1. Force, Work, Power and Energy

Unit 1: Topics covered

Force, Work, Power and Energy

(i) Turning forces concept; moment of a force; forces in equilibrium; centre of gravity; [discussions using simple examples and simple numerical problems].

Elementary introduction of translational and rotational motions; moment (turning effect) of a force, also called torque and its cgs and SI units; common examples - door, steering wheel, bicycle pedal, etc,; clockwise and anti-clockwise moments; conditions for a body to be in equilibrium (translational and rotational); principle of moment and its verification using a metre rule suspended by two spring balances with slotted weights hanging from it; simple numerical problems; Centre of gravity (qualitative only) with examples of some regular bodies and irregular lamina.

(ii) Uniform circular motion.
As an example of constant speed, though acceleration (force) is present. Differences between centrifugal and centripetal force.

(iii) Work, energy, power and their relation with force.

Definition of work. $W=FS\cos\theta$; *special cases* $\theta=0^0$, 90^0. $W=mgh$ *Definition of energy, energy as work done. Various units of works and energy and their relation with SI units. [erg, calorie, kWh and eV]. Definition of Power,* $P=W/t$; *SI and cgs units; other units, kilowatt (kW), megawatt (MW) and gigawatt (GW); and horse power (1hp=746W) [Simple numerical problems on work, power and energy].*

(iv) Different types of energy (e.g. chemical energy, mechanical energy, heat energy, electrical energy, nuclear energy, sound energy, light energy).

Mechanical energy: potential energy $U=mgh$ *(derivation included) gravitational PE, examples; kinetic energy* $K= ½ mv^2$ *(derivation included); forms of kinetic energy: translational rotational and vibrational - only simple examples. [Numerical problems on K and U only in case of translational motion]; qualitative discussions of electrical, chemical, heat, nuclear, light and sound energy, conversion from one form to another; common examples.*

(v) Machines as force multipliers; load, effort, mechanical advantage, velocity ratio and efficiency; simple treatment of levers, pulley systems showing the utility of each type of machine.
Functions and uses of simple machines: Terms- effort E, load L, mechanical advantage $MA = L/E$, *velocity ratio* $VR=V_E/V_L=d_E/d_L$, *input* (W_1), *output* (W_0), *efficiency* (η), *relation between* η *and MA, VR (derivation included): for all practical machines* $\eta<1$; $MA<VR$.
Lever: principle. First, second and third class of levers; examples: MA and VR in each case. Examples of each of these classes of levers as also found in the human body.
Pulley system: single fixed, single movable, block and tackle; MA, VR and η *in each case.*
Gears; *types of gear, applications, analysis of gear pair, gear ratio, uses*

(vi) Principal of Conservation of energy.

Statement of the principle of conservation of energy; theoretical verification that $U + K = constant$ *for a freely falling body. Application of this law to simple pendulum (qualitative only; [simple numerical problems].*

I orce

Turning forces concept; moment of a force; forces in equilibrium; centre of gravity; [discussions using simple examples and simple numerical problems]

Elementary introduction of translational and rotational motion; moment (turning effect) of a force, also called torque and its cgs and SI units; common examples – door, steering wheel, bicycle pedal etc.; clockwise and anti-clockwise moments; conditions for a body to be in equilibrium (translational and rotational); principle of moment and its verification using a metre rule suspended by two spring balances with slotted weights hanging from it; simple numerical problems; centre of gravity (qualitative only) with examples of some regular bodies and irregular lamina.

GENERAL

Force[1]

In simple terms, a **force** is a push or pull acting upon an object as a result of its interaction with another object. In most cases, the action results in a 'change' of physical position of the object(s)

Alternatively,

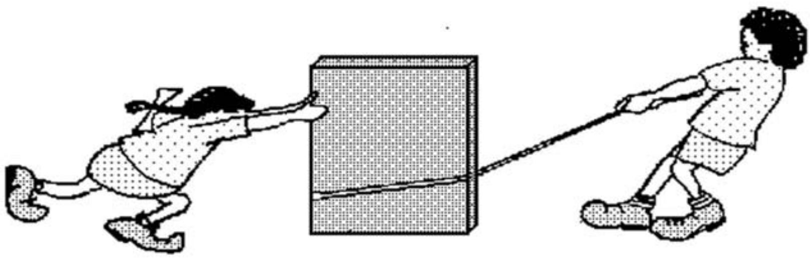
the concept of force can be best comprehended by reference to Newton's first law of motion stated as
"an object will remain at rest or in a constant state of motion unless acted upon by net external forces".

If interpreted differently, the object may be imagined to be stationary, that is with no or 'zero' motion being at rest, and would continue to be in that state unless made to change its

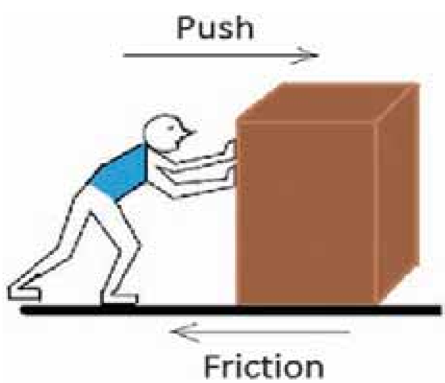
[1]From Latin *fortis* meaning "strong". In English, the word "force" can assume numerous meaning and connotation; for example, coercion, strength, organised body of soldiers in army or police, influence, drive etc. to name few. Also, colloquially, "a force to reckon", "force the issue", force a person's hand" (make a person act prematurely or unwillingly) etc.

position. The agency or physical entity that can bring about the change of state of an ob ect from rest or *uniform* motion can be termed or defined as C . In terms of action, this may entail a push or pull of the ob ect and hence force can be defined as a **p s** or **p** in a layman s language.

ome examples of push and or pull force are illustrated below.



(a) push and pull forces



(b) push force
[observe the action of friction force]

(c) pull force

ig. . xamples of pull and push forces

ote that the tendency of an ob ect to be at rest or in uniform motion can also be recognised as the concept of inertia due to Galileo , often termed simply the aw of Inertia .

ontact and on contact Force

ontact orce

In physics, a **contact orce** is a force that acts at the point of contact between two ob ects, without coming physically in contact with them.

Galileo Galilei Italian polymath - .

The arious Types of Contact Force

- Applied Force
- Frictional Force
- Air esistance Force
- pring Force
- Tension Force
- Normal Force

Applied Force**:**
Force which is applied to an object by another object. A person pushing his car is an example of applied force. When the person pushes the car, there is a force acting upon the car. The applied force is the force exerted on the car by the person.

Frictional Force**:**
Friction force is the result of two surfaces being pressed together closely.

This causes intermolecular attractive forces between molecules of different surfaces resulting in friction. The frictional forces depend upon the nature of the surfaces. The more rough the surface the more the frictional force.

Air esistance Force**:**
The Air resistance force acts upon objects as they travel through the air. This force opposes the motion of the object in the air; for example, a skydiver diving in the sky or a train running at hundreds of km hr speed.

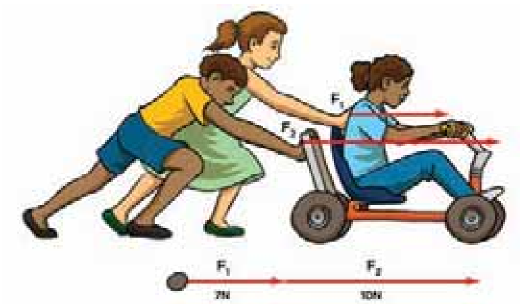
pring Force**:**
The pring force is the force exerted by a compressed or stretched spring upon any object which is attached to it. ere the magnitude of the spring force is directly proportional to the amount of stretch or compression of the spring.

Tension Force**:**
The tension force is transmitted through a string, rope or cable. The tension force is directed along the length of the wire. This force pulls the objects equally on the opposite ends of the wire.

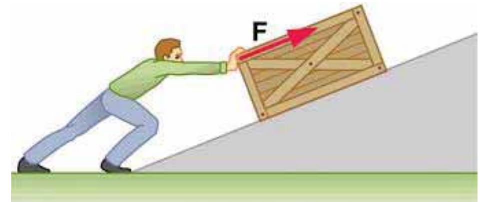
Normal Force**:**
This is also called the support force. If an object is resting upon a table then the table is exerting an upward force upon the object in order to support the weight of the object. If a person leans against a wall, the wall pushes horizontally on the person.

ome Images of Contact Force

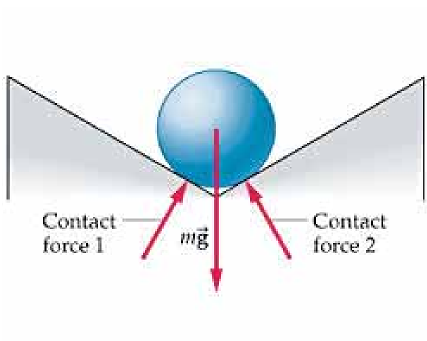
A few images of contact force are illustrated in Fig.1. .



