1

D.C. CIRCUIT CONCEPTS AND CIRCUIT ELEMENTS - I

1.1 INTRODUCTION TO BASICS OF ELECTRICAL ENGINEERING

Electrical Engineering forms the foundation of Electrical, Electronics, Communications, Controls, Computers, Information, Instrumentation, etc. Hence a good grasp of the fundamentals of Electrical Engineering is an absolute necessity to become a good engineer in any discipline.

In this chapter we discuss the basics of Electrical Engineering like sources of electrical energy-voltage and current sources and their conversion, Ohm's law, calculation of electrical power and energy and DC circuit analysis using mesh and nodal analysis.

1.1.1 Circuit Concepts - Concepts of Networks

An Electrical Circuit or an Electrical Nework consists of one or more Electical Energy Sources connected to a number of circuit elements like active or passive elements or both in such a way that there is a connection between the different elements causing a current to flow through the elements. The electrical circuit should be a closed circuit so that current can flow through it. If an Electrical Circuit is open then no current can flow through the circuit.

1.2 CURRENT FLOW

1.2.1 Potential and Potential Difference

An electrically charged particle sets up an electric field around it. If the particle is stationary, then the field set up by it is said to be ***Electrostatic Field***. The electric field lines of a positive charge **+q** will be radial and directed away from the charge. The field set up by the negative charge **–q** will be radial and directed towards the negative charge. Like charges repel and unlike charges attract. The force of attraction or repulsion between two charges q_1 and q_2 will be governed by ***Coulomb's Law*** which states that the force will be proportional to product of the charges $q_1 \times q_2$ and inversely proportional to the square of the distance 'R' between them and depends upon the medium in which the charges are placed.

$$F = \frac{q_1 q_2}{4\pi \in R^2} \qquad \text{.......... (1.2.1)}$$

where F is the force in newtons

q_1 and q_2 are the charges in coulombs

R is the distance in meters

$\in$ is the permittivity of the medium in farads / meter

The Absolute Permittivity for free space or vacuum,

$$\in_o = 8.854 \times 10^{-12} = \frac{1}{36\pi} \times 10^{-9} \text{ Farads / meter}$$

For other media the Absolute Permittivity,

$$\in = \in_o \times \in_r$$

where $\in_r$ is the relative permittivity of the medium which is a mere number.

$\in_r = 1$ for Air and $\in_r = 1$ for Mica

The force will be attractive if q_1 and q_2 are opposite charges and repulsive if q_1 and q_2 are like charges.

If we want to place any charge from one point to another point in the electrostatic field work has to be done against the electrostatic force or coulomb force experienced by that charge.

The work done in bringing a unit positive charge from infinity upto a given point p in an electrostatic field is defined as the potential at that point in the electro-static field. The unit for potential will be Joules/Coulomb. The unit is also called Volts.

The work done in moving a unit positive charge from one point in the electric field to another field in the electric field is known as the potential difference between the two points and is measured in volts. If V_A is the potential at point A and V_B is the potential at point B then the potential difference between the two points A and B will be $V_{AB} = V_A - V_B$. If $V_A > V_B$ then V_{AB} will be positive and is known as potential drop from point A to point B.

If $V_A < V_B$ then V_{AB} will be negative and is known as voltage rise from point B to point A. The voltage rise from point B to point A is generally denoted by the letter E.

$$E_{AB} = (E_A - E_B) = (V_B - V_A) = V_{BA} = -V_{AB} \qquad \text{...... (1.2.2)}$$

1.2.2 Electric Current

An electron placed in an electric field will experience a force and move towards the positive potential of the field since it is negatively charged. Continuous flow of electrons constitute a current flow from negative potential to positive potential of the field. This current is known as ***electron current***. The conventional current flow is opposite to that of the electron current in direction. The conventional current flow which is in opposite direction to electron current flow, will be flowing from a point of higher potential to a point of lower potential.

In metals (conducting materials), a large number of free electrons are available which move from one atom to the other at random when a potential difference is applied between two points of the conducting material and the current starts flowing.

The rate of flow of charges through any cross-section of a conductor is called a ***current*** and is denoted as 'i'. Current is expressed in terms of ***amperes***. Ampere is denoted by A or sometimes by α.

$$i = \frac{dq}{dt} \text{ Amperes} \qquad \text{.......... (1.2.3)}$$

where i is the instantaneous value of the current (value at any particular instant of the current)

The ***steady current*** 'I' is given as,

$$I = \frac{Q}{t} \text{ Amperes} \qquad \text{.......... (1.2.4)}$$

where Q is the charge flowing through the cross section of the conductor in time 't', if the flow of the charges is uniform.

Otherwise,

$$Q = \int i \, dt \text{ coulombs} \qquad \text{.......... (1.2.5)}$$

A wire is said to carry a current of ***one ampere*** when charge flows through it at the rate of one coulomb per second. Hence, one ampere is the current which flows when a charge of one coulomb moves across the cross-section of a conductor in one second.

1.3 ACTIVE AND PASSIVE ELEMENTS

The elements of an electric circuit can be classified into active and passive elements. Active elements are the sources that supply electrical energy to the circuit causing current flow through it. The energy sources can be independent or dependent sources. They may be voltage or current sources.

1.3.1 Sources of Electrical Energy - Voltage Source

The ***voltage source*** is assumed to deliver energy with a specified ***terminal voltage*** V_T, if it is a steady voltage source or $v(t)$ or simply v, if the voltage changes with respect to time. An ideal voltage source is expected to deliver a constant voltage to the outside circuit whatever be the amount of current drawn from the voltage source. The voltage of the source is called the ***Electro-Motive Force*** (**E.M.F.**) and is measured in Volts. It is denoted by the symbol E.

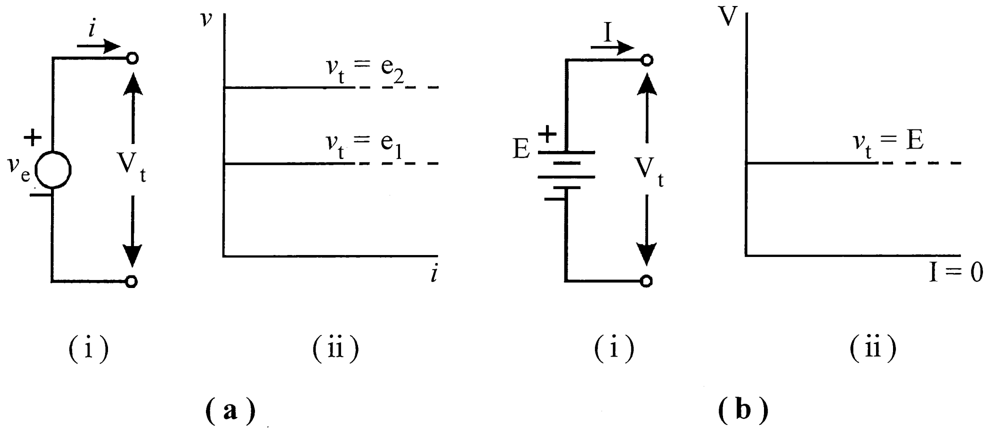
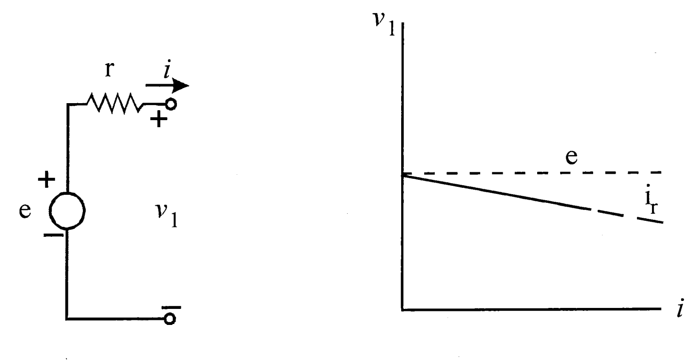


***Fig. 1.1 (a)** For a Time-Varying Voltage Source, **(b)** For a Time-Invariant Voltage Source (represented by a Battery)*



(i) (ii)

(c)

Fig. 1.1 (c)(i) *Model for a Voltage Source in which 'r' represents Source Resistance. The internal resistance 'r' will be very low. For the model of (i), the Terminal Voltage depends on source current as shown in (ii) where $v_t = v \;\; ir$*

If the source has an internal resistance γ then,

$$V = v(t) - i\,r \qquad \text{.......... (1.3.1)}$$

or

$$V_t = E - I\,r \text{ Volts} \qquad \text{.......... (1.3.1a)}$$

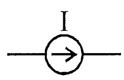
where I is the current drawn from the Voltage source in Amperes (A)

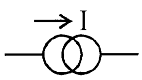
r is the internal resistance of the voltage source in Ohms (Ω).

In practice the terminal voltage of the voltage source will be decreasing as the current drawn from it is increased due to the voltage drop in the internal resistance of the voltage source. The internal resistance has to be very small in order that the voltage drop inside the source will be very small and maximum voltage may be available to the load.

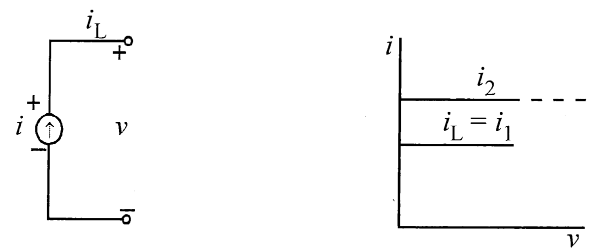
Note : *For any D.C. source, the polarities at the terminals will be same at all instants of time. For Time Varying Sources to polarities indicate the polarities at different terminals at any particular instant of time.*

1.3.2 Sources of Electrical Energy - Current Source

A current source is said to deliver a constant current $i_2 = I$ to the circuit thorugh the terminals, if it is a steady current source or i(t) or simply i, if the current changes with respect to time. An ideal current source is expected to deliver a constant current to the outside circuit whatever be the circuit. An ideal current source can be represented by the symbol 

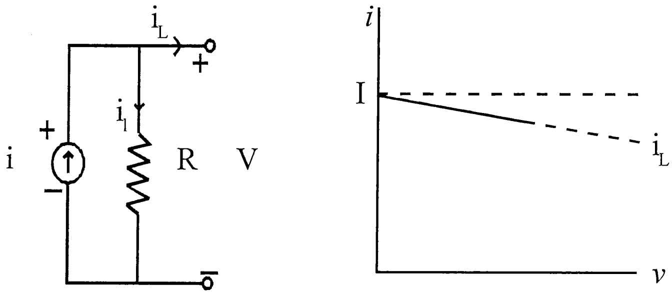
or 

In practice the current supplied by the current source will be decreasing as the voltage across the current source is increasing due to the internal resistance R of the current source, which is assumed to be across the current source. The internal resistance of the current source should be as high as possible so that maximum current will be delivered to the load connected across the current source with the current through the internal resistance being very very small.



(i) (ii)

Fig. 1.2 (a)(i) *Symbol for the Current Source for which i does not depend on v as shown in, (ii) Other lines may be drawn parallel to that shown for a specific current,* i_1.



(i) (ii)

Fig. 1.2 (b)(i) *Model for a Current Source in which R represents shunt resistance. For the model of 1.2 (i) the Terminal Current in 1.2 (ii) is given by* $i_L = i - (1/R)v$ *(ii) Other lines may be drawn parallel to that shown for a specific current,* i_L. The internal resistance 'R' will be very high.

