to a single element of set B.

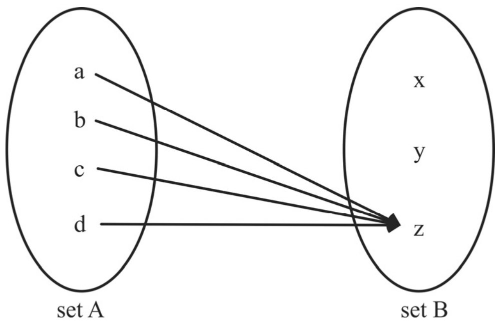


Fig. 1.8 Constant mapping

(i) Inverse function: A function f: $A \to B$ with another function g: $B \to A$ is said to be inverse function if each element $b \in B$ to a unique element $a \in A$ such that f(a) – B is called the inverse of f: $A \to B$

Solved Problems

1. If A = {0, 1} find the power set of A.

Ans: $P = 2^S = \{ \varnothing, \{0\}, \{1\}, \{0, 1\}$

2. Let A = {a, e, I, o, u}, B = {a, b, c…m} C = {n, o, p,….} and D = {d, I, s, c, r, e, t} find (a) $A \cup B$ (b) $B \cap C$ (c) A – B (d) B^i (e) (A – B) Δ (D – A) (f) $P(A \cap B)$

Ans: (a) $A \cup B$ = {a, b, c…,m, o, u}

(b) $B \cap C = \{\varnothing\}$

(c) A – B = {o, u}

(d) B^1 = {n, o, p,…z} = c

(e) $(A - B)\Delta(D - A) = \{o,u\}\Delta\{d,c,s,r,t\} = \{o,u\}$

(f) $P(A \cap B) = \{\varnothing$ {a}, {e}, {i}, {a, c}, {a, i}, {e, i} {a, e, i}}

3. Prove that

(a) $A \cap B = A$ (b) $A \cup B = B$ (c) $B' \subseteq A'$

Ans: (a) It is always true that $A \cap B \subseteq A$

Let $a \in A$, then $a \in B$ since $A \subseteq B$

So $a \in A \cup B$ thus $A \cap B = A$ since $A \subseteq A \cap B$

(b) Let $a \in A \cup B$, if $a \in B$, the inclusion is proved, otherwise $a \in A = A \cap B$, proving $a \in B$, thus $A \cup B \subseteq B$

(c) Let $a \in B'$, then $a \notin B = A \cup B$, so $a \notin A$ i.e., $a \in A'$ hence $B' \subseteq A'$

4. Show that

$$(A \cap B)\ [(B \cap C\ (C \cap D) \cup (C \cap D'))] = B \cap (A \cup C)$$

Ans:

$$\begin{aligned} LHS &= (A \cap B) \cup [B \cap C\ (C \cap D) \cup (C \cap D^1))] \\ &= (A \cap B) \cup [B \cap (C \cap (D \cup D')] \\ &= (A \cap B) \cup (B \cap C) \quad \text{(since } D \cup D' = U, C \cap U = C) \\ &= B \cap (A \cup C) \\ &= RHS \end{aligned}$$

5. If $A \cup B = A \cup C$ and $A \cap B = A \cap C$, prove that B = C

Ans:
$$
\begin{aligned}
B &= (A \cup B) \cap B \\
&= (A \cup C) \cap B \\
&= (A \cap B) \cup (C \cap B) \\
&= (A \cap C) \cup (B \cap C) \\
&= (A \cup B) \cap C \\
&= (A \cup C) \cap C \\
&= C \\
B &= C
\end{aligned}
$$

6. Using De Morgan's laws prove

$(\bar{A} \cap B) \cap (A \cup \bar{B}) \cap (A \cup C) = (A \cup \bar{B}) \cup (\bar{A} \cap (B \cup \bar{C})$

Ans: According to De Morgan's laws

$A \cup B = (\bar{A} \cap \bar{B}) = U - (\bar{A} \cap \bar{B})$

$$
\begin{aligned}
\text{L.H.S} &= \overline{(\bar{A} \cap B) \cap (A \cup \bar{B}) \cap (A \cup C)} \\
&= (\bar{A} \cap B) \cup \overline{(A \cup \bar{B})} \cup \overline{(A \cup C)} \\
&= (A \cup \bar{B}) \cup ((\bar{A} \cap B) \cup (\bar{A} \cap \bar{C})) \\
&= (A \cup \bar{B}) \cup (\bar{A} \cap (B \cup \bar{C}))
\end{aligned}
$$