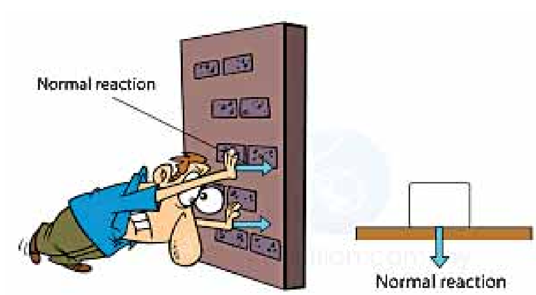
(a) a boy and girl pushing a child in a cart



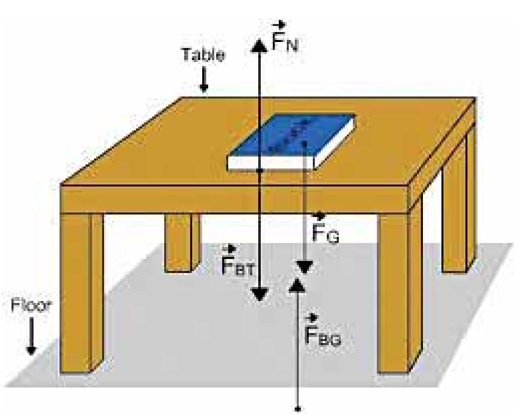
(b) a man pushing a box up an incline support



(c) a ball resting in a cut section



(d) a man pushing against a vertical wall



(e) a book resting on a table

(f) a boy kicking a ball

Fig.1. Images of contact force

on contact orce

The force which acts on an ob ect without coming physically in contact with it is called **non contact orce**.

ome of the commonly known non-contact forces

- Gravity

A well-known example is the force due to gravity or gravitational force; the force that is exerted by way of a pull on a body such as the one falling from a height. The force is given by

force mass of the body x acceleration due to gravity

that is obtained in newton if the mass is in kg and acceleration, g, is m s s. The other example is that of the force of attraction that exists among all bodies that have mass. The force exerted on each body by the other through weight is proportional to the mass of the first body times the mass of the second body divided by the square of the distance between them.

- Magnetic orce

A permanent magnetic has a field of its own and therefore can exert a force of push or pull, that is attraction or repulsion, on a piece of iron or steel, or even another magnet, that would depend on the distance of the piece from the magnet.

- orce due to lectromagnetism

An electromagnet when excited, that is, when a current flows through its winding, exerts a pull or push, or a force of attraction or repulsion on a ferromagnetic body, similar to that of a permanent magnet. The force comes into play when the body is not in contact with the electromagnet, the strength of the force being inversely proportional to the distance, in general.

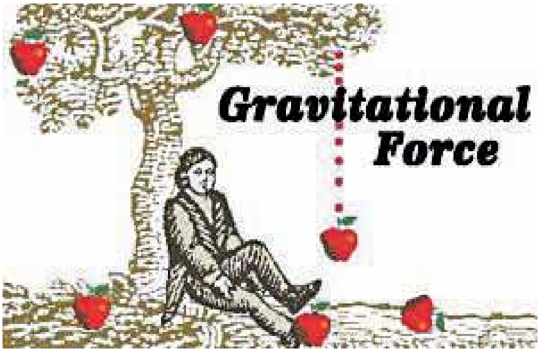
- lectrostatic orce

A body possessing electrostatic charge can exert an attractive or repulsive force on a body at a distance by virtue of electrostatic induction.

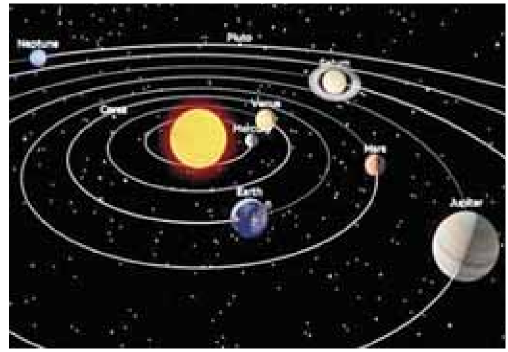
- uclear orce

Unlike gravity and electromagnetism, the strong nuclear force is a short- distance force that takes place between basic particles within a nucleus. It is charge independent and acts equally between a proton and a proton, a neutron and a neutron, and a proton and a neutron. The strong nuclear force mediates both nuclear fission and fusion reactions.

ome images of non-contact force are illustrated in ig. . .



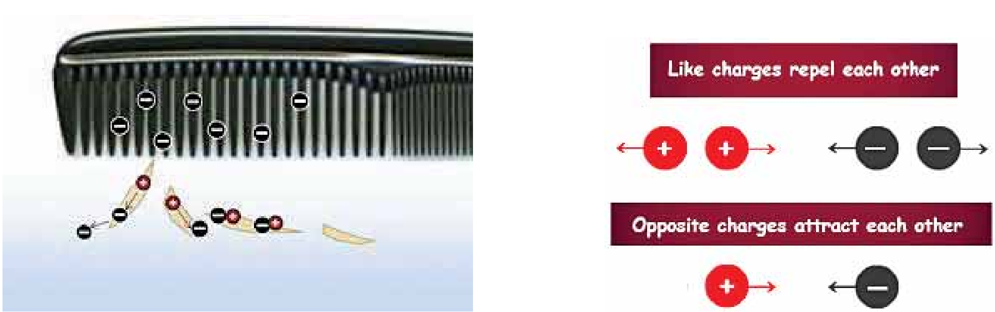
(a) force due to gravitation



(b) inter-planetary gravitational force

(c) force due to permanent magnet(s)

(d) magnetic pull of an electromagnet



(e) electrostatic force

Fig.1. Images of non-contact force

Friction

An important aspect often associated with force is **friction.** The tendency of friction is to oppose the applied force, in general; that is, to act in a direction *opposite* to the direction of force. Whilst friction is essential in relation to various activities in day-to-day life, including making it possible for one to walk, its detrimental effect, particularly in moving or rotating machines such as a motor, is to act in retarding the motion and thus lower the performance and efficiency of the machine; the frictional power loss being subtracted from the useful power produced in the machine. In such applications, it is much desirable to reduce friction by, for example, lubricating the moving parts like bearings; in the same vein apply oil to rusted hinges of a door to make it open or close smoothly.

antitative Interpretation of Force

uantitatively, the first factor to specify the quantum of force required to change the state of an object or body would be the *mass* of the body, m greater the mass, greater would be the force. The second aspect is the phenomenon that follows the application of the force. If the body is at rest in a given position, the force applied to it at a point or surface will result in its movement to acquire a new position, the body moving with a velocity. owever, the velocity (v) is not reached instantaneously, but takes some time (t). The same arguments apply to a body already in motion at an initial velocity v_o that is changed to v_1 on application of the force. This change of *velocity* from initial (for body at rest) to final brings in the physical entity called acceleration, defined by "rate of change of velocity with time", or dv dt, and denoted by f or a. Thus, the term force required to change the state of the body will be a product of the mass of the body and the acceleration caused by the force, or simply

F m x f

in any system of units.

Newton s econd Law of otion

The concept of force is also brought out by Newton's second law of motion stated as "The relationship between an object s mass m, its acceleration a, and the applied force F is F ma both, acceleration and force being vector quantities, the latter directed *in the direction of acceleration*" .

ir Isaac Newton, F , English mathematician 1 -1 .

This is the most powerful of Newton s three Laws, because it allows quantitative calculations of force in the dynamics how do velocities change when forces are applied. In contrast, according to Aristotle [Greek philosopher (BC) (BC)], the force is defined by F m x v where v is the *velocity* acquired by the body under the action of the force. Thus, according to Aristotle there is only one velocity if there is a force, but according to Newton an object with a certain velocity maintains that velocity *unless* a force acts on it to cause an acceleration (that is, a change in the velocity).

r

"The acceleration of an object is directly related to the net force acting on it and inversely proportional to its mass".

Unit of Force

The I unit of force is newton, named after the great British scientist; the symbol of the unit being N.

By definition,

ne unit of force or one newton (in I units) one unit of mass in kg x one unit of acceleration in m s s

and similarly in other systems of unit .

Example

Two bodies have masses in the ratio of . When a force is applied to the first body, it moves with an acceleration of m s s. What will be the acceleration produced in the second body by the same force

et the mass of body A be kg and that of body B be kg.
The force acting on body to produce an acceleration of m s s will be
x or
The same force when applied to body B will result in an acceleration of
α or . m s s

Trans ationa and otationa otions

Trans ationa otion

Translational motion is the motion by which a body shifts from one point in space to another. ne example of translational motion is the motion of a bullet fired from a gun. The other common examples of translational motion are

- a car moving on a road in a given direction

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or example, in CG system the unit is called dyne defined as a mass of one gramme x acceleration of one cm s .