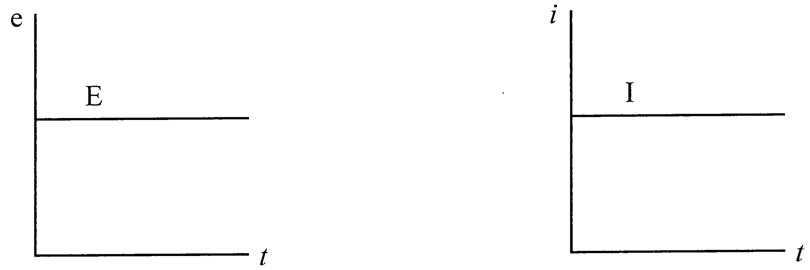
1.3.3 D.C. & A.C. Sources

If the voltage or current supplied by an electrical energy source is constant with respect to time as shown in Fig. 1.3 (a)(i) or Fig. 1.3 (a)(ii) then it is known as ***D.C. Voltage Source*** or ***Direct Current Source*** (or ***Steady Current Source***). D.C. stands for ***Direct Current.***

A D.C. Source has two terminals from which energy is supplied to the outside load. They are known as ***Positive Terminal*** which supplies the positive ions and ***Negative Terminal*** which receives the returning current or which can be assumed as supplying negative charges called electrons in the direction opposite to the conventional current direction. D.C. supply is provided by batteries or D.C. generators. The battery converts chemical energy into electrical energy. The generator converts mechanical energy into electrical energy.

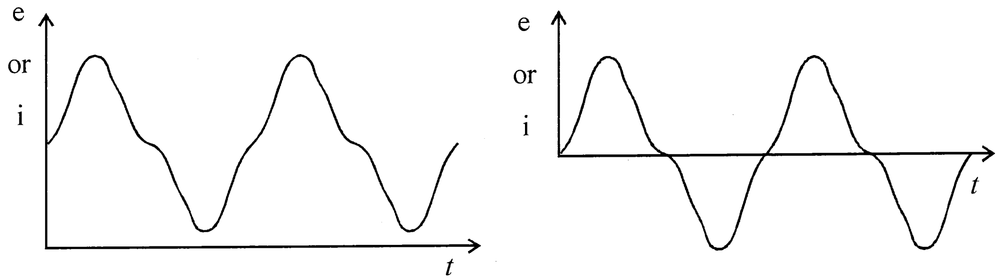
If the voltage or current is varying with respect to time but, has the same polarity as shown in Fig. 1.3(b)(i) then it is known as ***Unidirectional Source***. If the polarity is positive, it is known as ***Positive Source***. If the polarity is negative, it is known as ***Negative Source***.

If the voltage or current supplied by an electrical energy source varies in both magnitude and polarity with respect to time as shown in Fig. 1.3 (b)(ii) then it is known as ***A.C. Voltage Source*** or ***Alternating Current Source***. A.C. stands for ***Alternating Current***.



(i) D.C. Voltage Source (ii) D.C. Current Source

(a) D.C. Voltage & Current Sources



(i) Uni directional voltage or current (ii) A.C. Voltage or Current

(b) A.C. Uni-directional Voltage & Current Sources

Fig. 1.3 *A.C and D.C. Voltage & Current Sources*

The value of the voltage or current of an A.C. supply at any instant is called ***Instantaneous Value of Voltage or Current*** and is denoted as $v(t)$ or $i(t)$.

In general, the instantaneous values may also be denoted as v or i. **Generally no polarities will be marked for A.C. voltage or current. If at all polarities are marked for A.C. voltage or current, they mean the polarities of the voltage or current at the marked terminals, at any one particular instant and will be changing from time to time.**

1.3.4 Unilateral and Bilateral Elements

Elements in which current flow is in only one direction are called unilateral elements. Eg. Diode. Elements in which current flow can be in both directions are called bilateral elements. Eg. Resistance, Inductance, Capacitance etc.

i) Passive Elements - Resistance or Resistance Parameter

When a potential difference is applied across a conductor (or wire), the free electrons start moving in a particular direction. While moving through the material, these electrons collide with other atoms and molecules. They oppose this flow of electrons (or current) through it. This opposition is called ***Resistance***. Heat is produced because of the collisions of moving electrons with the other atoms and molecules. Thus whenever a current flows through a conductor, heat is produced in the conductor and this heat has to be dissipated fully. Otherwise, the insulation of the conductor (the Sheath made of insulating material covering the conductor) will get damaged.

The opposition offered to the flow of current (free electrons) is called ***Resistance***. Resistance is denoted by R and is measured in ***ohms*** named after a German mathematician ***George Simon Ohm*** and is represented by the Greek symbol Ω. For very high resistance we use large units such as kilo-ohms (kΩ which is equal to 10^3 Ω) or Mega-ohms (MΩ which is equal to 10^6 Ω) while for small resistances we use smaller units such as milli-ohms (mΩ which is equal to 10^{-3} Ω) or micro-ohms (μΩ which is equal to 10^{-6} Ω).

In electronic circuits, the current will generally be very small in milli-Amperes (mA or 10^{-3}A), micro-Amperes (μA or 10^{-6}A) or nano-Amperes (nA or 10^{-9}A) and hence, the resistance or resistors used will be in kilo-ohms or Mega-ohms and will be denoted simply as k or M (Ω is understood). They will be made of carbon resistors and will have color codes for different digits. They will not be accurate and will have a tolerance limit denoted by another color band and the wattage of the resistors will also be specified as ½ Watt or 1 Watt. For higher wattages of 5 or 8 or 10 Watts etc., wire wound resistors will be used.

If the resistance or resistivity is less the current flowing will be more. The resistance of a material is given by

$$R = \frac{\rho l}{a} \ \Omega \qquad \text{.......... (1.3.2)}$$

where ρ is the specific resistance of the material of the conductor in Ω-meters

l is the length of the conductor in meters

a is the area of cross-section of the conductor in square meters.

Not only conductors, but a coil wound with a conductor or any other electrical equipment offers resistance to current flow. A wire wound coil with two fixed terminals is called a ***resistor*** or ***resistance***. A coil with two fixed terminals and a variable contact terminal which makes contact with the body of the coil is called a ***Rheostat*** or ***Variable Resistor***. A rheostat can be connected in two ways as,

1. ***Series Resistance*** *(as shown in Fig. 1.4 (a))*

   If the moving contact is very near the starting terminal the resistance offered by the rheostat will be minimum and if it is nearer to the farthest end terminal, the resistance offered by the rheostat will be maximum. Sometimes the moving contact and one of the end terminals will be connected together in which case, the resistance offered will be the resistance of the remaining part of the winding. As the moving contact is varied the resistance offered by this part *will be varying, as the moving contact is moved away from or towards the second terminal. If the moving contact is towards the starting terminal then the resistance offered by the rheostat will be less.*

2. ***Potential Divider*** *(as shown in Fig.1.4 (b))*

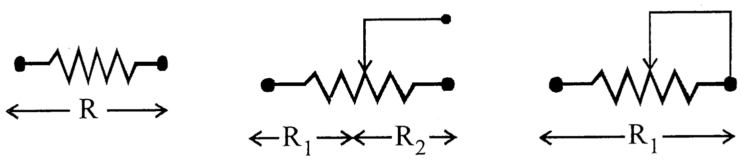


Fig. 1.4 (a) *Series Rheostat*

The two ends of the rheostat are connected across a voltage source which constitutes the input to the potential divider. The output is tapped between the moving contact and one of the end terminals, in which case, part of the input voltage will be the output voltage. The voltage tapped is given by,

$$V_{out} = V_{in} \times \frac{R_t}{R} \text{ Volts} \qquad \text{.......... (1.3.3)}$$

where R is the total resistance of the rheostat and R_t is the reistance of the tapped part of the rheostat winding.

Example 1.1:

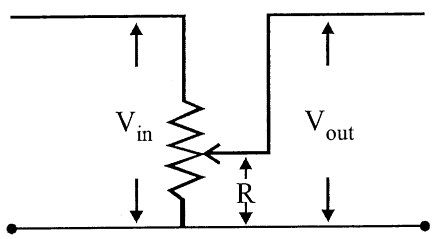


Fig. 1.4 (b) *Rheostat as Potential Divider*

Find the resistance of a coil of mean diameter 4 cm containing 400 turns of manganese wire 0.05 cm in diameter. The resistivity of manganese is 42 μΩ - cm.

Solution:

Here $\rho = 42\ \mu\Omega\text{-cm}.$

$= 42 \times 10^{-6}\ \Omega\text{cm}$

$= 42 \times 10^{-8}\ \Omega\text{ - m}$

$$a = \frac{\pi}{4} d^2$$

$$= \frac{\pi}{4} (0.05)^2\ \text{cm}^2$$

$$= \frac{\pi}{4} (0.05)^2 \times 10^{-4}\ \text{m}^2.$$

Number of turns (N) of the coil,

$N = 400$

Length per turn of the coil is $\pi \times D$

where D is the diameter of the coil.

Length (l) of the conductor of the coil,

$l = \pi \times D \times N$

$= \pi \times 4 \times 400$ cm

$= 16\ \pi$ m

Resistance of the coil,

$$R = \frac{42 \times 10^{-8} \times 16\pi \times 4}{\pi (0.05)^2 \times 10^{-4}} = 107.52\ \Omega.$$

1.3.5 Effect of Temperature on Resistance

As temperature increases the resistance of most of the conducting materials increase while for some material like carbon, electrolytes, insulators the resistance decreases as the temperature increases. The change in resistance depends upon the temperature-coefficient of resistance, which will be positive if the resistance increases with temperature and negative if the resistance decreases with temperature. The change in resistance per ohm per degree temperature change is called ***temperature-coefficient of resistance*** and its symbol is α.

If a metallic conductor of resistance R_0 at 0°C is heated to a temperature t_1, then the resistance R_1 at temperature t_1 is given by

$$R_1 = R_0 (1 + \alpha_0 t_1) \qquad \text{.......... (1.3.4)}$$

Since the temperature-coefficient itself varies with temperature, it does not have the same value at all temperatures. Thus if R_1 and R_2 are the resistances of a conductor at temperatures t_1 and t_2, we have

$$R_2 = R_1 (1 + \alpha_1 (t_2 - t_1)) \quad .... (1.3.5)$$

where α_1 is the temperature-coefficient of resistance at t_1°C.

Variation of α is obtained as

$$\frac{1}{\alpha_2} = \frac{1}{\alpha_1} + (t_1 - t_2)$$

or

$$\alpha_2 = \frac{\alpha_1}{1+\alpha_1(t_2 - t_1)} \quad .... (1.3.6)$$

Example 1.2:

The resistance of a coil decreases from 70Ω at 75°C to 50Ω at 15°C. Calculate the value of temperature-coefficient of resistance of the material of the coil at 0°C. Find the resistance at 0°C.

Solution:

Let α_0 be the temeperature-coefficient of resistance and R_0 be the resistance at 0°C.

$$70 = R_0 (1 + 75 \alpha_0) \quad .... (1)$$

and

$$50 = R_0 (1 + 15 \alpha_0) \quad .... (2)$$

Dividing Eq. (1) by Eq. (2) we get,

$$\frac{70}{50} = 1.4 = \frac{1 + 70\alpha_0}{1 + 15\alpha_0}$$

$$54 \alpha_0 = 0.4$$

or

$$\alpha_0 = \frac{0.4}{54} = 0.0074074$$

$$R_0 = \frac{70}{1+75\alpha_0} = \frac{70}{1.5555} = 45 \, \Omega$$

1.3.6 Electrical Conductance

The reciprocal of resistance of a conductor is called ***Conductance*** of the conductor and is denoted by 'G' and is expressed in terms of Siemen abbreviated as "S" or mhos (℧).

$$G = \frac{1}{R}$$

$$= \frac{1}{\rho l / a}$$

$$= \frac{a}{\rho l} = \sigma \frac{a}{l} \text{ Siemens} \qquad \text{.......... (1.3.7)}$$

where σ is called specific conductance or conductivity and is measured in Siemens per meter (S/m).

