Section - I

Heat Transfer

"The inside of a tube is, without a doubt, the poorest place in the world where a person could affect heat transfer, ... because it has a uniform cross section, it does not create randomness ... and it tends to give you the least for your money when you circulate a fluid through it."

Donald Q Kern., disparaged conventional tubes for heat transfer and emphasizing the need of extended surfaces.

Kern, D. Q., Speech delivered to Process Heat Exchanger Society, Houston, TX, January 20, 1958.

1

Thickness of a Cylindrical Pipe Insulation

Objective: Emphasize the following principle:

- Steady state conduction in composite wall (Temperature profiles with radius);

Problem Statement

The wall of a small cylindrical test chamber is made of stainless steel and glass, as shown in Fig.1.1. It is desired to reduce the heat loss from a chamber by adding an Insulation layer of 85% magnesia. Of interest is determining where to insert the insulation layer so that the rate of heat loss is lowest. One student recommended adding the 85% magnesia layer on the cold side (glass) while a second student suggested the hot side (steel). A third student claimed that it does not matter where to add the insulation. You are asked to carry out a study to determine the option that will result in the lowest rate of heat loss from the chamber. Also draw the temperature profile across the composite wall for best option.

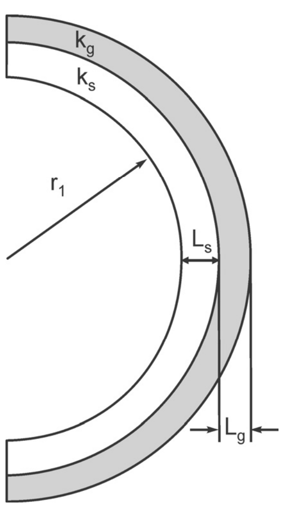


Fig. 1.1 Chamber Wall – Steel inside, glass outside

***Known quantities*:**

Chamber's inside radius, r_1 = 8.5 cm

Stainless steel thickess, L_s = 1.5 cm

Glass thickness, L_g = 1 cm

85% magnesia thickness, L_m = 4 cm

$T_{\infty,\ steel}$ = 100 ^oC

$h_{\ steel}$ = 8 W/m^2–^{o}C

$T_{\infty\ glass}$ = 10^{o}C

$h_{\ glass}$ = 16 W/m^2–^{o}C

NAME	EXPRESSION	VALUE	DESCRIPTION
k_ s	43 [W/(m*deg C)]	43[W/(m-K)]	1% Carbon Steel @ 20C
k_ g	0.7[W/(m*deg C)]	0.7[W/m-K)]	Window Glass @ 20C
K_ m	0.065[W/(m*deg C)]	0.065[W/(m-K)]	85% Magnesia @ 20C
T _ infl	100[deg C]	373. 15[K]	T Inside Chamber
T _ inf4	10[deg C]	283. 15[K]	T Outside Chamber
h _ 1	8[W/(m^2*deg C)]	8[W/(m^2-K)]	h inside Chamber
h_4	16[W/(m^2*deg C)]	16[W/(m^2-K)]	h Outside Chamber

Theory

Conduction in Cylinder

Consider a hollow cylinder of length L (shown below), whose inner and outer surfaces are exposed to fluids at different temperatures. The system is analyzed by the standard method as follows:

- The heat transfer rate is obtained by using the temperature distribution with Fourier's law:

$$q_r = -kA\frac{dT}{dr} = -k(2\pi rL)\frac{dT}{dr}$$

$$= \frac{2\pi Lk(T_{s,1} - T_{s,2})}{\ln(r_2/r_1)}$$

- Above Equation shows that the heat transfer rate q_r is a constant in the radial direction.

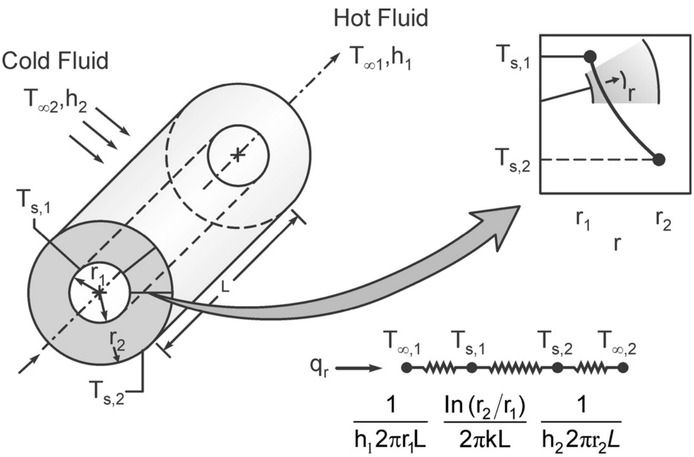


Fig. 1.2 Composite cylindrical wall resistance

- From equation, the thermal resistance for radial conduction in a cylindrical wall is

$$R_{t,cond} = \frac{\Delta T}{q_r} = \frac{\ln(r_2/r_1)}{2\pi L k}$$

Composite Cylindrical Wall

Consider a composite cylindrical wall of length L shown below.