- a train travelling on a ‘straight’ railway track

Recti inear otion

Recti inear otion is another name for straight-line motion of a body or object. A body is said to experience rectilinear motion if *any two* particles of the body travel the same distance along two parallel straight lines as depicted in Fig.1. .

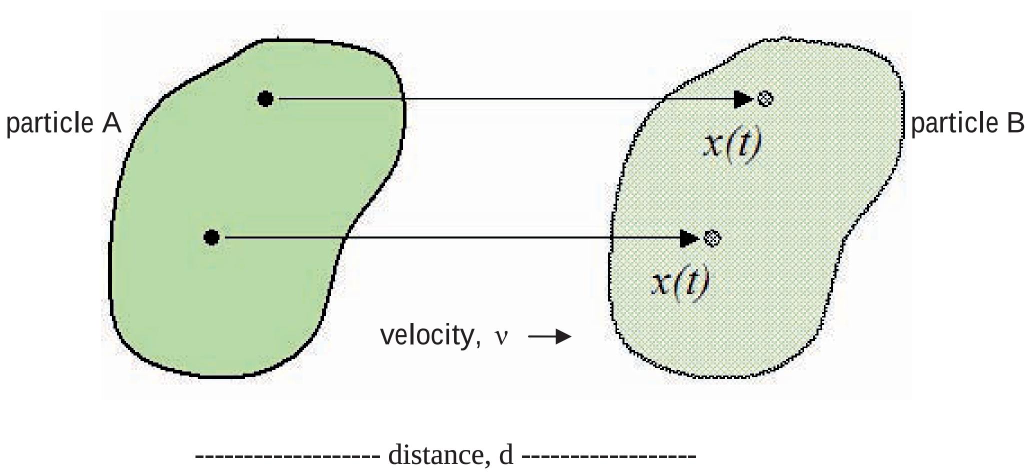


Fig.1. ectilinear motion

As shown in the figure, particles A and B of a body *move together* in direction x and reach a position defined by x(t) at time t, having moved a distance d units. For a body in rectilinear motion as a whole, the relationship of the distance in terms of a *constant* velocity, v, and time t is expressed by a linear equation for a rectilinear motion, given by

$$d \quad v \text{ x } t$$

in a given system of units.

It is important to note that a rectilinear motion is characterized, in general, by

- a constant velocity
- a varying velocity due to a constant acceleration
- a varying velocity accompanied by a varying acceleration
  with time, having their own implications in practice to relate the quantities with displacement, velocity and acceleration.

E ations governing recti inear otion

In case of a *constant* acceleration, the U physical quantities, vi . acceleration, velocity, time and displacement, can be related by using the equations

$$_{(final)} = \ _{(initial)} + at$$

$$D = \ _{(initial)} + - at$$

$$_{(final)} = \ _{(initial)} + \ aD$$

$$D \ - [\ _{(final)} \ \ _{(initial)}] \ t$$

where

$_{(initial)}$ is the initial velocity
$_{(final)}$ is the final velocity
a is the acceleration
D is the displacement, and
t is the time

in appropriate system of units, pertaining to the body in motion.

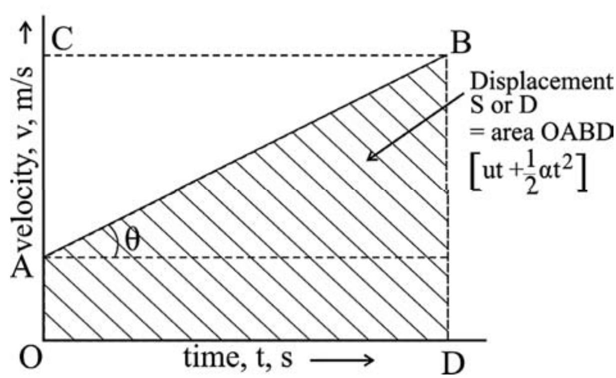
These relationships can be demonstrated graphically as depicted in ig. . . The gradient of a line on a displacement time graph represents the velocity. The gradient of the velocity time graph gives the acceleration while the area under the velocity time graph gives the displacement. Thus, in ig. .

A initial velocity u at t
C velocity ν at time t

u α t

α (constant) acceleration

slope of line AB
[tan θ]



ig. . Graphical representation of velocity time relationship and equations of motion in rectilinear mode

Thus, referring to the figure, line AB represents the motion with an initial velocity u at t and increasing with a constant acceleration a. At the end of time t, the velocity is v. The slope of the line being (v-u) t is simply the acceleration, giving v u at. The area of the trapezium ABD represents the distance traversed in time t, given by (u v) x t resulting in s (u v) as the distance. ther expressions involving u, v, a and s can be obtained by simple manipulation of the above equations.

Example

1. A car of mass kg moving at km hr is brought to rest after covering a distance of 1 m. Calculate the retarding force acting on the car.

Mass of the car kg
Initial velocity, u km hr; final velocity, v .
Distance covered, s 1 m
Using the expression v u a s
- 1 x a x 1 , giving a - m s s [negative sign denoting retardation]
From F m a,
retarding force 1 N

. A small (toy) car of mass g travels with a uniform velocity of m s for s. The brakes are then applied and the car is uniformly retarded and comes to rest in further 1 s. Calculate (i) the retardation, (ii) the distance that the car would travel after the brakes are applied, (iii) force exerted by the brakes.

The mass of the car is g or . kg
Initial velocity, u m s; final velocity, v .
The time of travel being s, the distance covered, s x or 1 m
The time for car to come to rest, t 1 s
(i) from equation α (v u) t by substitution
α (.) 1 or the retardation, α . m s s
(ii) using the equation α s v u and substituting
s (x) x . or the distance to brake 1 m
[Note that the velocity m s corresponds to km hour and sudden application of brakes to the car would still drag the car for a distance of *125 m before it come to rest*]
(iii) the force applied would simply be, F m x α or . x . 1. N

. A cricket ball of mass 1 g strikes the hand of a player with a velocity of m s and is brought to rest in . 1 s. Calculate (i) the force applied by the hand of the player, (ii) the acceleration of the ball during the action.

ere the "initial" velocity of the ball, u, is m s. As the ball comes to rest in the player's hand its final velocity, v, is zero, the time, t, taken being . 1 s.
Using the expression v u α t and substituting

Acceleration of the ball α — - m s s
.

[the negative sign indicates *retardation* of the ball]
Then, the force acting on the ball is

m x α
. x or

. A lead bullet of mass g, moving with a velocity of m s, comes to rest after penetrating cm in a still target. Calculate (i) the resistive force offered by the target, (ii) the retardation produced.

ere, initial velocity of the bullet, u m s
final velocity, v
distance traversed, s cm or . m [penetration in the target]

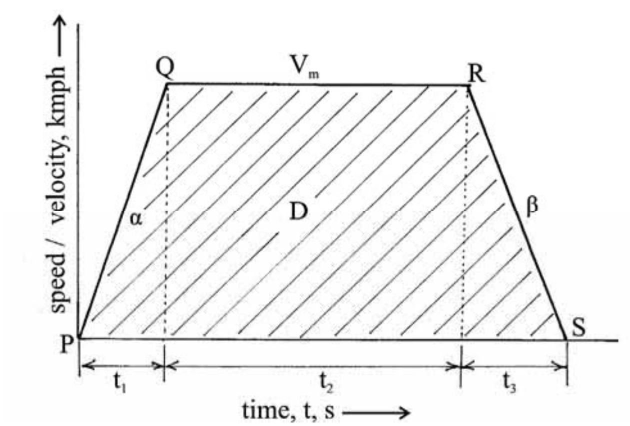
∴ using the expression v - u α s

the retardation, $\alpha = \frac{-(\quad)}{\times \ .}$ or . × m s s (magnitude alone)

and the resistive force by the target
. x . x or .

Speed-time graph of a typical electric train

An example of much importance involving the above quantities and equations is the depiction of speed-time graph of a typical electric traction or train movement, esp. the suburban services, as given in ig. . .



α acceleration, kmphps
m maximum, steady speed, kmph
β deceleration, kmphps
D distance travelled, km
(area under P)

ig. . peed-time graph of electric traction

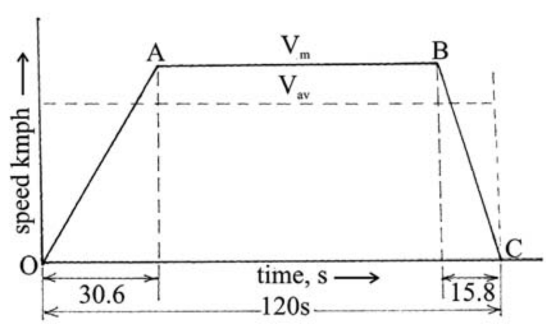
In the diagram shown in Fig.1. , the time, t, in *seconds* is plotted against abscissa and train speed, v, in *km/s* is plotted against the ordinate. The train starts from rest at a given station, P, picks up speed, reaches a 'maximum' speed and travels at that speed for a distance. At that point brakes are applied so that the train retards and comes to halt at the following station, . The entire movement is depicted in the graph by four points, P , and three sections

P related to acceleration of the train from rest at a *constant* acceleration, α, in *km/hr/s*;
denoting the train travel at a constant (or 'maximum') speed, v, in *km/hr*, reached at point B or instant of time;
the interval expressing deceleration (or retardation) of the train, β, in *km/hr/s*.