If the conductance or conductivity is less the current flowing will be less.

1.4 V-I RELEATIONSHIP FOR RESISTANCE – OHM'S LAW

The relationship between the current flowing through a conductor and the potential difference across the conductor 47 is given by ***Ohm's Law***.

The *Ohm's Law* states that *the potential difference across a conductor is directly proportional to the current flowing through the conductor, the temperature of the conductor remaining constant.* The constant of proportionality is R, the resistance.

$$V = I \times R \text{ Volts} \qquad \text{.......... (1.4.1)}$$

or

$$V = R \times I \text{ Volts} \qquad \text{.......... (1.4.1a)}$$

Here, V is the voltage drop across the conductor.

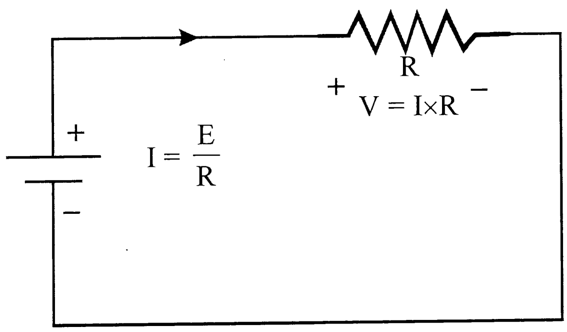


Fig. 1.5 *Circuit for Ohm's Law*

Note: *In the circuit, the voltage drop caused in the conducting wires connecting the battery and resistance are assumed to be zero as it will be negligible because the wire is a conducting material.*

Note: *While writing the equations for Volt-Ampere relationships in matrix form the second form i.e. Eq. (1.4.1a) will be applicable and can be expressed as,*

$$[V] = [R][I] \qquad \text{.......... (1.4.1b)}$$

Ohm's Law can also be expressed as

$$E = I \times R \text{ Volts} \qquad \text{.......... (1.4.1c)}$$

Here, E is the voltage rise across the conductor.

$$E_{AB} = -V_{AB}$$

The equation for Ohm's Law can also be written as,

$$I = \frac{V}{R} \text{ Amperes} \qquad \text{.......... (1.4.2)}$$

Ohm's Law also gives the Volt-Ampere relationship for an element. Ohm's Law can be applied to a part of a circuit or to the full circuit in which the current flows.

Ohm's Law can also be applied to A.C. Circuits or to circuits with Unidirectional Source in Laplace Transform domain for instantaneous values. For steady state conditions of A.C Ohm's Law using impedances and using RMS values for voltages and currents, all in complex form will be discussed later. However, for resistive circuits consisting of only resistances Ohm's Law can be written as,

$$v = i \times R \text{ Volts} \qquad \text{.......... (1.4.3)}$$

or

$$v = R \times i \text{ Volts} \qquad \text{.......... (1.4.3a)}$$

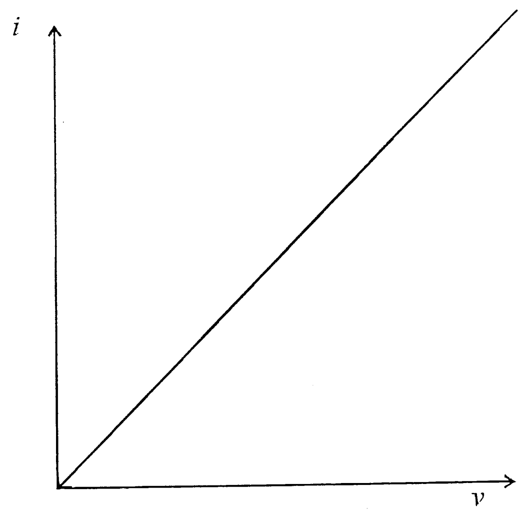


Fig. 1.6 *Linear V-I Characteristic for Resistance*

or $$i = \frac{v}{R} \text{ Amperes} \qquad \text{.......... (1.4.3b)}$$

where v and i are instantaneous values of Voltage and Current respectively.

1.4.1 Linear & Non-Linear Resistances

Those resistances in which the current flow changes in direct proportion with changes in the voltage applied across them are called ***Linear Resistances***. (curve 1 of fig. 1.6) The *v-i* characteristics for linear resistances will be current increasing as the voltage across increases. Ohm's Law is applicable as the resistance remains constant.

Those resistances for which the current through them does not vary in direct proportion are called ***Non-Linear Resistances***. For nonlinear resistances the *v-i* characteristics will be nonlinear. (curves 2 and 3 of fig.1.6)

In certain nonlinear resistances like Thyrite, the current increases more than proportionately with applied voltage with resistance decreasing rapidly like in curve 2 of Fig. 1.7. Hence, it is used in Lightning Arrestors.

In certain other nonlinear resistances like Semiconductors, Thermistors, the current decreases as the voltage across increases like in curve 3 of Fig. 1.6. Hence, thermistor is used in over current protection in Motors, etc.

Example 1.3:

A current of 0.75A is passed through a coil of nichrome wire which has an area of cross-section of 0.01 cm^2. If the resistivity of the nichrome is 108 ′ 10^{-6}W-cm and the potential difference across the ends of the coil is 81V. What is the length of the wire? What is the conductivity and conductance of the wire?

Solution:

Resistance,

$$R = \frac{\rho l}{a}$$

where, $$R = \frac{V}{I}$$

$$= \frac{81}{0.75}$$

$$= 108\ \Omega$$

$$a = 0.01 \text{ cm}^2 = 0.01 \times 10^{-4} \text{ m}^2$$

$$l = \frac{R \times a}{\rho}$$

$$= \frac{108 \times 0.01 \times 10^{-4}}{108 \times 10^{-8}} = 100 \text{ m}$$

Conductivity,

$$\sigma = \frac{1}{\rho}$$

$$= \frac{1}{108 \times 10^{-8}}$$

$$= 92.59 \times 10^4 \text{ ℧/m}$$

Conductance,

$$G = \frac{1}{R}$$

$$= \frac{1}{108}$$

$$= 9.259 \times 10^{-3} \text{ ℧ or Siemens}$$

1.5 ELECTRICAL POWER

Power is the rate of doing work and is expressed in Joules per second. When one coulomb of electrical charge moves through a potential difference of one volt in one second the work done is one Joule/sec and in electrical engineering it is expressed as one Watt and is denoted by the symbol P.

So Power supplied,

$$P = E \times I \text{ Watts} \qquad \text{.......... (1.5.1)}$$

where E is the source voltage.

Power expended,

$$P = V \times I \text{ Watts} \qquad \text{......... (1.5.1a)}$$

where V is the voltage drop.

Applying Ohm's Law for V,

$$P = (I \times R) \times I \text{ Watts}$$

$$P = I^2 \times R \text{ Watts} \qquad \text{.......... (1.5.2)}$$

or Applying Ohm's Law for I,

$$P = V \times \left(\frac{V}{R} \right) \text{ Watts}$$

$$P = \frac{V^2}{R} \text{ Watts} \qquad \left(\because I = \frac{V}{R} \right) \qquad \text{.......... (1.5.3)}$$

For A.C. circuits or circuits with unidirectional source, the equations for electrical power can be written using instantaneous values as,

$$p = e \times i \text{ Watts} \qquad \text{.......... (1.5.4)}$$

where e is the source voltage.

Power expended,

$$p = v \times i \text{ Watts} \qquad \text{.......... (1.5.4a)}$$

where v is the voltage drop.

Applying Ohm's Law for v,

$$p = i^2 \times R \text{ Watts} \qquad \text{.......... (1.5.5)}$$

or Applying Ohm's Law for i,

$$p = \frac{v^2}{R} \text{ Watts} \qquad \text{.......... (1.5.6)}$$

The power expended is also known as ***Power Loss*** or ***Copper Loss*** since the power loss takes place in the conductor (generally made of copper). It is called copper loss, even though the conductor material is not made of copper.

The power loss or copper loss appears in the form of heat. This heat has to be dissipated properly or else the insulation of the conductor or the insulation coating (varnish)of the coil will get damaged and there will be short circuits between turns of the coil and the coil may get burnt away in the case of machines and other equipments using coils.

1.6 ENERGY CALCULATIONS

Energy is the work done in a given time to achieve the required state of heating, lighting, lifting weights, moving the objects, etc. As such energy calculations are very important. Of late to have good efficiency in getting the work done and to have good economy Energy Auditing is resorted to in Industries and because energy charges are recurring charges involving expenditure.

According to the *Law of Conservation of Energy*, energy can neither be created nor destroyed. As such energy can atmost be converted from one form of energy into another form like converting mechanical energy into electrical energy and vice versa, converting electrical energy into heat energy and vice versa, etc. In this process the efficiency of the equipment used in conversion plays an important role. Also the constants of conversion are to be considered.

1.7 ELECTRICAL ENERGY

Electrical Energy is the total amount work done and is expressed in Joules or in Watt-seconds in electrical engineering. It is denoted by W. If E is the voltage rise or electromotive force and I is the current then, the energy generated or the energy supplied for a time t seconds is given as,

$$W = P \times t = E \times I \times t \text{ Watt-sec} \qquad \text{.......... (1.7.1)}$$

or

$$W = I^2 \times R \times t \text{ Watt-sec} \qquad \text{.......... (1.7.2)}$$

or

$$W = \frac{E^2}{R} \times t \text{ Watt-sec} \qquad \text{.......... (1.7.3)}$$

For A.C. circuits or circuits with unidirectional source,

$$w = p \times dt = e \times i \times dt \text{ Watt-sec} \qquad \text{.......... (1.7.4)}$$

or

$$w = i^2 \times R \times dt \text{ Watt-sec} \qquad \text{.......... (1.7.5)}$$

or

$$w = \frac{e^2}{R} \times dt \text{ Watt-sec} \qquad \text{.......... (1.7.6)}$$

where p, v, i all stand for instantaneous values and dt is the differential time

So,

$$W = \int p\,dt \text{ Watt-sec} \qquad \text{.......... (1.7.7)}$$

or

$$W = \int e \times i\,dt \text{ Watt-sec} \qquad \text{.......... (1.7.8)}$$

or $$W = \int \frac{e^2}{R} dt \text{ Watt-sec} \quad \text{.......... (1.7.9)}$$

The energy expended canl be obtained by using the above equations substituting V for E or *v* for *e*.

If the power is supplied for time t_1 to t_2 seconds then, the total energy W will be given as,

$$W = \int_{t_1}^{t_2} p \, dt \text{ Watt-sec} \quad \text{.......... (1.7.10)}$$

Since the Watt-sec is a small unit, for practical purposes, energy is expressed in terms of Kilo-Watt-Hour (KWH) or units.

1 unit of energy= 1 KWH

$$1 \text{ unit of energy} = \frac{\textit{Power in Watts} \times \textit{Time in seconds}}{1000 \times 60 \times 60}$$

Power distribution companies charge the electrical energy supplied to the consumer in terms of Standard Energy Units (Board of Trade Units) known as Kilo-Watt-Hours.