7. If A = {a, b}* and B = {b, c}* then $A \cap B$ is ?

Ans: A = {a, b}* = {a, b, ab, ba, aa, bb…}

B = {b, c}* = {b, c, bc, cb, bb, cc…}

$\therefore$ $A \cap B$ = b*

8. What is the reflexive closure of the relation $\{(1, 2), (2, 3), (3, 4), (5, 4)\}$ on the set $\{1, 2, 3, 4, 5\}$

Ans: $\{(1, 1), (1, 2), (2, 2), (2, 3), (3, 3), (3, 4), (5, 5), (5, 4)\}$

9. If $X = \{a, b\}$ then number of relations from X to X are ______?

Ans: 4, they are $\{(a, a), (a, b), (b, a), (b, b)\}$

10. If $R = (a, b), (b, c), (c, a)$ find R^+

Ans: $R^2 = R \; O \; R = \{(a, c) \; (b, a)(c, b)\}$

$R^3 = R^2 \; O \; R = \{(a, a) \; (b, b)(c, c)\}$

$R^4 = R^3 \; O \; R = \{(a, b) \; (b, c)(c, a)\}$

$\therefore \; R^+ = R^2 \cup R^3 \cup R^4 = \{(a, b) \; (b, c) \; (c, a) \; (a, c)$
$(b, a) \; (c, b) \; (a, a) \; (b, b) \; (c, c)\}$

Summary

1. *Set*: It is a collection of elements without any structure or order.
2. *Cardinality*: Number of distinct elements in a set is called cardinality of the set.
3. *Empty set*: Set with no elements.
4. *Relation*: A relation R from A to B, is a set of ordered pair where each first element comes from A and each second element comes from B.
5. *Equivalence relation*: A binary relation R on a set A is an equivalence relation if and only if R is reflexive, symmetric and transitive.
6. *Singleton set*: A set containing only one element.
7. *Power set*: The power set of a set S is denoted by 2^S
   If $S = \{x_1, x_2\}$ then $2^S = \{\varnothing, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}$
8. *Function*: A function is a process of correspondence between input and output.
9. *Cartesian product*: The set of all ordered pairs of elements (x, y) if $x \in A$ and $y \in b$ is called Cartesian product.

Review Questions

1. What is a set? List out the operations performed on sets?

Ans: Refer Section 1.2.2 Page No. 3

2. Explain set identities with examples?

Ans: Refer Section 1.3 Page No. 4

3. What is a relation? What are different types of relations?

Ans: Refer Section 1.7 Page No. 6

4. What is a function? What are different types of functions?

Ans: Refer Section 1.9 Page No. 8

5. Give out closures of relations with examples?

Ans: Refer Section 1.8.1 Page No. 8

Objective Type Questions

1. $A \cap (B \cup A')$ is equal to

   (a) A' (b) B' (c) A (d) none

2. If $A = \{a, b\}^*$ and $B = b^*$, then $A - B$ is

   (a) (a, ab, ba)* (b) b*

   (c) a* (d) (a, ab, ba) (a, ab, ba)*

3. The function $f : R^+ \to R^+$ defined over $f(n) = n + x$ is

   (a) onto (b) into (c) one to one (d) none

4. If set X has m elements and set Y has n elements then how many relations are there from X to Y?

   (a) nm (b) 2^{mn} (c) 2^{m+n} (d) none

5. Power set of $\varnothing$ is

   (a) $\{\varnothing, \{\varnothing\}\}$ (b) $\{\varnothing\}$ (c) $\{\varnothing, \varnothing\varnothing\}$ (d) none

6. $A^* - A^+ =$

   (a) $\in$ (b) A (c) $\varnothing$ (d) none

7. If R is a non empty relation on collection of sets defined by ARB, if and only if $A \cap B = \varnothing$. Which is true

   (a) R is reflexive and transitive

   (b) R is symmetric and not transitive

   (c) R is an equivalence relation

   (d) R is not reflexive and not symmetric

8. If R = (a, b) (b, c) (c, a) then which is true

   (a) $R^2 = R^5$ (b) $R = R^4$

   (c) $R^3 = R^6$ (d) all the above

9. If R = (a, b) (b, c) (c, a) then R^+ is

   (a) $R \cup R^2 \cup R^3$ (b) $R \cap R^2 \cap R^3$

   (c) $R^2 \cup R^4 \cup R^6$ (d) none

10. If f and g are two functions on set x = {2, 3, 4} given by f = {(2, 3) (3, 4) (4, 2)} and g = {(2, 3) (3, 2) (4, 4)}, find fog

    (a) (2, 2) (3, 4) (4, 3)
    (b) (2, 2) (3, 3) (4, 4)
    (c) (2, 3) (3, 4) (4, 2)
    (d) none