Contents

Preface ..... (vii)
Symbols ..... (ix)
Standard Results ..... (xi)

UNIT - I | Differential Equations of First Order and First Degree

1.1 Introduction 2
1.2 Formation of an Ordinary D.E 3
1.3 The Differential Equations of the First Order and of the First Degree 9
1.3.1 Separation of Variables 9
1.3.2 Homogeneous Equations 13
1.3.3 Non-homogeneous Differential Equations 17
1.4 Linear Differential Equations 24
1.4.1 Non-linear Differential Equation of First Order 30
1.5 Exact Differential Equations 36
1.5.1 Integrating Factors 39
1.5.2 First Method to Find an Integrating Factor 40
1.5.3 Second Method to Find the Integrating Factor 41
1.5.4 Third Method to Find the Integrating Factor 43
1.5.5 Fourth Method to Find the Integrating Factor 45
1.5.6 Fifth Method for Finding Integrating Factor 47
1.6 Applications : Law of Natural Growth and Decay, Newton's Law of Cooling and Chemical Reactions 50
1.6.1 Orthogonal Trajectories 62
1.6.2 Orthogonal Trajectories : Cartesian Coordinates 62
1.6.3 Orthogonal Trajectories : Polar Coordinates 63
1.6.4 Applications of Electric Circuits 70
Summary **78**
Solved University Questions **80**
Objective Type Questions **119**

UNIT - II | Linear Differential Equations of Higher Order

2.1 Linear Differential Equations of Higher Order with Constant Coefficients 124
2.1.1 Complementary Function (C.F) 124
2.2 Particular Integral 130
2.2.1 Methods of Finding P.I. 134
2.3 Method of Variation of Parameters 151
2.4 Applications of Linear Differential Equation of Higher Order : LCR Circuits 158
2.5 Applications of Linear Differential Equation of Higher Order : Simple Harmonic Motion 164
Summary 172
Solved University Questions 174
Objective Type Questions 195

UNIT - III | Laplace Transforms

3.1 Introduction 204
3.1.1 Definition 204
3.1.2 Conditions for L.T to Exist 205
3.1.3 Linearity Property 205
3.1.4 First Shifting, Second Shifting and Change of Scale Properties 205
3.1.5 Laplace Transforms of Standard Functions 207
3.1.6 Multiplication by 't' 212
3.1.7 Division by 't' 214
3.1.8 Laplace Transform of Derivatives 217
3.1.9 Laplace Transform of Integrals 220
3.1.10 Laplace Transforms of Periodic Functions 223
3.1.11 Laplace Transform of the Unit Step Function (Heavisides unit function) 230
3.1.12 Heaviside Shift Theorem 230
3.1.13 Dirac Delta Function (Unit impulse function) 231
3.2 Inverse Laplace Transforms 234
3.2.1 Table of Inverse Transforms 236
3.2.2 Method of Partial Fractions 239
3.2.3 First Shifting Theorem 243
3.2.4 Second Shifting Theorem 247
3.2.5 Change of Scale Property 251

3.2.6 Inverse Laplace Transform of Derivatives 254
3.2.7 Inverse Laplace Transform of Integrals 255
3.2.8 Multiplication by powers of 's' 255
3.2.9 Division by Powers of 's' 256
3.2.10 Theorem 256
3.2.11 Convolution 257
3.2.12 Convolution Theorem (convolution property) 258
3.2.13 Application of Laplace Transform to Solutions of Ordinary Differential Equations 281

Summary 290

Solved University Questions 293

Objective Type Questions 339

UNIT - IV | Partial Differentiation

4.1 Partial Differentiation 354
4.1.1 Composite Functions - Total Derivative - Chain Rule 367
4.1.2 Homogeneous Function 375
4.1.3 Euler's Theorem 376
4.1.4 General Formula 377
4.2 Mean Value Theorems 389
4.2.1 Rolle's Theorem 389
4.2.2 Geometrical Interpretation of Rolle's Theorem 390
4.3 Lagrange's Mean Value Theorem (First mean value theorem) 396
4.3.1 Geometrical Interpretation of Langrange's Mean Value Theorem 397
4.4 Cauchy's Mean Value Theorem 402
4.5 Generalized Mean Value Theorems 405
4.5.1 Higher Mean Value Theorem with Lagrange's form of remainder (Taylor's theorem with Lagrange's form of remainder) 405
4.5.2 (a) Higher Mean Value Theorem with Cauchy's form of remainder (Taylor's theorem with Cauchy's form of remainder) 406
4.5.3 Maclaurin's Theorem with Lagrange's form of Remainder 408
4.5.4 Alternate form of Lagrange's Mean Value Theorem 409
4.5.5 Alternate form of Cauchy's Mean Value Theorem 409

4.6 Jacobian 414
4.6.1 Some Properties of Jacobian 415
4.6.2 Jacobian of Composite Functions 423
4.6.3 Jacobian of Implicit Functions 424
4.6.4 Functional Dependence and Independence of Functions 427
4.7 Taylor's Theorem for a Function of Two Variables 435
4.7.1 Maxima and Minima of Functions of Two Variables 441
4.7.2 Langrage's Method of Undetermined Multipliers 445
Summary 455
Solved University Questions 457
Objective type Questions 468

UNIT - V | First Order Partial Differential Equations

5.1 Partial Differential Equations 474
5.1.1 Introduction 474
5.1.2 Formation of a Partial Differential Equation by the Elimination of Arbitrary Constants 474
5.2 Formation of the Partial Differential Equations by the Elimination of Arbitrary Functions 478
5.3 First Order Linear Partial Differential Equations 483
5.3.1 Lagrange's Equation 483
5.3.2 Multipliers Method to Solve the Lagrange's Partial Differential Equations 489
5.4 Non-linear Partial Differential Equations of Order One 497
5.4.1 Definitions 497
5.4.2 Standard Form I (Equations involving p and q only) 497
5.4.3 Standard Form II 502
5.4.4 Standard Form III 506
5.4.5 Standard Form –IV (CLAIRAUT'S TYPE) 509
Summary 513
Solved University Questions 517

UNIT - VI | Higher Order Partial Differential Equations

6.1 Linear Partial Differential Equations with Constant Coefficients Homogeneous Linear Equations 534
6.1.1 Linear Partial Differential Equation with Constant Coefficients 534
6.1.2 Homogeneous Linear Partial Differential Equation with Constant Coefficients. 535
6.1.3 Solution of a Partial Differential Equation 535
6.1.4 Complementary Function 535
6.1.5 Particular Integral 539
6.1.6 A Short Method 542
6.1.7 Exceptional Case When F (a, b) = 0 548
6.1.8 General Method of Finding the P.I 553
6.1.9 Another Method of Complementary Function. 561
6.1.10 Non-homogeneous Linear Equations 564
6.1.11 Complementary Function 566
6.1.12 Particular Integral 568
6.2 Classification of Second Order Partial Differential Equation 580
Summary **582**

Model Papers of Engineering Mathematics - I **585**
Gate Previous Questions with Answers **591**