1.8 KIRCHOFF'S LAWS

There are two more important laws governing the performance of a circuit known as

1. *Kirchoff's Voltage Law (KVL)*
2. *Kirchoff's Current Law (KCL or KIL)*

where I stands for current in KIL.

1.8.1 Kirchoff's Voltage Law (KVL)

Kirchoff's Voltage Law states that, *in a closed electric circuit the algebraic sum of E.M.F.s and Voltage drops is zero.*

By convention, the E.M.F.s or voltage rises are taken to be positive and voltage drops are taken to be negative.

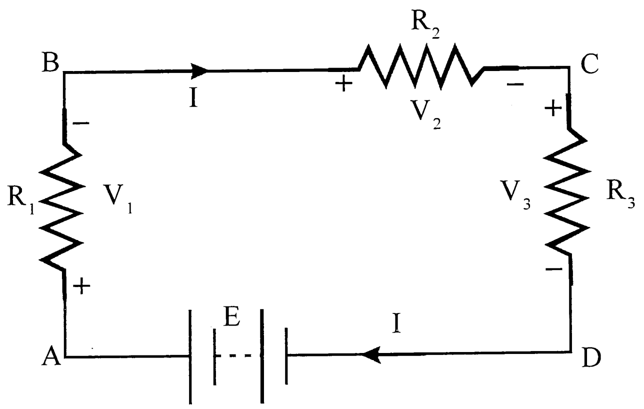


Fig. 1.7 *Circuit for Kirchoff's Voltage Law*

In the closed circuit ABCDA given in Fig. 1.7, applying Kirchoff's Voltage Law, we have,

$$E - V_1 - V_2 - V_3 = 0 \qquad \text{.......... (1.8.1)}$$

$$E - IR_1 - IR_2 - IR_3 = 0 \qquad \text{.......... (1.8.1a)}$$

or

$$IR_1 + IR_2 + IR_3 = E \qquad \text{.......... (1.8.1b)}$$

Sum of voltage drops = Sum of E.M.F.s or Voltage rises

Kirchoff's Voltage Law can be applied to any closed loop (closed circuit) even if there is no voltage source in which case the right hand side of Eq. (1.7.1b) will be zero for several loops. KVL in matrix form is given as,

$$[\,R\,][\,I\,] = [\,E\,] \qquad \text{.......... (1.8.1c)}$$

1.8.2 Kirchoff's Current Law (KCL or KIL)

Kirchoff's Current Law states that, *at any junction (or node) at which different elements are connected, the algebraic sum of the current at the junction is zero A junction or node is the meeting point of more than one element in a circuit. For eg. point B in the circuit given for Example 1.9 is a junction or node.*

By convention, a currents entering the junction are taken to be positive and currents leaving the junction are taken to be negative.

The currents of the current sources entering the junction are positive.

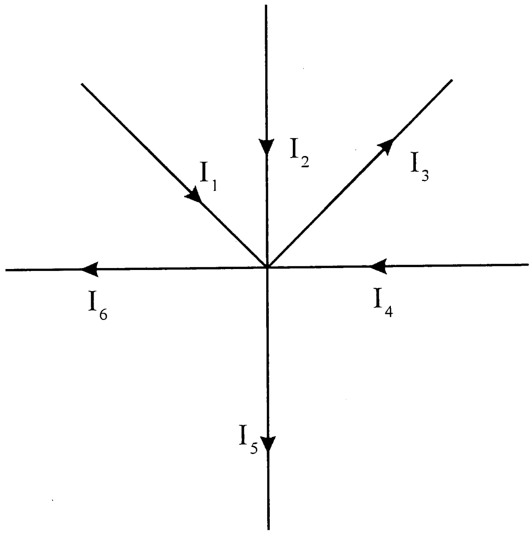


Fig. 1.8 *Kirchoff's Current Law*

In the Fig.1.8

applying Kirchoff's Current Law to the junction A, we have,

$$I_1 + I_2 + I_4 - I_3 - I_5 - I_6 = 0 \qquad \text{.......... (1.8.2)}$$

or

$$I_3 + I_5 + I_6 = I_1 + I_2 + I_4 \qquad \text{.......... (1.8.2a)}$$

Sum of currents leaving the junctions = Sum of currents entering the junction

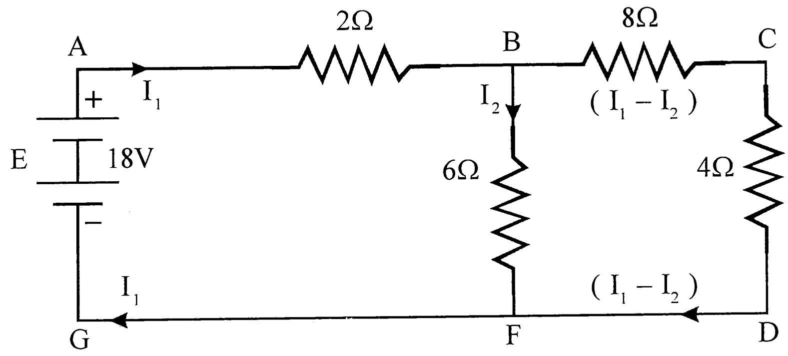
Note: Kirchoff's Voltage Law and Kirchoff's Current Law can also be applied to A.C. Circuits or to circuits with Unidirectional Source using instantaneous values for Voltages and currents. In A.C., for steady state values using impedances and using RMS values for voltages and currents, all in complex form will be discussed later. However, for resistive circuits consisting of only resistances Kirchoff's Voltage and Current Laws can be written as,

KVL: $$iR_1 + iR_2 + iR_3 = e \qquad \text{.......... (1.8.2c)}$$

KCL: $$i_3 + i_5 + i_6 = i_1 + i_2 + i_4 \qquad \text{.......... (1.8.2d)}$$

Example 1.4:

Applying KCL and KVL, find the currents in the various elements of the circuit given in Fig. 1.8. Find the power delivered by the battery and the energy supplied by the battery for a period of half an hour. Also calculate the power loss in the 6Ω resistor.



Solution:

Let the current supplied by the battery to junction A be I_1. The same current I_1 flows through the 2Ω resistance towards junction B. At B a part of this current of I_1 flows through the 6Ω resistance towards junction F. Let it be I_2.

Applying Kirchoff's Current Law to junction A, the current 8Ω resistance will be $(I_1 - I_2)$ towards junction C.

Applying Kirchoff's Voltage Law to the loop ABFGA,

$$2 \times I_1 + 6 \times I_2 = 18$$

or $$2I_1 + 6I_2 = 18 \qquad \text{.......... (1)}$$

Applying Kirchoff's Voltage Law to the loop BCDFB,

$$8 \times (I_1 - I_2) + 4 \times (I_1 - I_2) - 6 \times I_2 = 0$$

i.e., $$12I_1 - 18I_2 = 0 \qquad \text{.......... (2)}$$

Solving Eq. (1) and (2) we obtain,

$$I_1 = 3 \text{ A}$$

and $$I_2 = 2 \text{ A}$$

Also, Eq. (1) and (2) can be solved using Cramer's Rule,

$$I_1 = \frac{\begin{vmatrix} 18 & 6 \\ 0 & -18 \end{vmatrix}}{\begin{vmatrix} 2 & 6 \\ 12 & -18 \end{vmatrix}}$$

$$= \frac{18\times(-18) - 6\times 0}{2\times(-18) - 6\times 12}$$

$$= \frac{-324}{-72} = 3\text{ A}$$

Similarly,

$$I_2 = \frac{\begin{vmatrix} 2 & 18 \\ 12 & 0 \end{vmatrix}}{\begin{vmatrix} 2 & 6 \\ 12 & -18 \end{vmatrix}}$$

$$= \frac{2\times 0 - 18\times 12}{2\times(-18) - 6\times 12}$$

$$= \frac{-216}{-108} = 2\text{ A}$$

or $2I_1 + 6I_2 = 18$

Substituting the values of $I_1 = 3$ we obtain,

$I_2 = 2$ Amperes

Current in 2Ω resistor,

$I_1 = 3$ A

Current in 6Ω resistor,

$I_2 = 2$ A

Current in 8Ω resistor,

$I_1 - I_2 = 1$ A

Current in 4Ω resistor,

$I_1 - I_2 = 1$ A

Current supplied by the battery,

$P = EI_1$

$= 18 \times 3 = 54$ Watts

Energy, $W = P \times t$

Energy supplied by the battery for half an hour (1800 sec),

$$W = 54 \times 1800 = 97200 \text{ Watt-sec.}$$

$$= \frac{97200}{60 \times 60}$$

$$= 27 \text{ Watt-hours}$$

$$= \frac{27}{1000}$$

$$= 0.027 \text{ KWH}$$

Power Loss,

$$P = I^2R \text{ Watts}$$

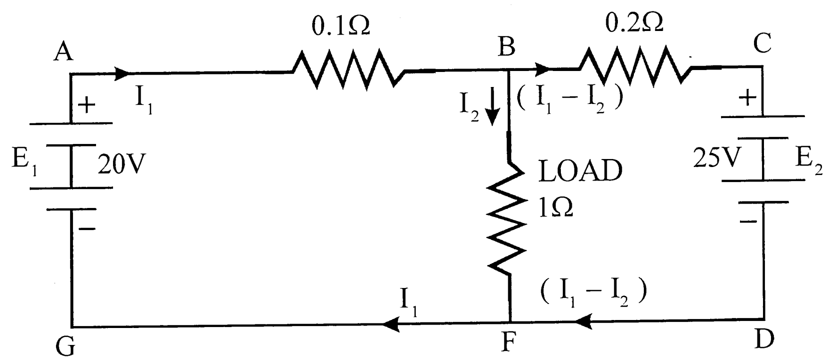
Power Loss in the 6Ω resistor $= (I_2)^2 \times 6$

$$= 2^2 \times 6$$

$$= 24 \text{ Watts}$$

Example 1.5:

In the circuit of given figure, find the power supplied to the load. Find also the voltage at the load using KCL and KVL Equations. Also find the current through 0.2Ω resistance.



Solution:

Let the current entering node B be I_1 and let the current flowing through the resistance 1Ω resistance be I_2. Applying KCL for node B, the current through 0.2Ω resistance will be $(I_1 - I_2)$.

Writing KVL for loop ABFGA,

$$0.1I_1 + 1 \times I_2 = 20$$

i.e., $$0.1I_1 + I_2 = 20 \qquad \text{.......... (1)}$$

Writing KVL for loop BCDFB,

$$0.2x(I_1 - I_2) + 1x (- I_2) = -25$$

i.e., $$0.2I_1 - 1.2I_2 = -25 \qquad \text{.......... (2)}$$

Solving Eq. (1) and (2) we get,

$$I_1 = -3.125 \text{ A}$$

$$I_2 = -20.3125 \text{ A}$$

Hence, the current in the load is I_2

$$= 20.3125 \text{ A.}$$

Current through 0.2Ω resistance is,

$$(I_1 - I_2) = (-3.125 - 20.3125) = -23.4375 \text{ A}$$

The negative signs for I_1 , I_2, $(I_1 - I_2)$ mean that the direction of the current flow is opposite to the assumed direction i.e., the current flows from B to A and not from A to B as assumed, I_2 from C to B and $(I_1\text{-}I_2)$ from F to B.