The distance P on the time axis denotes the total time of travel of the train *in seconds* whilst the area of the graph, bound by sections P and time line P , or area of the quadrilateral P , will work out to be the distance between the stations P and *in km*, depicted by the hatched or 'cross' lines.
uch speed-time diagrams are of immense importance in planning of train movements pertaining to a number of stations along a railway route. The parameters involved in the train movement, viz., acceleration, maximum speed, retardation are worked out with great care, including the halt of the train at intermediate stations (in seconds) to arrive at the most efficient and cost-effective overall movement of a given train; and there may be several trains moving on a given track with optimum time intervals in between the consecutive trains.

Example

An electric train has a maximum speed of kmph and an average speed of kmph on a level track between two stops 1, m apart. It is accelerated at 1. kmphps and braked at . kmphps. Draw the speed-time curve for the run.



Acceleration α 1. kmphps

∴ time of acceleration

$_m$ α . or . s
etardation, β . kmphps

∴ time of retardation or braking

$_m$ β , or . s
Distance of run , m . km

∴ time of run distance average speed . hr (.) x s or s

The speed-time curve is shown in the ad oining figure

otationa otion

As the name implies, rotational motion relates to motion of an object or body around an axis or point whilst maintaining constant distance from the axis of rotation. It deals only with rigid bodies'. In mechanics, a rigid body is an object that retains its overall shape, meaning that the particles that make up the body remain in the same position relative to one another. A rotating wheel of a motor is a common example of a rigid body that exhibits a rotational motion; on a much larger scale an example of rotational motion is the earth rotating about its axis.

Translational Motion versus otational Motion

There is a strong analogy between rotational motion and standard translational motion. Indeed, each physical concept used to analyse rotational motion has its translational concomitant. This is illustrated in the following table.

Table 1.1 Comparison of translational and rotational motion

Translational Motion		otational Motion	
Displacement	D	Angular displacement	ϕ
elocity	$v = \frac{dD}{dt}$	Angular velocity	$\omega = \frac{d\phi}{dt}$
Acceleration	$a = \frac{dv}{dt}$	Angular acceleration	$\alpha = \frac{d\omega}{dt}$
	$= \frac{d\ D}{dt}$		$= \frac{d\ \phi}{dt}$
Power	P Fv	Power	P Tω

TURNING F R E N EPT

This concept is applicable when a force is made to act on a body that is free to rotate, usually about a pivot or axis of rotation as depicted in Fig.1. where a force F (N) is acting at a distance D (m) at the end of a 'bar' from its pivoted end or "turning point" .

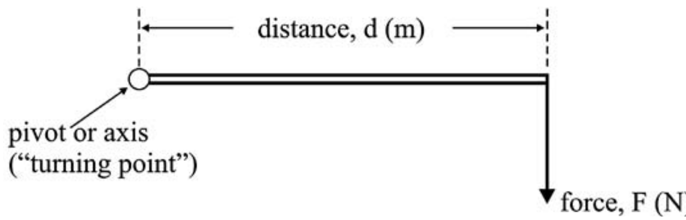


Fig.1. Concept of turning force

Force F is known as t rning force.

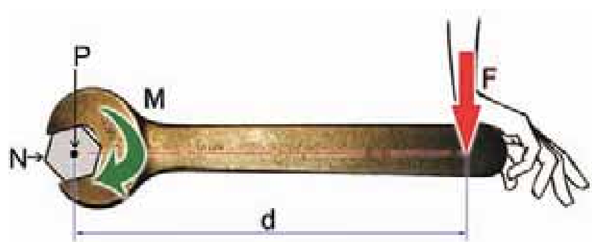
ENT F A F R E

More scientifically, the concept of turning force relates to a term called "moment of a force" that can be understood as *turning effect* of the force, defined as product of the **force** multiplied by the perpendicular distance from the line of action of the **force** to the pivot, point or the axis about which the object will turn.
Note that a more technical term often used, esp. in engineering, to express moment of a force is **tor e,** defined once again as

Torque force x its perpendicular distance from the point of (free) rotation of the body to which the force is applied

A simple example of production of torque is in the use of tightening (or opening) of a nut by means of a wrench as illustrated in Fig.1.

ere F represents the force applied by the hand in a clockwise direction to imply tightening of a right-handed bolt fitted with the nut, N; an anticlockwise application of the force would result in opening of the nut. Point P signifies the axis of rotation of the nut, whilst d denotes *the perpendicular* distance of the force from the axis of rotation.
The, moment of the force or torque F x d



ig. . A wrench tightening a nut

Unit of tor e

In the above example, and in general, if the force F is expressed in newton (N) and distance d in metre (m), the torque is obtained in newton-metre or simply Nm. Thus, the I unit of torque or moment of a force is Nm.

Likewise, in CG units when the force is given in dyne and distance in cm, the unit of moment of the force would be dune-cm.

t er co on e a p es o o ent o orce

A door being opened (or closed) about its hinges

As shown in fig.1. , , represent two hinges fitted on the door at appropriate distance each from the top and bottom edge. F is the force applied on the external surface of the door (implying closing action) at an angle θ from the vertical as shown such that its component perpendicular to the door surface would be F sin θ and *its* perpendicular distance from the axis of rotation, that is the hinges, is r as indicated. The moment of force in this case would be given by

F sin θ x r units .

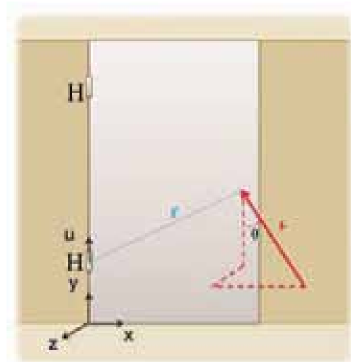


Fig.1. Closing of a door

teering wheel of a car

A photographic illustration of a steering wheel common to all motor vehicles is given in Fig.1.1 .

Note the position of hands on the steering wheel which should correctly be the " - 'clock" position for efficient manoeuvre of the vehicle. ere, the forces, F, on the wheel are applied by the driver's hands on the wheel's rim in the opposite directions at points P P, separated by a distance d; usually nearly equal to the diameter of the wheel. The force on the right hand is applied in anti-clockwise direction (assuming a "right-hand" driven vehicle) to turn the car to the left and *vice versa*.


Fig.1.1 Torque(s) applied to steering wheel of a car

This is a special case of production of torque by the force F, at a perpendicular distance of d from the axis of rotation of the wheel; in one direction on the 'right' side and in the opposite direction on the left with both the forces being applied simultaneously. If both the forces are assumed to be of same magnitude a situation not

What is the importance of the location of hinges with respect to door edges If the door is very heavy, should the number of hinges be increased to , or even necessarily obtainable in practice the phenomenon of production of *two* torques in the above fashion is termed as resulting in a "couple" as explained in mechanics. This is dealt with later.

Bicycle pedal(s)

A schematic of pedal arrangement in a typical bicycle is shown in Fig.1.11.

The picture in this case shows two 'key' wheels or sprockets that are responsible for producing forward movement of the bicycle as the rider pushes on the two pedals, marked P P. Considered individually, each pedal , the right one in the given instant, when pressed by the rider with a force F produces a moment or torque T given by F x D units, acting about the axis of rotation at the axle A.

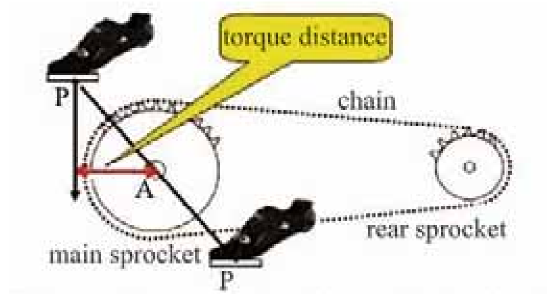


Fig.1.11 : Torques produced by pedals in a bicycle

The other pedal, or the left one, is very nearly idle being in the position shown in the figure till it reaches the upward position, similar to the right pedal, by virtue of forward motion of the bicycle and pressed by the rider downward, resulting in a torque as in the previous stage.