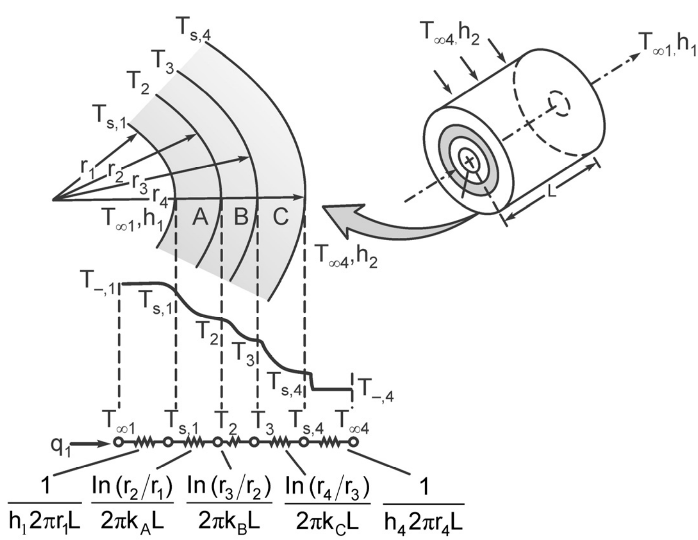


Fig. 1.3 Temperature profile across composite cylindrical wall

- Neglecting interfacial contact resistances, the heat transfer rate may be expressed as:

$$q_r = \frac{T_{\infty,1} - T_{\infty,4}}{\frac{1}{2\pi r_1 L h_1} + \frac{\ln(r_2/r_1)}{2\pi k_A L} + \frac{\ln(r_3/r_2)}{2\pi k_B L} + \frac{\ln(r_4/r_3)}{2\pi k_C L} + \frac{1}{2\pi r_4 L h_4}}$$

W M rite the following function in MATLAB editor

```
function [q T]=compositewall(Ti,To,hi,ho,ri,x,k)
% function [q T]=compositewall(Ti,To,hi,ho,ri,x,k)
% Ti, To: Inside and outside Temperature, in K
% hi, ho: Convective heat transfer coefficients, W/mK
% ri: Inner radius, m
% x, k: thickness, thernal conductivities of composite wall
% q: heat loss rate per unit length, W
wallcount=length(x);
DT=Ti-To;
InnerConvRes=1/(2*pi*ri*hi);
OuterConvRes=1/(2*pi*ri*ho);
r(1)=ri;
for i=1:wallcount
    r(i+1)=r(i)+x(i);
end
CondRes=0;
for i=1:wallcount
    wallRes(i)=log(r(i+1)/r(i))/(2*pi*k(i));
    CondRes=CondRes+wallRes(i);
end
q=DT/(InnerConvRes+CondRes+OuterConvRes);
T(1)=Ti-q*InnerConvRes;
for i=1:wallcount
    T(i+1)=T(i)-q*wallRes(i);
end
T(i+2)=T(i)-q*OuterConvRes;
```

Execution Procedure: Execute the following commands from MATLAB command prompt & observe the results

```
>> [q T]=compositewall(373, 273, 8, 16, 0.085, [0.015 0.01 0.04], [43 0.7 0.065])

q =

   88.2788


T =

  352.3382  352.2851  350.3721  283.3309  340.0412

>> [q T]=compositewall(373, 273, 8, 16, 0.085, [0.015 0.04 0.01], [43 0.065 0.7])

q =

   83.9468
```

```
T =

  353.3521  353.3016  284.1408  282.8239  274.3168

>> [q T]=compositewall(373, 273, 8, 16, 0.085, [0.04 0.015 0.01], [0.065 43 0.7])

q =

   76.2490


T =

  355.1538  283.1512  283.1192  281.9231  274.1961

>>
```

Draw Temperature profile with respect to radius once q and k are known:

Step 1: Write the following function and save the files as compwallode.m to evaluate ODE

```
function dtdr=compwallode(r,T)
global k;
q=76.249; % Watts
dtdr=-q/(k*2*pi*r);
```

Step 2: Write another function and saveit as compTprofile.m to solve ODE for the given composite wall with values specified for q, k and the radius range:

```
function compTprofile()
global k;
k=0.065; %W/mC
[r1 T1]=ode45('compwallode',[0.085 0.125],100);
len=length(T1);
k=43;
[r2 T2]=ode45('compwallode',[0.125 0.145],T1(len));
len=length(T2);
k=0.7;
[r3 T3]=ode45('compwallode',[0.14 0.15],T2(len));
r=[r1;r2;r3];
T=[T1;T2;T3];
plot (r*100, T);
title('Temperature profile of composite wall);
xlabel('radius, cm');
ylabel('Temperature, C');
```

Step 3: From Command prompt, execute the program:

Following graph will be the result:

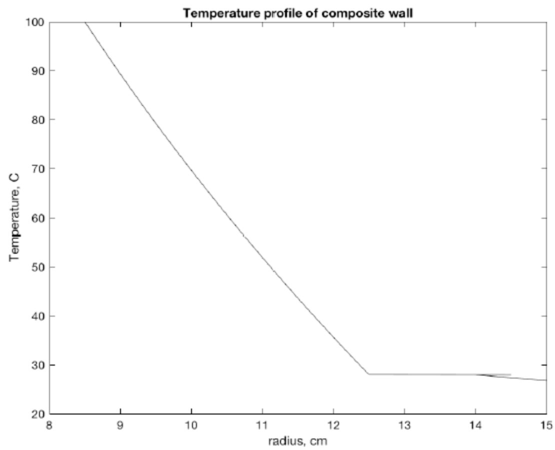
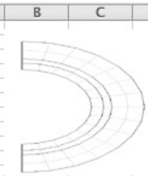
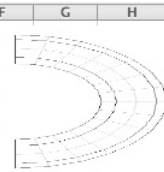
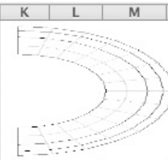


Fig. 1.4 Temperature profile across composite wall

Open an Excel sheet. Observe that the spreadsheet consists of multiple cells where you can enter the data (with columns numbered with Alphabets and the rows numbered with numerical digits).

Throughout this book, the cell numbers are used to explain the formulae used. For example A1 is a cell of Ath Column and 1st row. User is allowed to enter any text in the cells, however if it starts with '=' symbol, it will treat it as formula.

Excel Solution for the Problem:

Magnesia added on Cold side (Glass)			
radius, m		K, W/mC	
r_1	0.085	k_1	43
r_2	0.1	k_2	0.7
r_3	0.11	k_3	0.065
r_4	0.15		

Resitance 1= 0.000602 C/W
Resitance 2= 0.02167 C/W
Resitance 3= 0.759424 C/W
q1= 115.1343 W

Magnesia added between Glass & Steel			
radius, m		K, W/mC	
r_1	0.085	k_1	43
r_2	0.1	k_2	0.065
r_3	0.14	k_3	0.7
r_4	0.15		

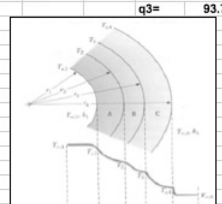
Resitance 1= 0.000602 C/W
Resitance 2= 0.823863 C/W
Resitance 3= 0.015686 C/W
q2= 107.1236 W

Magnesia added inside Steel			
radius, m		K, W/mC	
r_1	0.085	k_1	0.065
r_2	0.125	k_2	43
r_3	0.14	k_3	0.7
r_4	0.15		

Resitance 1= 0.944307 C/W
Resitance 2= 0.000419 C/W
Resitance 3= 0.015686 C/W
q3= 93.71 W

From Above calculations, we can conclude that the heat transfer rates are different
with the location of insulation layer and
Insulation layer (Magnesia) should be added inside the steel wall
to minimize the heat losses. Counter

$$q_r = \frac{T_{\infty,1} - T_{\infty,4}}{\frac{\ln(r_2/r_1)}{2\pi k_A L} + \frac{\ln(r_3/r_2)}{2\pi k_B L} + \frac{\ln(r_4/r_3)}{2\pi k_C L}}$$



Formulae Used for the Calculations:

Cell ID Formula used

C19 = (LN(B15/B14))/(2*3.1416*D14*1)

C20 = (LN(B16/B15))/(2*3.1416*D15*1)

C21 = (LN(B17/B16))/(2*3.1416*D16*1)

Note that the formula can be entered for C19 and the formulae for C20, C21 can be obtained by holding the right bottom corner of C19 cell and drag to copy the formula for C20, C21

C19 = (LN(B15/B14))/(2*3.1416*D14*1)

C20 = (LN(B15/B14))/(2*3.1416*D14*1)

C21 = (LN(B17/B16))/(2*3.1416*D16*1)

Note that the formula can be entered for C19 and the formulae for C20, C21 can be obtained by holding the right bottom corner of C19 cell and drag to copy the formula for C20, C21

B22 = (100-10)/SUM(C19:C21)

H19 = (LN(G15/G14))/(2*3.1416*I14*1)

H20 = (LN(G16/G15))/(2*3.1416*I15*1)

H21 = (LN(G17/G16))/(2*3.1416*I16*1)

***Note*:** Select cells C19 to C21, Copy them and past them in cell H19 also will give you the results same as per the mentioned formulae.

G22 = (100-10)/SUM(H19:H21)

M19 = (LN(L15/L14))/(2*3.1416*N14*1)

M20 = (LN(L16/L15))/(2*3.1416*N15*1)

M21 = (LN(L17/L16))/(2*3.1416*N16*1)

N22 = (100-10)/SUM(M19:M21)

References

Chapter 2 One dimensional steady state conduction,

faculty.kfupm.edu.sa/CHE/Shammakh/files/myfiles/300-chapter-2.pdf.

COMSOL Multiphysics Instruction Manual – eportfolio.lib.ksu.edu.tw.