Voltage at the load is

$$V = IR$$

$$= 20.3125 \times 1$$

$$= 20.3125 \text{ Volts}$$

Load Power,

$$P = I^2R$$

$$= (20.3125)^2 \times 1$$

$$= 412.598 \text{ Watts.}$$

1.9 RESISTANCES IN SERIES

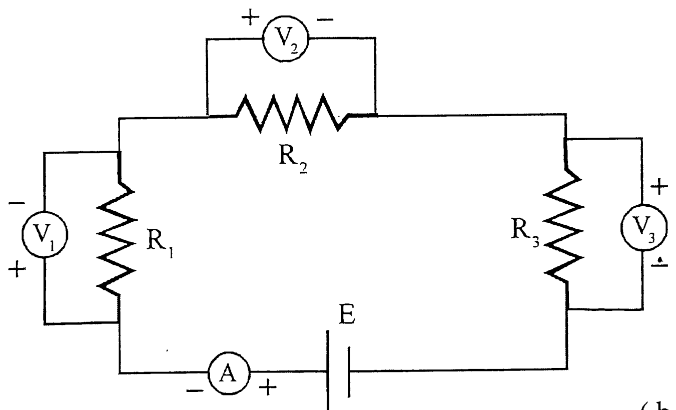
If the ending terminal of the resistance R_1 is connected to the beginning terminal of the resistance R_2 and the ending terminal of R_2 is connected to the beginning terminal of the resistance R_3 and so on then the resistances R_1, R_2, R_3, etc., are said to be connected in series.

In series circuits, the elements in the series can be connected in any order. For example, R_2, R_3, R_1, etc., instead of R_1, R_2 R_3 etc. In series circuits, the same current will flow through all the elements in series.

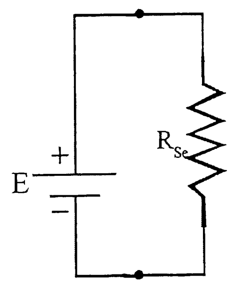
In D.C. series circuits, while connecting the elements in series, one should be very careful of the polarities of the meters used to measure the currents or voltages or the polarities of the equipment. Positive polarities of the meters or the equipments should always be connected to the positive of the supply point and the negative terminals should be connected to the negative of the supply point. While two equipments are connected in series, the positive of the first equipment should be connected to the positive terminal of the supply point. Negative terminal of the first equipment should be connected to the positive terminal of the second equipment and the negative terminal of the second equipment should be connected to the positive terminal of the third equipment and so on.

Ammeters are used to measure the currents. The ammeters should always be connected in series in the circuit so that the current to be measured flows through the ammeters. In order that, the voltage drop across the ammeter to be very very small so that full current flows through the circuit, the resistance of the ammeter should be very very small. Hence, if the ammeter is connected across the supply or across two points having large voltage drop, very heavy current will flow through the ammeter and the ammeter will get burnt.

Voltmeters are used to measure the voltage of the supply or voltage drop between two points inorder that the voltmeter does not draw more current so as to allow full current in the circuit, the voltmeters should have very high resistance. If the voltmeter is connected in series, it causes high voltage drop across it and the voltage supplied to the remaining circuit will be less. Hence, voltmeters should be connected only in parallel and not in series.



(a) Series Circuit



(b) Equivalent circuit for Fig. (a)

Fig. 1.9 Circuit with Resistances in Series

In the closed circuit ABCDA given in Fig. 1.9 (a), applying Kirchoff's Voltage Law, we have,

$$E - V_1 - V_2 - V_3 = 0 \qquad \text{.......... (1.9.1)}$$

$$E - IR_1 - IR_2 - IR_3 = 0 \qquad \text{.......... (1.9.1a)}$$

or $$IR_1 + IR_2 + IR_3 = E \qquad \text{.......... (1.9.1b)}$$

or $$I(R_1 + R_2 + R_3) = E \qquad \text{.......... (1.9.1c)}$$

For the equivalent circuit of Fig. 1.9 (b),

$$IR_{Se} = E \qquad \text{.......... (1.9.2)}$$

Comparing Eq. (1.9.1c) and (1.9.2) we have,

$$R_{Se} = (R_1 + R_2 + R_3)$$

In general for n resistances in series,

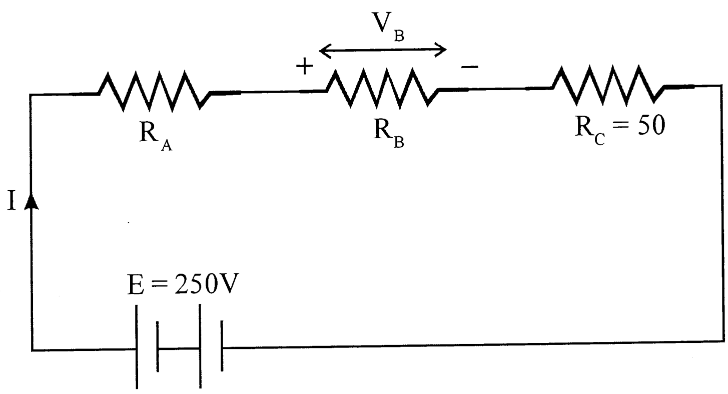
$$R_{Se} = \sum_{i=1}^{n} R_i \qquad \text{.......... (1.9.3)}$$

In terms of conductances for resistances in series,

$$\frac{1}{G_{Se}} = \sum_{i=1}^{n} \frac{1}{G_i} \qquad \text{.......... (1.9.4)}$$

Example 1.6:

Fig. below shows three resistors R_A, R_B and R_C connected in series to a 250V source; Given $R_C = 50\Omega$, and $V_B = 80$Volts when the current is 2 Amperes, calculate the total resistances, R_A and R_B.



Solution:

Since $I = 2$ Amperes

$$V_B = IR_B$$

$$= 80V$$

$$R_B = 40\Omega$$

Also, $$I = \frac{E}{(R_A + R_B + R_C)}$$

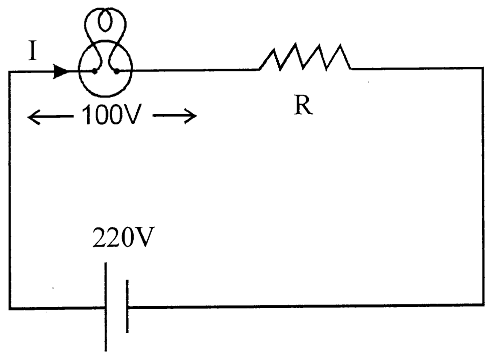
Therefore,

$$R_{Se} = R_A + R_B + R_C = \frac{E}{I} = \frac{250}{2} = 125\Omega$$

Therefore, $R_A = R_{Se} - (R_B + R_C) = 35\Omega.$

Example 1.7:

A lamp rated 500W, 100V is to be operated from 220V supply. Find the value of the resistor to be connected in series with the lamp. What is the power lost in the resistance.



Solution:

$$\text{Current in the lamp} = \frac{W}{E}$$

$$= \frac{500}{100} = 5 \text{ Amperes}$$

Since the Voltage drop across the lamp is 100 V, Voltage to be dropped in the series resistor is 120V.

$$\text{Therefore, Value of the resistor} = \frac{120}{5}$$

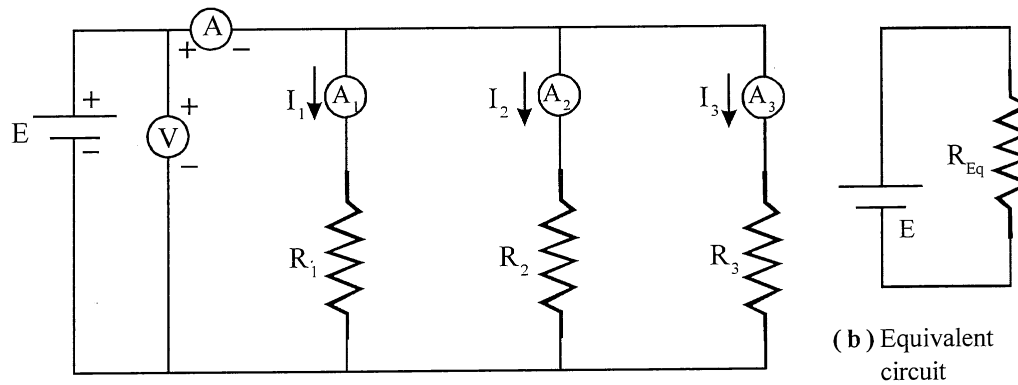
$$= 24\ \Omega$$

$$\text{Power Lost in this resistor} = I^2R$$

$$= 5^2 \times 24 = 600\text{W}$$

1.10 RESISTANCES IN PARALLEL

If the starting terminal of two or more elements are connected together and the ending terminals of these elements are connected together then the elements are said to be connected in parallel. A parallel element may also be known as a Shunt Element.



(a) Parallel circuit

(b) Equivalent circuit

Fig. 1.10 *Circuit with Resistances in Parallel*

The voltage across all the elements that are connected in parallel will be same.

In D.C. circuits, if the elements of the meters or the equipments with polarities marked are connected in parallel then terminals of the same polarities should be connected together.

In the parallel circuit given in the Fig. 1.10(a), applying Kirchoff's Current Law to the junction A,

$$I = I_1 + I_2 + I_3 \qquad \text{.......... (1.10.1)}$$

Applying Ohm's Law,

$$I = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3} \qquad \text{.......... (1.10.1a)}$$

For the equivalent circuit given in Fig. 1.14b,

$$I = \frac{E}{R_P} \qquad \text{.......... (1.10.2)}$$

Comparing Eq. (1.16.2) and (1.16.1a) we have,

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad \text{.......... (1.10.3)}$$

or In terms of conductances,

$$G_P = G_1 + G_2 + G_3 \qquad \text{.......... (1.10.4)}$$

In general for parallel circuits with n resistances in parallel,

$$\frac{1}{R_P} = \sum_{i=1}^{n} \frac{1}{R_i} \qquad \text{.......... (1.10.5)}$$

or

$$G_P = \sum_{i=1}^{n} G_i \qquad \text{.......... (1.10.6)}$$

If two resistances R_1 and R_2 are in parallel,

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \text{.......... (1.10.7)}$$

or

$$R_P = \frac{R_1 R_2}{R_1 + R_2} \qquad \text{.......... (1.10.7a)}$$

If the two resistances are equal and in parallel

i.e., $R_1 = R_2 = R$

then,

$$R_P = \frac{R}{2} \qquad \text{.......... (1.10.7b)}$$

If three resistances R_1, R_2 and R_3 are parallel,

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad \text{.......... (1.10.8)}$$

or

$$R_P = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1} \qquad \text{.......... (1.10.8a)}$$

If three resistances are equal, i.e., $R_1 = R_2 = R_3 = R$ then,

$$R_P = \frac{R}{3} \qquad \text{.......... (1.10.9)}$$

In general, if n resistances, each of value R are in parallel the,

$$R_P = \frac{R}{n} \qquad \text{.......... (1.10.10)}$$

1.10.1 Division of Currents in Parallel Circuits

If two resistances R_1 and R_2 are connected in parallel and if the total current entering the parallel combination is I then this current I divides into two parts I_1 flowing through R_1 and I_2 flowing through R_2.