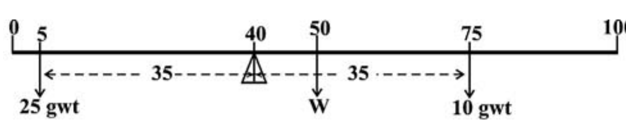
The pedals thus produce a forward torque alternately leading to forward motion of the bicycle. The torque(s) are then 'transferred' by means of the chain to rear sprocket that is actually connected to the rear wheel of the bicycle through the rear axle. bserve that although there are two pedals fitted at the same distance from the axis of rotation in opposite direction, they do not form a couple in the usual sense as, for example, in the case of the steering wheel of a car.

Example

1. A uniform metre scale is balanced on a fulcrum at cm mark when 'forces' of g wt. and 1 g wt. are suspended at cm and cm marks of the scale, respectively. Calculate the mass of the metre scale.

The configuration of the problem is given in the figure below.

ere, the forces expressed as g wt. (acting downward) are in effect masses in gramme, multiplied by



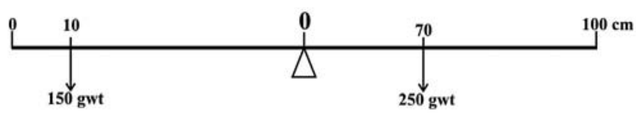
To be understood from the simple dictionary meaning "two" or a "pair".

The rear sprocket is designed to "free-wheel" such that the produced torque(s) result in motion of the bicycle *in forward direction only* and N movement in the reverse direction.

acceleration due to gravity, . 1 m s s in I units. If the masses were expressed in kg the forces would be obtained in newton. In the configuration shown, the forces acting at respective distances produce moments in clockwise and anticlockwise directions.
Let the mass of the scale be M g, the force being M g wt., assumed to be concentrated at mid point and acting downward, at a distance of 1 cm from the fulcrum and to its right as shown.

Therefore, clockwise moment M g wt. x 1 1 g wt. x
(1 M) g wt.-cm
and anticlockwise moment (x) g wt.-cm
Equating the two at balance
(1 M) g wt.-cm g wt.-cm
whence
M . g

. A uniform metre scale weighing g is pivoted at its mid-point, . Two forces of g wt. and g wt. are suspended from the scale as shown in the figure below. Calculate (i) total clockwise moment, (ii) total anticlockwise moment about . Also, calculate the distance from where a force of g wt. may be suspended to balance the scale hori ontally.



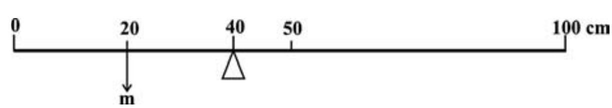
(i) Total clockwise moment x or g wt.-cm
(ii) Total anticlockwise moment x or g wt.-cm

The anticlockwise moment is greater than the clockwise moment by g wt.-cm. ence to balance the scale the g wt. force should be suspended at a distance of or cm *to the right of* .

. A uniform metre rule of mass g is balanced hori ontally on a fulcrum at cm mark. An unknown mass m is suspended from the rule at the cm mark. ind m . To which side the rule will tilt if mass m is moved towards cm mark. What is the resultant moment in this case ow can the rule be balanced by making use of another mass of g

The configuration related to the problem is as shown

The rule causes a clockwise moment of x or g wt.-cm
Correspondingly, the mass m should provide a counter-clockwise moment of m x gwt.-cm to balance the load



∴ m x
giving m g

If the mass m is moved to cm mark, the anticlockwise moment will be x or

1 gwt.-cm [see the figure]. ince this is greater than the clockwise moment, the rule would tilt left or in anticlockwise direction.

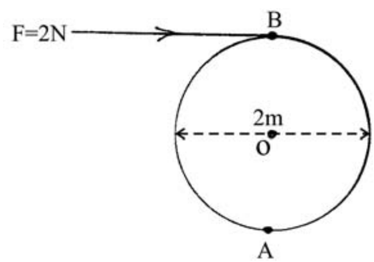
Let the distance at which the mass of g be placed to the right of the fulcrum to balance the rule in the last condition. Then the new balance equation will be

x 1 1

whence x 1 cm

that is, the mass (of g) should be suspended at " cm" mark of the scale.

. A wheel of diameter m is shown in the figure with axle at . A force F N is applied at B in the direction shown. Calculate the moment of the force about (i) centre ; (ii) point A.



(i) the moment at is given by

moment force x perpendicular distance
N x 1 (m)
Nm

(ii) moment at point A N x (m)
Nm

Bending Moment

A particular variation of moment of force is the "bending moment" (BM), an entity of significant importance in the design of weight-carrying protruding beams encountered in building construction and other engineering structures. The aspect is illustrated in Fig.1.1 where the beam B is rigidly fitted to a wall, carrying a weight W at the end. The moment of this weight, or force F, is given by

BM F x D

where D is the perpendicular distance of F from the wall as shown. bserve that the moment is proportional to D, varying from zero at the wall surface to a maximum at the point of location of F as depicted by the lower diagram in Fig.1.1 . The BM will lead to bending of the beam if the latter is not suitably designed for the purpose.

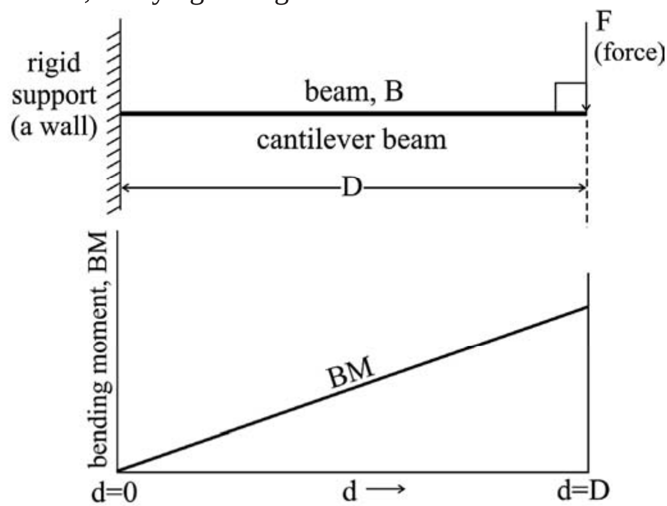


Fig.1.1 Bending moment of a cantilever beam

o p e

A couple is a pair of forces, *equal in magnitude*, oppositely directed, and displaced by a perpendicular distance between them as depicted in Fig.1.1 . The forces may be coplanar or positioned differently.

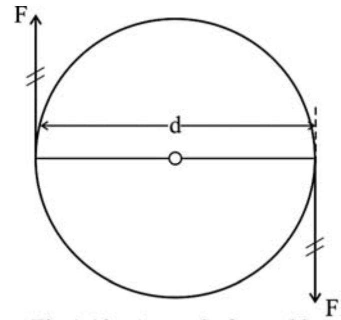


Fig.1.13 : A couple formed by two (coplanar) forces

This is also called a simple couple in which each force has a turning effect or moment, or a torque, about an axis which is *normal* (perpendicular) to the plane of the forces. The I unit for the couple is newton-metre (Nm) as for torque.
If the two forces are *F* and *F*, the negative sign implying the force being oppositely directed, then the magnitude of the torque is given by

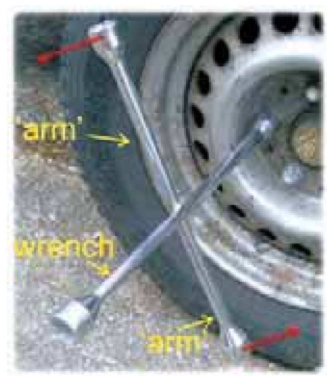
$$\tau \quad Fd$$

where

τ is the torque
F is the magnitude of each of the forces
d is the perpendicular distance between the forces, also called the *arm* of the couple.

An example of useful application of couple of forces is illustrated in ig. . for changing stepnee or spare wheel of a car using (box) spanner or wrench.

ote the position of wrench fitting the nut and the two arms at tight angles to the axis of rotation.



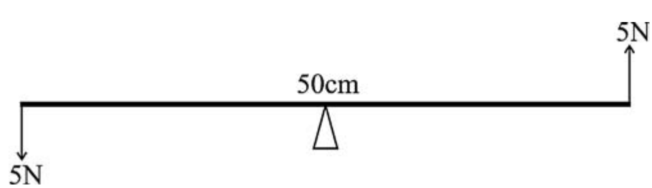
ig. . Changing stepnee or spare wheel of a car

Example

Two forces, each of act vertically upwards and downwards respectively at the two ends of a uniform metre scale which is suspended at its mid-point. Determine the magnitude of the resultant moment of these forces about the mid-point.

[IC]

The metre scale on its fulcrum is shown in the figure.



esultant moment of the two forces
moment of couple

force x perpendicular distances between them
x (x .)
m [anti-clockwise]

ockwise and ntic ockwise o ents

If the moment of a force turns or rotates the body in clockwise direction, then it is called as clockwise moment.
If the moment of a force turns or rotates the body in anti-clockwise direction, then it is called anti-clockwise moment.

efer to Fig.1. which shows the use of a wrench for opening or tightening a nut fitted on a bolt. The movement of hand in the figure *downward* would result in rotation or turning of the wrench clockwise; and this means a clockwise moment or torque. This also indicates tightening of the nut on a "right-handed" bolt. If the wrench is turned towards left or anticlockwise; this would result in an anticlockwise moment or torque, leading to opening of the nut.

eri ication o Princip e o o ent

The principle of moment can be explained in terms of balancing of clockwise and anticlockwise moments in a system of forces such that the net moment would be zero. pecifically, assume a 'rigid' rod or beam suspended at its mid-point from a rigid support, or supported on a fulcrum at mid-length. Then if two weights, W_1 and W are suspended from the beam at distances D_1 (on the right) and D (on the left) from the point of support, the weight (or force) W_1 would result in a clockwise moment of W_1 x D_1 Nm whilst the anticlockwise moment resulting from the weight on the left would be W x D Nm, assuming the forces to be in newton and distances in metre. Now if the two moments are equal such that there is no net moment and the beam remains horizontal, that is in the original condition, it would corroborate the principle of moment.

The principle can be verified by means of a simple experiment using a metre scale and a few weights that might be suspended from the scale variously, similar to the example discussed

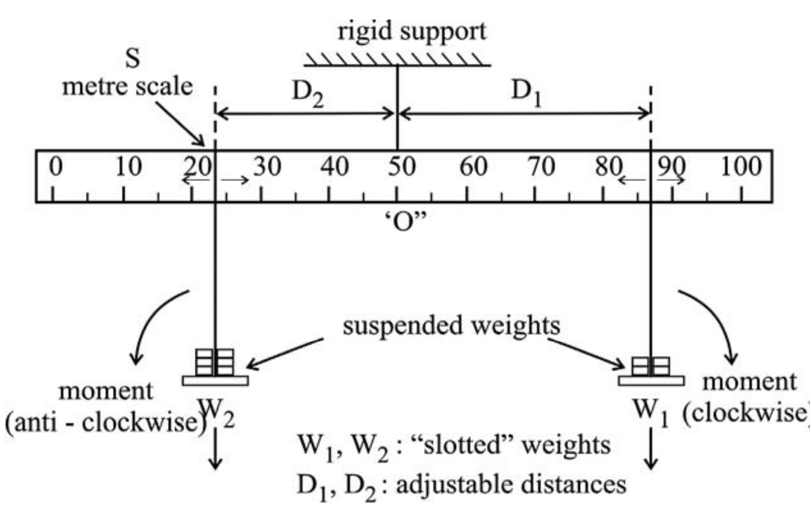


Fig.1.1 Experiment for verification of principle of moments

above. . The schematic of the set-up is shown in Fig.1.15 in which S is the metre scale suspended from a rigid support at its mid-length. The scale carries two "pans" having provision to add "slotted" weights of varying masses on either side of the "zero" position of the scale, each being adjustable from the mid-point and the distance measured from the mid-point.

For given weights W_1 and W_2, their relative distances from either side being D_1 and D_2 are adjusted such that the metre scale is horizontal as it would be without any weights. Under this conditions

the clockwise moment due to $W_1 = W_1 \times D_1$

and anticlockwise moment due to $W_2 = W_2 \times D_2$

and $W_1 \times D_1 = W_2 \times D_2$

This is verification of the principle of moments.

MOMENTUM

An important quantity in kinematic that is associated with mass AND velocity in the motion of a body is identified as **momentum**[9], defined as

A commonest example of momentum of a body is a moving train, having a mass of hundreds of killogramme (or a few tonnes), consisting of mass of the coaches and the engine of the train as well as all the passengers, the velocity of the train reaching over a hundred km/hr. The train thus acquires tremendous momentum whilst in motion resulting in a situation that makes it extremely difficult to stop the train in an emergency within a desirable distance.

The most common symbol for momentum in physics is p. The SI unit for momentum is kg m/s when mass is expressed in kg and velocity in m/s. Clearly, if velocity is in km/hr, the momentum will be given in kg km/hr.

Momentum is a *vector quantity* since it involves the quantity velocity that itself is a vector quantity.

Observe that momentum is a function of both mass and velocity and can be 'controlled' by either; see, for example, Fig.1.16.

[9]It is interesting that the word "momentum" also finds frequent use in idiomatic English to refer to impetus to continuity of efforts in an activity or 'project'.

Two bodies moving with (same) velocity,

s a ass, s a er o ent arge ass, arger o ent

Fig.1.1 Momentum of bodies of different mass and same velocity

Example

A force of 1 kg wt. is applied on a body of mass 1 g initially at rest for .1 s. Calculate (i) momentum acquired by the body, (ii) distance traversed by the body in .1 s. Assume acceleration due to gravity, g 1 m s s.

The mass of the body is 1 g or .1 kg
The force applied is 1 kg wt. This amounts to 1 x g or 1 x 1 1 N

∴ acceleration acquired by the body, α force mass or 1 .1 1 m s s
From equation v u α t and with initial velocity being . (at rest), the final velocity
v . 1 x .1 or 1 m s
Therefore (i) momentum m x v or .1 x 1 1 kg-m s
(ii) from the equation α s v u ,
the distance traversed s (v u) α and substituting
(1 x1) x1 or m

ange of o ent

This is defined as the difference of final and initial momentum of a body. Thus if mass of a body is M_1 at time T_1 and its velocity is $_1$, its momentum is p_1 M_1 x $_1$. If the mass is M and velocity at time T , the momentum would be p M x . Then the difference Δp M - M_1 $_1$, assuming 1, is the change of momentum in the time interval ΔT T T_1.

In most cases, the mass of the body may remain unchanged whilst the velocity may change increasing or decreasing on various counts; for example in the case of a moving train referred to earlier, the speed of the train may be *reduced* as a station approaches before applying brakes and the final momentum may be less than that before reduction of speed, the change of momentum may then be termed negative .

Rate of change of momentum

A term variously in use in kinematics is the rate of change of momentum and is defined as the ratio of "change of momentum and time interval" or simply

$$\Delta p/\Delta T$$

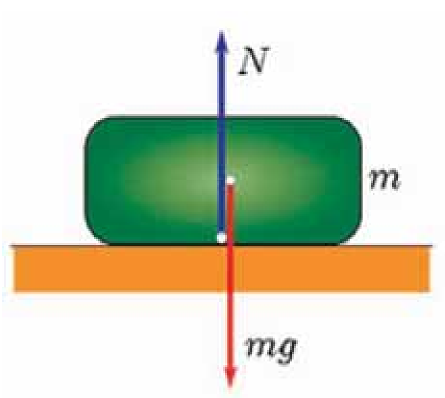
with the appropriate unit associated with it.

FORCES IN EQUILIBRIUM

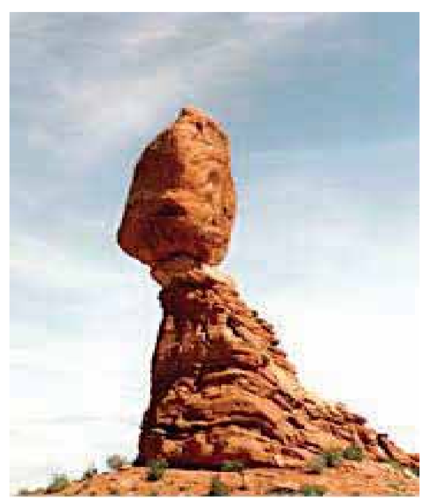
Static Equilibrium

A concept of much importance in mechanics or physics when dealing with forces is the idea of **equilibrium** or **balance of the forces**. In general, an object or body can be acted on by several forces at the same time. A force being a vector quantity has both a magnitude and a direction associated with it. If the magnitude and direction of the forces, or their 'appropriate' components acting on the body are exactly balanced, then there is no **net force** acting on the body and it is said to be in **equilibrium**. In the absence of a net force acting on an object in equilibrium, an object at rest will stay at rest, and an object in motion will continue to be in motion according to Newton's first law of motion.

Two examples in practice from common experience are illustrated in Fig.1.17.



(a)



(b)

Fig.1.17 : Images of static equilibrium

In Fig.1.17(a). a body of mass m is at rest on a table top. If acted upon vertically upward by a force, N, it may have a tendency to lift up. However, if the magnitude of N is exactly equal to the body's weight, mg, and the direction of the two forces match, the body will continue to remain “at rest” on the table. This represents a simple example of “static equilibrium”

Fig.1.1 (b) shows a nature’s creation in stones, precariously remaining balanced over ages, exhibiting another example of static equilibrium.

In general, the forces are considered to be balanced if the rightward forces are balanced by the leftward forces and the upward forces are balanced by the downward forces. This however does not necessarily mean that all the forces are equal to each other. Consider the two objects pictured in the ‘force diagram’ shown in Fig.1.1 . Note that the two objects are at equilibrium because the forces that act upon them are balanced; however, the individual forces are not equal to each other. The N force is not equal to the N force.