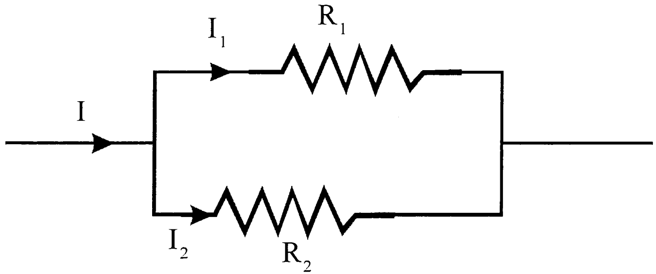


Fig. 1.11 *Division of Currents in Parallel Circuits*

The Voltage Drop across the parallel combination will be

$$V_P = I \times R_P = I \times \left(\frac{R_1R_2}{R_1 + R_2}\right) \text{ Volts}$$

The current I_1 flowing through R_1 will be given as,

$$I_1 = \frac{V_P}{R_1} = \frac{1}{R_1} \times I \times \left(\frac{R_1 R_2}{R_1 + R_2} \right) \quad \text{Amps}$$

$$I_1 = I \times \left(\frac{R_2}{R_1 + R_2} \right) \quad \text{Amps} \qquad \text{.......... (1.10.11)}$$

When two resistances R_1 and R_2 are in parallel, Current through R_1 is given as,

$$I_1 = \text{Total Current} \times \frac{\text{Second Resistance}}{\text{Sum of the Two Resistances in Parallel}}$$

.... (1.10.12)

This form is used frequently in Electronic Circuits.

Similarly,

$$I_2 = \frac{V_P}{R_2}$$

$$= \frac{1}{R_2} \times I \times \left(\frac{R_1 R_2}{R_1 + R_2} \right) \quad \text{Amps}$$

$$I_2 = I \times \left(\frac{R_1}{R_1 + R_2} \right) \quad \text{Amps} \qquad \text{.......... (1.10.13)}$$

Also, $\quad I_2 = (I - I_1)$ Amps

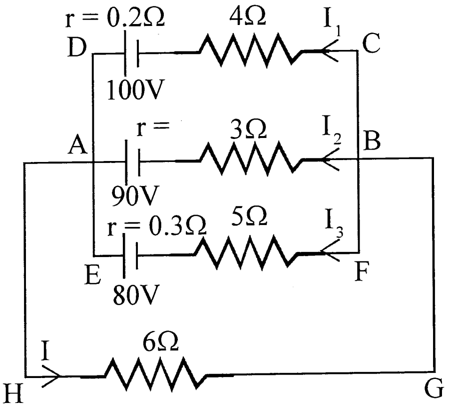
In general, *the current through any parallel path is given as the product of the total current and the parallel equivalent resistance divided by the resistance of that path.*

Example 1.8:

Solve the network shown in the figure for the current through 6Ω resistor.

Solution:

Let the current flowing through various branches be as marked in the figure. Applying Kirchoff's Voltage Law to the following closed circuits,



Circuit CDAHGBC,

$$-4I_1 - 0.2I_1 + 100 - 6(I_1 + I_2 + I_3) = 0$$

or $$10.2\,I_1 + 6I_2 + 6I_3 = 100 \qquad \text{..........}(1)$$

Circuit BAHGB,

$$-3I_2 - 0.25I_2 + 90 - 6(I_1 + I_2 + I_3) = 0$$

or $$6I_1 + 9.25\,I_2 + 6I_3 = 90 \qquad \text{..........}(2)$$

Circuit FEAHGBF,

$$-5I_3 - 0.3I_3 + 80 - 6(I_1 + I_2 + I_3) = 0$$

or $$6I_1 + 6I_2 + 11.3\,I_3 = 80 \qquad \text{..........}(3)$$

Subtracting (2) from (1), we get,

$$4.2\,I_1 - 3.25\,I_2 = 10 \qquad \text{..........}(4)$$

Eqn.(2) x 11.3 - Eqn. (3) x 6 gives

$$31.8\,I_1 + 68.525\,I_2 = 537 \qquad \text{..........}(5)$$

Eqn.(5) x 4.2 - Eqn.(4) x 31.8 gives

$$391.51I_2 = 1937.4$$

$$I_2 = 4.953\text{A}$$

From Eqn.(5) substituting for I_2

$$I_1 = 6.21\text{A}$$

From Eqn. (3), substituting for I_1 and I_2

$$I_3 = 1.15A$$

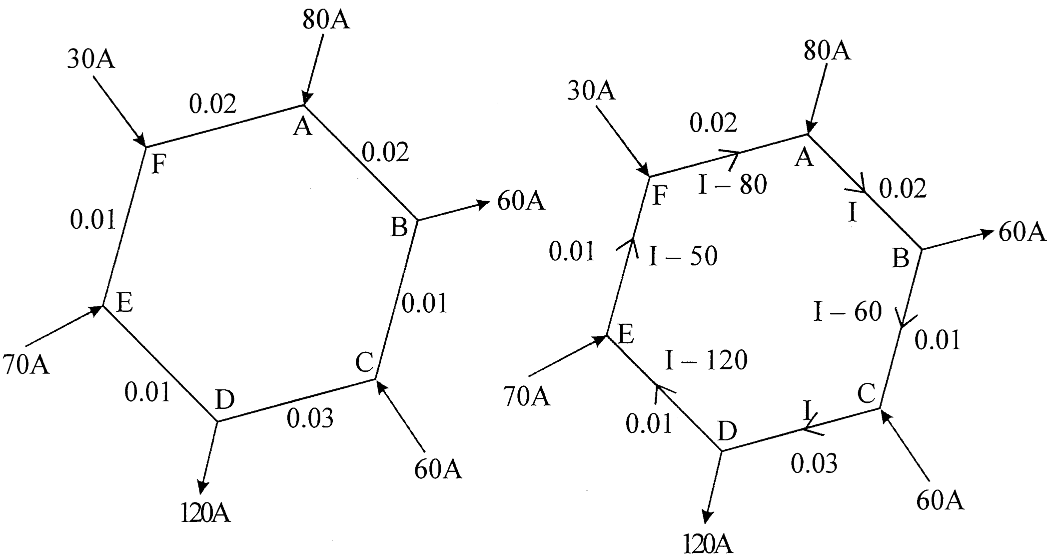
Current in 6Ω resistor,

$$I = I_1 + I_2 + I_3$$
$$= 6.21 + 4.953 + 1.15$$
$$= 12.313A$$

Example 1.9:

Find the magnitude and direction of the currents in all branches of the circuit shown in the figure using Kirchoff's Laws. All resistances are in Ohms.

Solution:



Let current from A to B junctions be I Amps. Applying Kirchoff's First Law, let current flowing through various branches be as shown in the figure.

Applying Kirchoff's Current Law to current in each branch and Kirchoff's Voltage Law to a closed loop ABCDEFA, we get,

$$-0.02I - 0.01(I - 60) - 0.03I - 0.01(I - 120) - 0.01(I - 50)$$
$$- 0.02(I - 80) = 0$$

or $0.02I + 0.01I + 0.03I + 0.01I + 0.01I + 0.02I$

$= 0.6 + 1.2 + 0.5 + 1.6$

or $0.1I = 3.9$

or $I = 39A$

Current in various branches is as under :

I_{AB}	$= 39A$		(i.e., A to B)
I_{BC}	$= I - 60$	$= -21A$	(i.e., C to B)
I_{CD}	$= I$	$= 39A$	(i.e., C to D)
I_{DE}	$= I - 120$	$= -81A$	(i.e., E to D)
I_{EF}	$= I - 50$	$= -11A$	(i.e., F to E)
I_{FA}	$= I - 80$	$= -41A$	(i.e., A to F)

1.11 SERIES-PARALLEL RESISTANCES

In the case of series-parallel resistances, the parallel equivalent of the resistances in parallel are obtained first as a single resistance which will be in series with the other resistances, thus bringing the circuit into a single series circuit. After finding the current flowing through this equivalent series circuit again the parallel equivalent resistance may be replaced with the corresponding parallel circuit and the current in the parallel paths are calculated as given in the Section 1.10, *the current through any parallel path is given as the product of the total current and the parallel equivalent resistance divided by the resistance of that path.*

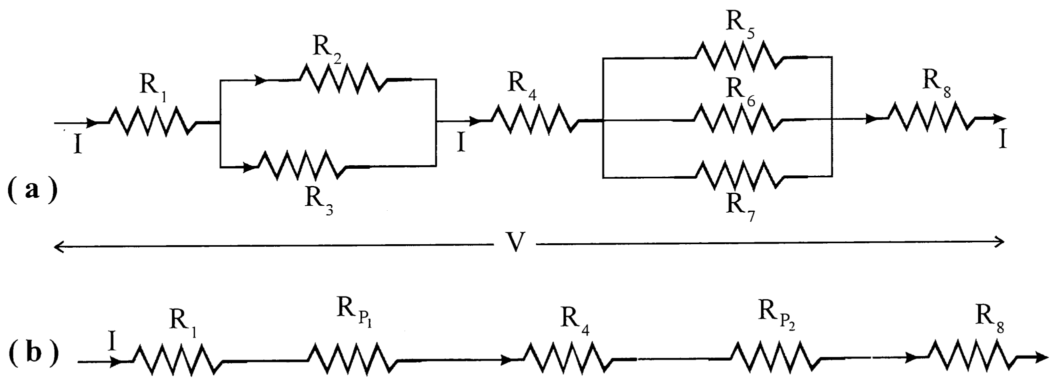


Fig. 1.12 *Resistance in Series-Parallel*

Typical values of currents are given below for the above circuit,

$$R_{P_1} = \frac{R_2R_3}{(R_2+R_3)}$$

$$R_{P_2} = \frac{R_5R_6R_7}{R_5R_6+R_6R_7+R_7R_5}$$

$$I = \frac{V}{R_1 + R_{P_1} + R_4 + R_{P_2} + R_8}$$

$$I_2 = I \times \frac{R_3}{(R_2+R_3)}$$

$$I_6 = \frac{I}{R_6}\left(\frac{R_5R_6R_7}{(R_5R_6+R_6R_7+R_7R_5)}\right)$$

Example 1.10:

A Wheatstone Bridge consists of AB = 4Ω, BC = 3Ω, CD = 6Ω and DA = 5Ω. A 2V Cell is connected between B and D and a Galvanometer of 10Ω resistance between A and C. Find the current through the Galvanometer.

Solution:

The circuit is shown in the figure. Applying Kirchoff's Current Law at junction B, A and C, the current in various branches is marked.

Applying Kirchoff's Voltage Law to various closed loops and considering loop BACB, we get,

$$-4I_1 - 10I_3 + 3I_2 = 0$$

or $$4I_1 + 10I_3 - 3I_2 = 0 \quad \text{.......... (1)}$$

Considering loop ADCA, we get

$$-5(I_1 - I_3) + 6(I_2 + I_3) + 10I_3 = 0$$

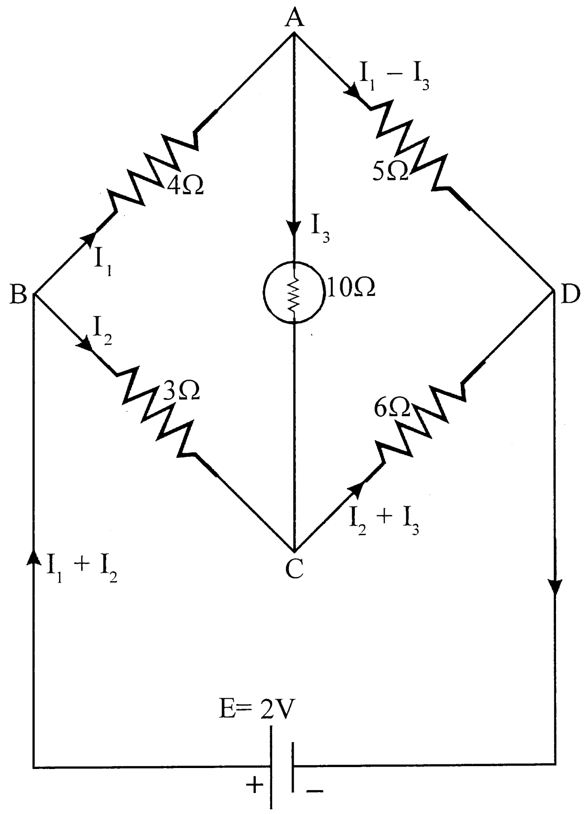
or $$5I_1 - 6I_2 - 21I_3 = 0 \quad \text{.......... (2)}$$

Considering loop BADEB, we get,

$$-4I_1 - 5(I_1 - I_3) + 2 = 0$$

or $$4I_1 - 5I_1 + 5I_3 = -2 \qquad \text{.......... (3)}$$

or $$9I_1 - 5I_3 = 2$$



Multiplying Eq. (1) by Eq. (2) and Subtracting from Eq. (2), we get,

$$\begin{array}{rcl} 5I_1 - 6I_2 - 21I_3 & = & 0 \\ 8I_1 - 6I_2 + 20I_3 & = & 0 \\ \hline -3I_1 \quad - 41I_3 & = & 0 \\ \hline \end{array}$$

or $$I_1 = -\frac{41}{3} I_3$$

Substituting the value of I_1 in Eq. (3), we get,

$$9\left(-\frac{41}{3}I_3\right) - 5I_3 = 0$$

or $$-123I_3 - 5I_3 = 2$$

or $$I_3 = -\frac{1}{64}\text{ A}$$

Example 1.11:

A resistance of 15Ω is connected in series with two resistances each of 30Ω arranged in parallel. A voltage source of 30V is connected to this circuit.

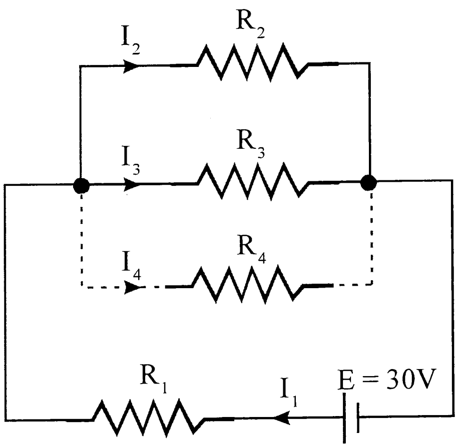
(a) What is the current drawn from the source.

(b) What resistance should be placed in shunt (parallel) with the parallel combination in order that the current drawn from the source is 1.2 A.

Solution:

The circuit is as shown in the figure given below.

(a) R_1 is in series with the parallel combination of R_2 and R_3.



Hence, Total Resistance R_t is,

$$R_t = R_1 + \frac{R_2R_3}{R_2+R_3}$$

$$= 15 + \frac{30\times30}{30+30}$$

$$R_t = 30\Omega$$

Hence current,

$$I_1 = \frac{E}{R_t}$$

$$I_1 = \frac{30}{30} = 1\ A$$

(b) With $I_1 = 1.2A$, Voltage across R_1

$$= I_1R_1 = 1.2 \times 15$$

$$= 18\ V$$

Voltage Drop across parallel combination of R_2, R_3 and R_4 is,

$$V_P = 30 - 18$$

$$V_P = 12\ V$$

Hence, Voltage Drop across each resistor R_2, R_3, R_4 is 12 V.

Hence, $$I_2 = \frac{V_P}{R_2} = \frac{12}{30}$$

$$I_2 = 0.4\ A$$

$$I_3 = \frac{V_P}{R_3} = \frac{12}{30}$$

$$I_3 = 0.4\ A$$

$$I_4 = I_1 - (I_2 + I_3)$$

$$= 1.2 - (0.4 + 0.4)$$

$$I_4 = 0.4\ A$$

$$I_4 = \frac{V_P}{R_4}$$

$$0.4R_4 = 12$$

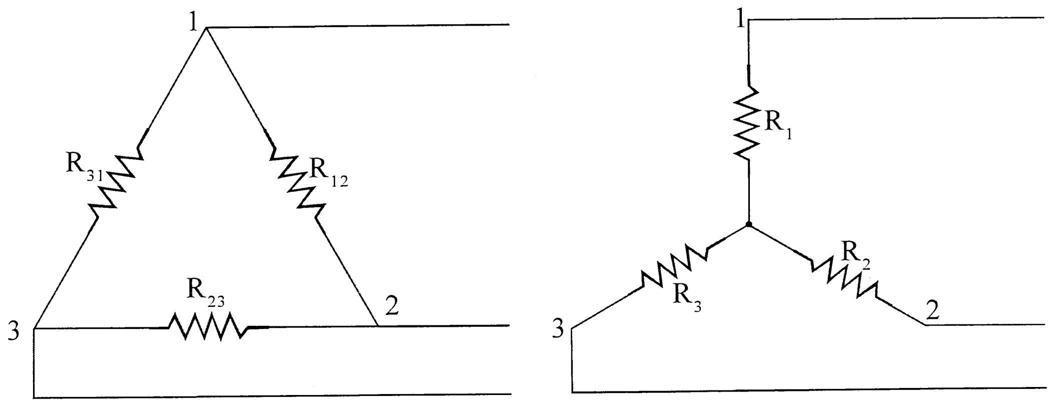
Hence, $$R_4 = \frac{12}{0.4}$$

$$R_4 = 30\ \Omega$$

1.12 RESISTANCES IN STAR OR DELTA CONNECTIONS

If the end of R_1 is connected to beginning of R_2, the end of R_2 is connected to beginning of R_3 and the end of R_3 is connected to beginning of R_1 then, the resistances R_1, R_2, R_3 are said to be connected in Delta (Δ) and the three points common to any two resistances are connected to the remaining part of the circuit as given in the Fig. 1.13 (a).

If one end of the resistances R_1, R_2, R_3, are all connected together and the other ends of the three resistances are connected to different points of the circuits, as given in the Fig. 1.13 (b), then the resistances R_1, R_2, R_3 are said to be connected in Star or WYE (Y).



(a) Delta Connection **(b)** Star Connection

Fig. 1.13 *Resistance in Star-Delta Connection*

In general, more than three resistances can be connected in Star.

In general, more than three resistances can also be connected in Mesh.

In some cases, the circuits cannot be solved by means of simple series, parallel or parallel combinations but by finding the equivalent Delta for the given Star connection or equivalent Star connection for the given Delta connection of resistances, the circuit can be resolved into simple Series, Parallel or Series-Parallel combinations.

1.12.1 The Equivalent Star Resistances For Given Delta Connected Resistances

Referring to Fig. 1.13 (a), in Delta Connection, we observe that between the terminals 1 and 2 the resistance R_{12} is in parallel with the series combination of R_{23} and R_{31}.

Referring to Fig. 1.13 (b), in Star Connection, we observe that between the terminals 1 and 2 the resistance R_1 is in series with R_2.

Equating the resistances between terminals 1 and 2 of Star and its equivalent Delta, we have R_{12} in parallel with $(R_{23} + R_{31})$

$$R_1 + R_2 = R_{12} + \frac{R_{23}R_{31}}{R_{23} + R_{31}}$$

Rewriting the above equation we have,

$$R_1 + R_2 = \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}}$$

$$R_1 + R_2 = \frac{R_{12}(R_{23} + R_{31})}{\sum R_{12}} \quad \text{.......... (1.12.1)}$$

Similarly equating the resistances between terminals 2 and 3, we have,

$$R_2 + R_3 = \frac{R_{23}(R_{31} + R_{12})}{\sum R_{12}} \quad \text{.......... (1.12.2)}$$

Similarly equating the resistances between terminals 3 and 1, we have,

$$R_3 + R_1 = \frac{R_{31}(R_{12} + R_{23})}{\sum R_{12}} \quad \text{.......... (1.12.3)}$$

Subtracting Eq. (1.18.2) from Eq. (1.18.3), we have,

$$R_1 - R_2 = \frac{R_{12}(R_{31} - R_{23})}{\sum R_{12}} \quad \text{.......... (1.12.4)}$$

Adding Eq. (1.18.1) and Eq. (1.18.4), we have,

$$2R_1 = \frac{R_{12}(2R_{31})}{\sum R_{12}}$$

Hence, $R_1 = \dfrac{R_{12}R_{31}}{\sum R_{12}}$

Rewriting the above equation, we have,

$$R_1 = \frac{R_{12}R_{13}}{\sum R_{12}} \qquad \text{.......... (1.12.5)}$$

Similarly, $$R_2 = \frac{R_{21}R_{23}}{\sum R_{12}} \qquad \text{.......... (1.12.6)}$$

$$R_3 = \frac{R_{31}R_{32}}{\sum R_{12}} \qquad \text{.......... (1.12.7)}$$

Note that from Eq. (1.12.5), Eq. (1.12.6) and Eq. (1.12.7), we observe that, *the equivalent star resistance connected to a given terminal is given by the product of the two Delta resistances that are connected to that terminal divided by the sum of the three Delta connected resistances.*

The same statement can be extended to more than three resistances connected in Mesh and its equivalent Star !! (check up)

1.12.2 The Equivalent Delta Resistances For Given Star Connected Resistances

Multiplying Eq. (1.12.5) and Eq. (1.12.6), we have,

$$R_1R_2 = \frac{R_{12}^2R_{23}R_{31}}{\left(\sum R_{12}\right)^2} \qquad \text{.......... (1.12.8)}$$

Multiplying Eq. (1.12.6) and Eq. (1.12.7), we have,

$$R_2R_3 = \frac{R_{12}R_{23}^2R_{31}}{\left(\sum R_{12}\right)^2} \qquad \text{.......... (1.12.9)}$$

Multiplying Eq. (1.12.7) and Eq. (1.12.5), we have,

$$R_3R_1 = \frac{R_{12}R_{23}R_{31}^2}{\left(\sum R_{12}\right)^2} \qquad \text{.......... (1.12.10)}$$

Adding Eq. (1.12.8), Eq. (1.12.9) and Eq. (1.12.10), we have,

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}R_{23}R_{31}(R_{12} + R_{23} + R_{31})}{\left(\sum R_{12}\right)^2}$$

$$= (R_{12}R_{23}R_{31})\frac{\left(\sum R_{12}\right)}{\left(\sum R_{12}\right)^2}$$

$$= R_{12}\left[\frac{R_{23}R_{31}}{\sum R_{12}}\right]$$

$R_1R_2 + R_2R_3 + R_3R_1 = R_{12}R_3$ by using R_3 from eqn.1.12.7 for the term inside the bracket

Hence, $$R_{12} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

or $$R_{12} = R_1 + R_2 + \frac{R_1R_2}{R_3} \qquad \text{.......... (1.12.11)}$$

Similarly, $$R_{23} = R_2 + R_3 + \frac{R_2R_3}{R_1} \qquad \text{.......... (1.12.12)}$$

$$R_{31} = R_3 + R_1 + \frac{R_3R_1}{R_2} \qquad \text{.......... (1.12.13)}$$

Note that from Eq. (1.12.11), Eq. (1.12.12) and Eq. (1.12.13), we observe that, *the equivalent Delta resistance connected between any two given terminals is given by the sum of the two Star resistances that are connected between those two terminals plus the product of those two resistances connected between those two terminals divided by the third Star resistance.*

In terms of conductances the equivalent mesh element can be given in terms of the star elements for star or delta having three or more resistances as

$$G_{ij} = \frac{G_i G_j}{\sum G_i} \quad \text{!! (check up)} \qquad \text{.......... (1.12.14)}$$