These two objects are at equilibrium since the forces are balanced.

Fig.1.1 orizontal and vertical forces on two bodies and equilibrium

Clearly, if there is an imbalance between the forces on opposite sides such that one is greater than the other, the body would begin to move in an appropriate direction and would no longer be in the state of equilibrium.

Triang e o Forces

A very common occurrence in practice, requiring static equilibrium, for example in trusses in engineering design of building construction, is the balance of *three* forces that may be simply coplanar but in different directions. The balancing of forces in such cases follows the concept of “triangle of forces”. As a simple illustration of this concept, consider the diagram of Fig.1.1 .

hown in the figure is a body of mass M, suspended by two strings , tied to two parallel, vertical walls at points P. Both the strings are equal in length and tied to the walls at the same height from the ground. Then the forces F_1 and F balance the weight of the body, expressed as a vertically downward force F , such that the body hangs in the middle, equidistant from the walls, the three forces being *coplanar*.

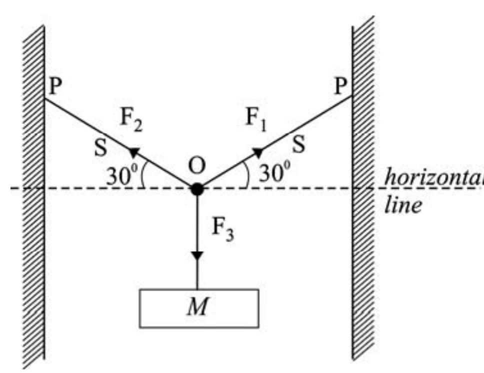


Fig.1.1 Concept of triangle of forces

Note that in this special case of three forces, the vertical components of forces F_1 and F a*dded together* exactly balance the body's weight, that is force F , whilst the horizontal components along the horizontal line, balance each other; cancelling being oppositely directed[1] .

What would happen if the strings were unequal in lengths or were tied at different heights on the walls

yna ic E i i ri

If instead of being in static equilibrium under the action of balancing forces (or their components), a body is in motion at steady speed, for example a train, continues to remain in that state, following Newton's first law of motion, the body is said to be in "dynamic equilibrium". The various forces acting on the body again balance to maintain dynamic equilibrium; for example, in the case of the train a 'driving' force in the direction of its motion balancing against the force of friction on the track and that due to wind in opposite direction[11].

A classic example of dynamic equilibrium is the stable position of heavenly bodies in the universe, relative to each other, under the action of several forces.

ENTRE F GRA IT

The c**entre of gravity** of a body is that point through which the resultant of the system of parallel forces formed by the *weights* of all the particles, or sub-parts, constituting the body passes for all positions of the body. It is denoted as **G** or, simply **G** .
Alternatively

The centre of gravity is a point on any object where all the weight of the object is concentrated. The centre of gravity is the position of the object where we assume that the whole weight of the body is concentrated, in other words we say that centre of gravity is the assumed location of the whole weight of the body. It is always inside the body.

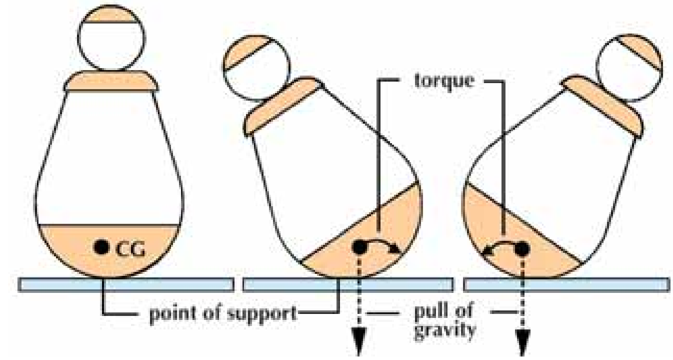
The concept of centre of gravity is explained qualitatively by reference to Fig.1.

---

[1] In general, if the two string lengths $_1$ and were unequal, suspended from different heights, the suspended weight take up a resultant position such that the three co-planer forces represented by $F_{1,}$ F and F would balance as sides of a triangle.

[11] If the train is moving up an incline, there would also be opposing force due to component of its weight along the incline that would have to be overcome by the driving force.

In the figure, a jar, partly filled with some solid or liquid, is shown supported on a table top. Fig.1.20(a) shows the jar standing upright, its bottom being resting at point P on the table and its CG passing through P, the point lying on a vertical line dividing the jar symmetrically as indicated[12].



(a) (b) (c)

Fig.1.20 : The concept of centre of gravity

In Fig.1.20(b), the jar is shown tilted slightly to the left, the tilt having been effected by a manual push, such that the CG is shifted to the right with respect to the point of support, not passing through it. The jar would exhibit a 'restoring' torque in clockwise direction, about the CG, in opposition to the push applied initially; if the push would be removed the jar would revert to the position shown in Fig.1.20(a) and CG aligning with the vertical as before. The condition shown in Fig.1.20(c) is the reverse of that depicted in Fig.1.20(b).

Note that in all the three cases the "pull of gravity" is through CG.

The table below lists the position of CG for a few bodies of common shape(s).

Table 1.2: Shape of body vs. centre of gravity

Shape of body	Position of CG
Thin uniform bar	Middle point of the bar
Circular ring	Centre of the ring
Circular disc	Centre of the disc
Solid or hollow sphere	Centre of the sphere
Cubical or rectangular block	Point of intersection of the diagonals
Triangular lamina	Point of intersection of the medians
Square, parallelogram and rectangular lamina	Point of intersection of the diagonals
Cylinder	Middle point of the axis
Cone or pyramid	On the line joining the apex of the centre of the base at a distance equal to ¼ of the length of this line from the base

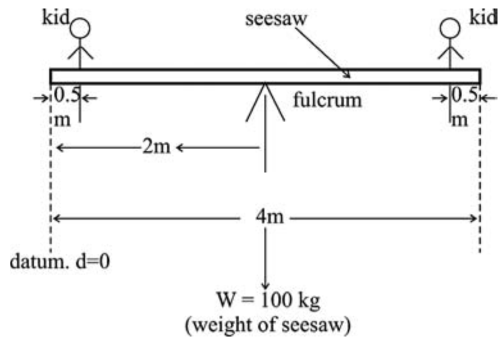
[12]This also shows condition of stable equilibrium of the jar resting on the table.

An e a p e of ca c ation of G

Based on "moment of forces"

Consider a seesaw of length m and of weight 1 kg on which are sitting two kids of kg and kg weight, respectively, at . m distance from both the ends as shown in Fig.1. 1.

(a) seesaw with kids



(b) schematic

Fig.1. 1 Calculation of centre of gravity

First, choose any arbitrary point called datum. Let it be at one end of the seesaw. o that the distances of both the kids from the datum are . m and . m, respectively. The distance of seesaw CG alone from the datum line is m. Now the moment of the seesaw from the datum is 1 Nm, moment of the first kid (at left) is . Nm and the moment of the second kid (sitting to the right) is . m 1 Nm, the total moments are 1 Nm. The total weights are 1 kg. The distance of the centre of gravity from the datum is then obtained as .1 m.

Note that if both kids weighed equally, the CG of the seesaw would be at the mid-point the location of the fulcrum - or m from the left as expected.

T eoretica Approac for a c ation of entre of Gravity

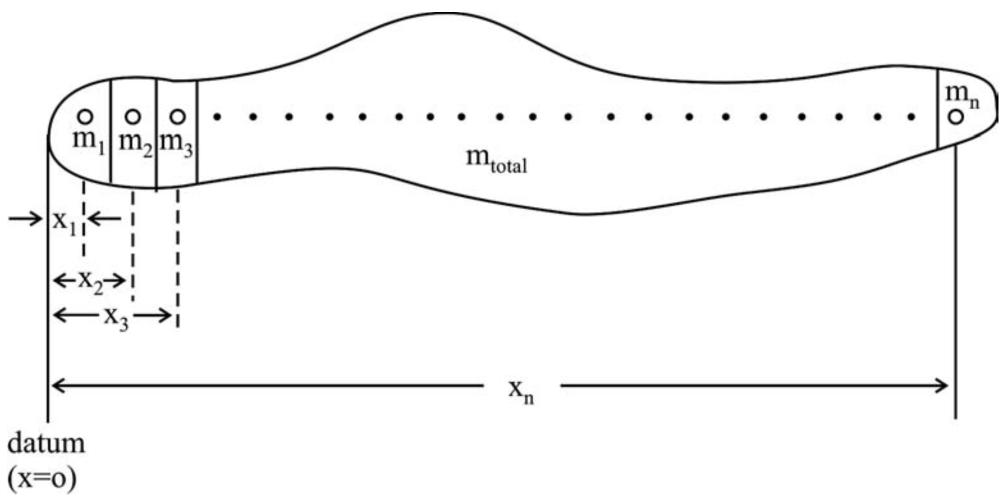
To illustrate the method, consider a solid object of arbitrary shape and size of which the CG is to be determined. Assume that the object can be divided into small 'particles' or sub-sections, n in numbers each having weights w_1, w , w , . . . , w_i, . . , w_n, located in three-dimensional space with respective coordinates given by (x_1, y_1, z_1), (x , y , z), (x , y , z), ... , (x_i, y_i, z_i), . . . , (x_n, y_n, z_n) etc. with respect to a (arbitrary) datum , .