If $R_{12} = R_{23} = R_{31} = R$ (say), then, the equivalent star resistances will be,

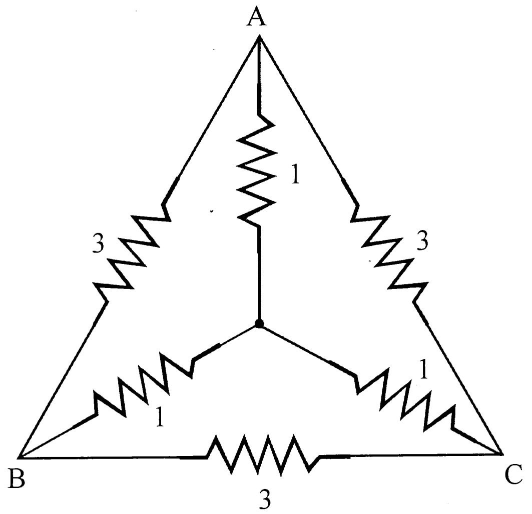
$$R_1 = R_2 = R_3 = \frac{R}{3} \qquad \text{.......... (1.12.15)}$$

If $R_1 = R_2 = R_3 = R$ (say), then, the equivalent delta resistances will be,

$$R_{12} = R_{23} = R_{31} = 3R \qquad \text{.......... (1.12.16)}$$

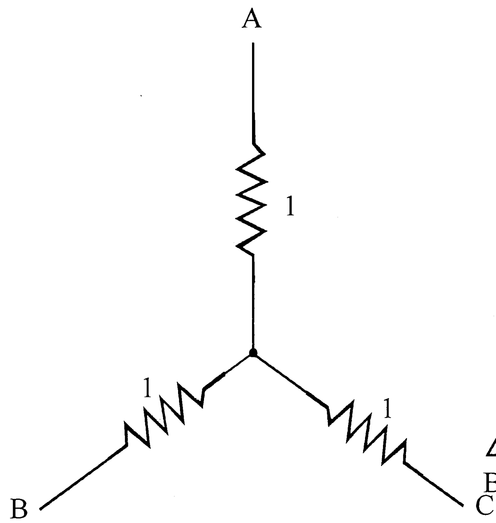
Example 1.12:

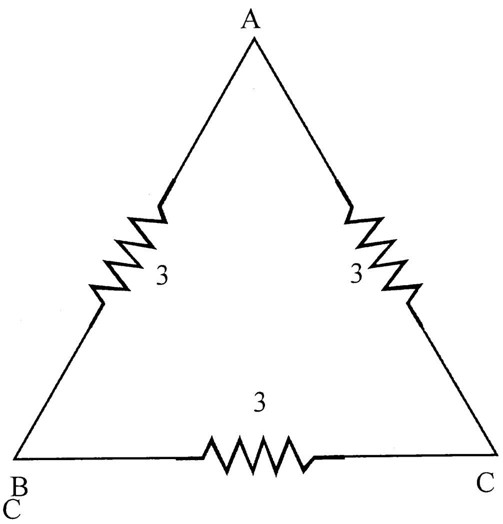
For the network shown in the figure below, find the equivalent resistance between the terminals B and C.



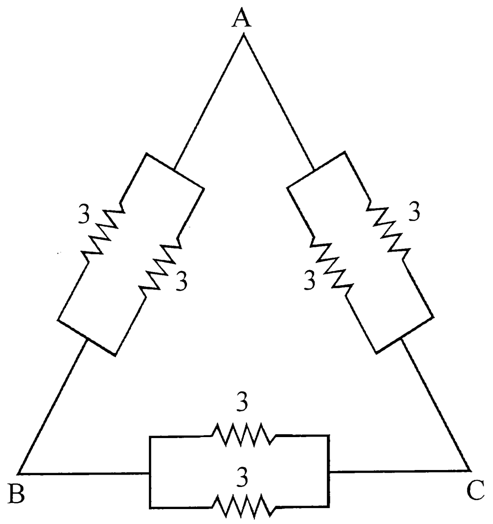
Solution:

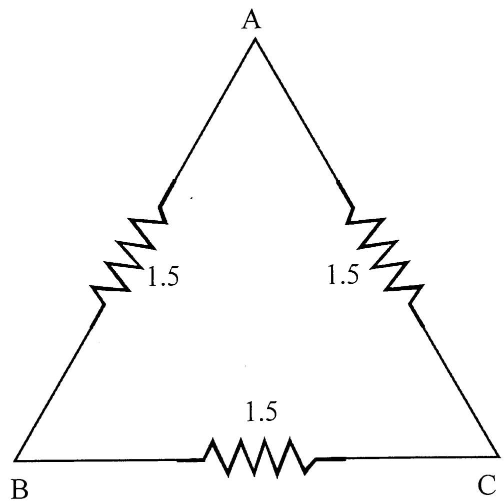
The given combination of resistances between the terminals B and C is neither series combinations nor parallel combination. If, the star connection between A, B and C is converted into equivalent delta, we will have a known combination which can be simplified by series parallel simplification.

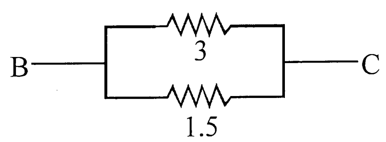


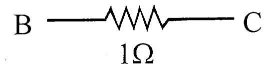


The given figure is redrawn after replacing the star by its equivalent delta.





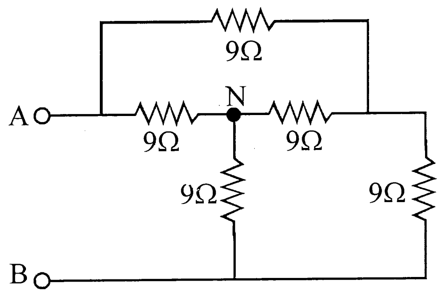




$$R_{BC} = \frac{3 \times 1.5}{3 + 1.5} = 1\Omega.$$

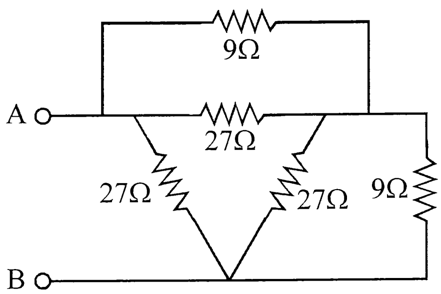
Example 1.13:

Determine the equivalent resistance between A and B.

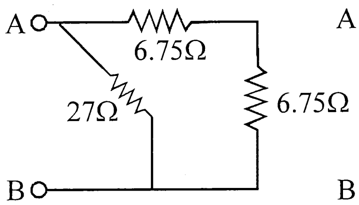


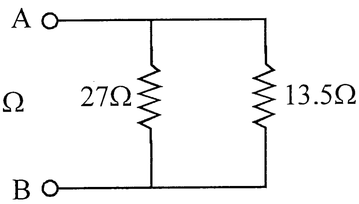
Solution:

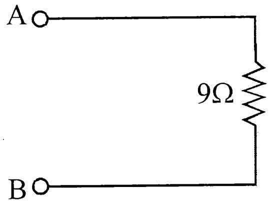
The combination is neither series nor parallel. There is Delta and Star connection. Converting the star connection for which N is the star point and redrawing the circuit, we get,



$$\frac{27\times 9}{27+9} = \frac{27\times 9}{36} = 6.75\Omega$$







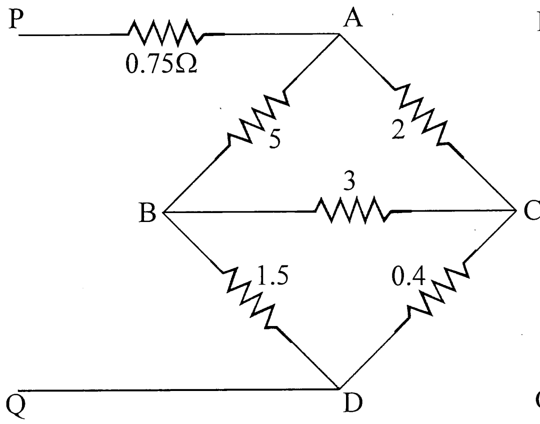
Therefore,

$$R_{AB} = 27 \parallel 13.5$$

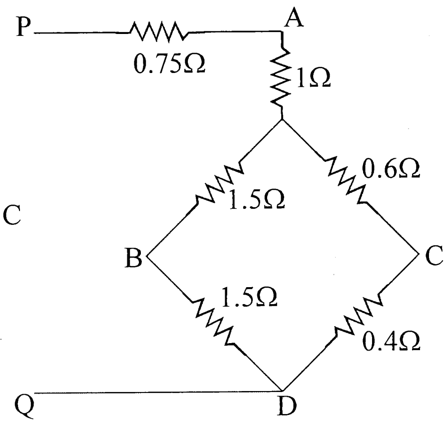
$$= \frac{27\times 13.5}{27+13.5} = 9\Omega.$$

Example 1.14:

In the Wheat stone Bridge Circuit of figure (a) given below, find the effective resistance between PQ. Find the current supplied by a 10V Batter connected to PQ.



(a)



(b)

Solution:

Converting the Δ formed by 5, 2 and 3Ω in a star of figure (b), we have,

$$R_A = \frac{5 \times 2}{5+3+2} = 1\Omega$$

$$R_B = \frac{5 \times 3}{10} = 1.5\Omega$$

$$R_C = \frac{3 \times 2}{10} = 0.6\Omega$$

The circuit then reduces to the following form.

The two 1.5 Ohm resistors are in series and this is in parallel with the 0.6 and 0.4 Ohm resistors which are in series.

Therefore, the total effective resistance between P and Q

$$= 0.75 + 1 + \frac{3 \times 1}{3+1}$$

$$= 2.5\Omega$$

If a 10V battery is attached to PQ, current drawn,

$$= \frac{10}{2.5} = 4A.$$

1.13 USES OF RESISTANCE

1. Resistance is used in any circuit to limit the current.
2. Resistance is used in series resonance circuits to limit the currents at resonance point.
3. Resistance will be used in Wave Shaping Circuits to obtain a different wave form from a given wave form.
4. Resistance can be used in analog circuits, to solve for other systems like Mechanical Systems, Hydraulic System, etc., in terms of Electrical Systems
5. Resistance will be used in Filter Circuits to select or reject certain frequencies.
6. Change of resistances as obtained in ***Resistance Strain Gauges***, which convert physical signals like temperature, strain, load, force etc., into electrical signals are used for measurement of the physical signals.
7. Resistance can also be used in Power Factor changing circuits but because it causes Power Loss and additional real power to be supplied, the power factor changing circuits generally do not employ change of resistances.
8. Resistances are used in quenching the arcs (extinguishing the arcs) in the case of circuit breakers, while breaking heavy current circuits in electrical transmission and distribution.
9. Resistance can be used in Measurement Circuits. For example, Variable Resistance is used to balance the bridge networks. Resistance shunt is used to extend the range of an ammeter or galvanometer, by diverting a major part of the circuit current through the shunt resistance so that only the allowable current flows through the ammeter or galvanometer. The actual current is calculated from the reading of the ammeter or galvanometer using a factor in terms of resistance of the galvanometer and the resistance of the shunt as in the case of parallel circuit. *A shunt is a very small pure resistance of the order 1Ω or less.*
10. Resistance shunt can also be used as a series resistance causing a small voltage drop across it, so that this voltage drop can be used in CRO (Cathode Ray Oscilloscope) to trace the current waveform which will be proportional to the voltage waveform across the resistance shunt.