Then, the coordinates of the CG of the body with respect to the chosen datum are given by

$$x_{CG} = \frac{\sum_{i=}^{n} w_i x_i}{w_{total}} \qquad y_{CG} = \frac{\sum_{i=}^{n} w_i y_i}{w_{total}} \qquad {}_{CG} = \frac{\sum_{i=}^{n} w_i \ {}_i}{w_{total}}$$

where w_{total} represents total weight of the particles given by
w_{total} w w w . . . w_i, . . w_n

To derive ust x_{CG} for an arbitrary body is depicted in ig. . .



ig. . To derive x_{CG} of a body

$$_{CG} = \frac{\sum_{i=}^{i=n} m_i x_i}{m_{total}}$$

Centre of Mass

In physics, the **centre o ass** of a distribution of masses in space is the unique point where the weighted relative position of the distributed masses sums to ero or the point where if a force is applied, causes it to move in the direction of force without rotation. The distribution of mass is balanced around the centre of mass and the average of the weighted position coordinates of the distributed mass defines its coordinates.

Alternatively

The centre of mass is the point where all of the mass of the ob ect is concentrated. When an ob ect is supported at its centre of mass, there is no net torque acting on the body and it will remain in static equilibrium

E ER I E

1. A force of dynes acts on a rigid body such that the perpendicular distance between the fulcrum and the point of application of the force is cm. Calculate the moment due to the force.

[dyne-cm

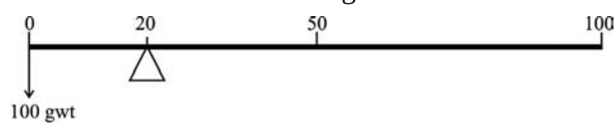
. A force of N produces a moment of 1 Nm in a rigid body. Calculate the perpendicular distance between the point of application of the force and the turning point.

[. m

. A couple of 1 N force acts on a rigid body such that the arm of the couple is cm. Calculate the moment of the couple in I units.

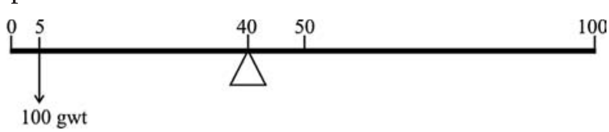
[1 . Nm

. A uniform metre scale is balanced at cm mark from its "zero" mark (at left end) when a force of 1 g wt. is suspended from the "zero" end. Draw the configuration of the problem and calculate the mass of the scale, assumed concentrated at its mid-point.



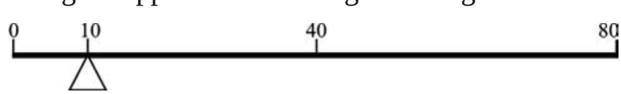
[. g

. A regular metre scale of mass g is placed on a fulcrum at cm mark from the zero at left end as shown. A force of 1 g wt. is suspended at cm mark of the scale to the left of fulcrum. Calculate the distance and location of a force of g wt. to balance the scale.



[. cm to the right of the fulcrum

. A uniform wooden beam AB, cm long is supported on a triangular wedge as shown. Calculate the maximum force (g wt.) that can be placed on end A to balance the beam. Assume the mass of the beam to be concentrated at its mid point.



[gwt.

. A body of mass 1. kg is dropped from a height of 1 m. What is the force acting on it during the fall Assume g . m s s.

[1 . N

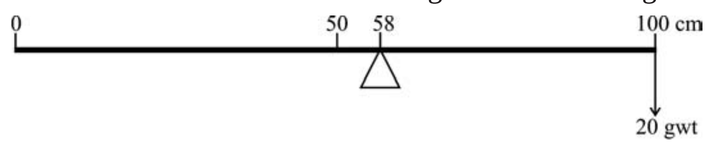
. Calculate the acceleration produced in a body of mass g when acted upon by a force of N.

[m s s

. A body of mass g is moving with a velocity of 1 m s. It is brought to rest by a resistive force of 1 N. Find (i) the retardation, (ii) the distance that the body will travel after resistive force is applied.

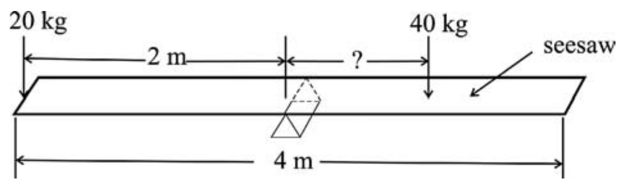
[m s s; cm

1 . A uniform metre rule balances horizontally on a knife edge placed at the cm mark when a weight of gwt. Is suspended from one end. Draw the diagram and find weight of the rule.



[1 gwt.

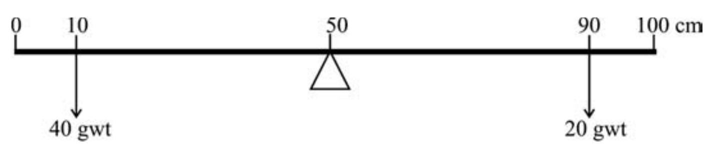
11. When a boy of kgwt. sits at one end of a m long seesaw as shown. Where should the man of kgwt. sit to balance the seesaw



[1 m to the right of the fulcrum

1 . The figure shows a uniform metre rule placed on a fulcrum at its mid point and having a weight of gwt. at the 1 cm mark and a weight of gwt. at the cm mark.

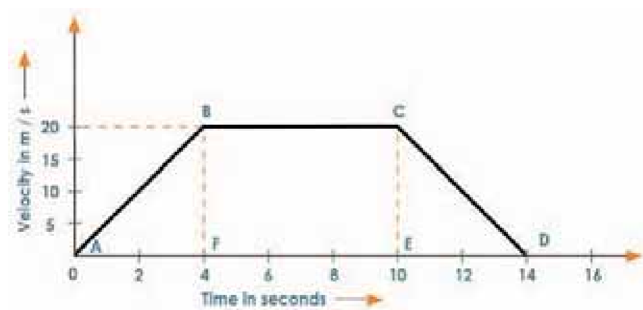
(i) Is the rule in equilibrium If not, how will it turn (ii) ow can it be balanced by using additional weight of gwt.



[(i) it is not in equilibrium, will turn anticlockwise, (ii) at cm mark

1 . The figure shows the velocity-time graph for a scooter, having a total mass of 1 kg. From the graph calculate

a) the acceleration
b) the distance covered
c) the force acting in first seconds.



[m s s; m; N

1 . A force of dynes acts on a rigid body such that the perpendicular distance between the fulcrum and the point of application of the force is cm. Calculate the moment due to the force.

[dyne-cm

1 . A force of N produces a moment of 1 Nm in a rigid body. Calculate the perpendicular distance between the point of application of the force and the turning point.

[. m

1 . A couple of 1 N force acts on a rigid body such that the arm of the couple is cm. Calculate the moment of the couple in I units.

[1 . Nm

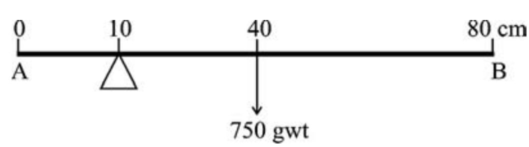
1 . A uniform metre scale is balanced at cm mark from its "zero" mark (at left end) when a force of 1 gwt. is suspended from the "zero" end. From the configuration of the problem and calculate the mass of the scale, assumed concentrated at its mid-point.

[. g

1 . A regular metre scale of mass g is placed on a fulcrum at cm mark from the ero at left end. A force of g wt. is suspended at cm mark of the scale to the left of fulcrum. Calculate the distance and location of a force of g wt. to balance the scale.

[. cm to the right of the fulcrum

1 . A uniform wooden beam AB, cm long is supported on a triangular wedge as shown. Calculate the maximum force (gwt.) that can be placed on end A to balance the beam. Assume the mass of the beam to be concentrated at its mid point.



[gwt.

. A body of mass . kg is dropped from a height of m. What is the force acting on it during the fall Assume g . m s s.

[.

1. Calculate the acceleration produced in a body of mass g when acted upon by a force of N.

[m s s

. A body of mass g is moving with a velocity of 1 m s. It is brought to rest by a resistive force of 1 N. Find (i) the retardation, (ii) the distance that the body will travel after resistive force is applied.

[m s s; cm