STEADY-STATE PSYCHROMETRIC ANALYSIS OF HVAC CIRCUITS

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ABSTRACT

An HVAC circuit is a collection of interconnected elements that can be modelled by interconnected blocks. Similar to electrical or hydraulic circuits analysis, we define the HVAC network analysis as the process of finding the temperatures and humidity ratios across, and the sensible and latent heat flow rates through every component in the network for a given mass flow rate of dry air. This implies formulating and solving a system of non-linear equations. The proposed solution is to linearize locally the vapour saturation curve and to decouple the sensible and the latent heat in order to obtain a system of linear equations. This steady-state psychrometric analysis is a computational alternative for the empirical usage of the psychrometric charts.

NOMECLATURE

A	Area $[m^2]$
c	Specific heat capacity [kJ/kg] − of dry air, if no subscript
E	Heat exchanger effectiveness [-]
h	Specific enthalpy [kJ/kg]
H	Heating
l	Latent heat for vaporization [kJ/kg]
m	Mass [kg]
M	Molar mass $[kg/kg_{da}]$
n	Molar fraction
NTU	Number of transfer units $[-]$
p	Pressure [Pa]
R	Ideal gas constant [kJ/kmol K]
T	Temperature [K]
U	Overall heat-transfer coefficient $[kW/m^2\ K]$
w	Humidity ratio $[kg_w/kg_{da}]$

Dotted variables

$\dot{H}$	Enthalpy rate [kW]
$\dot{m}$	Mass flow rate [kg/s] − of the dry air, if no subscript
$\dot{Q}$	Heat flow rate [kW]

Bold letters

$\mathbf{A}$	Square matrix of the coefficients of the model
$\mathbf{A}_{mn}$	Element (m, n) of the matrix of the coefficients of the model
$\mathbf{b}_m$	Element m of the vector of inputs
$\mathbf{b}$	Vector of inputs
$\mathbf{x}$	Vector of unknowns

Greek letters

β	By-pass factor $[-]$
Δ	Difference
ε	Mixing coefficient, contact factor $[-]$
θ	Temperature [°C]
φ	Relative humidity $[-]$

Subscripts

aux	Auxiliary (solar, occupants, electrical, etc.)
da	Dry air
inf	Air infiltration
l	Latent
o	Outdoor
s	Saturation (temperature, humidity ratio) Sensible (heat, enthalpy)
w	Water vapour
x	Exchanger

Superscripts

^{T}	Transpose of a vector or a matrix
$'$	Derivative of a function

INTRODUCTION

For any circuit, be it electric, hydraulic or HVAC, three things are associated with it: network elements, inputs (or excitations) and outputs (or responses) of the network. In the network analysis, the elements and the excitations are known and the task is to find the response (e.g. Clarke 2001, Bakshi and Bakshi 2008).

An HVAC circuit is a set of elements interconnected by air- and water-duct networks. The air circuit may be viewed as a pipeline that transfers mass and heat from one point to another. The elementary quantities used to describe these transport phenomena are the mass flow rate of the dry air for the mass transfer, and the air temperature and humidity ratio for the (sensible and latent) heat transfer. Aeraulic network analysis is the process of finding the pressures across and the mass flow rates through every component (e.g. Karnopp and Rosenberg 1975, McKew and Duda 2008, Kirby 2010); it requires the temperatures and humidity ratios of the air in order to find the mass flow rate of the air. We define the

psychrometric analysis of the HVAC networks as the process of finding the dry bulb temperatures and the humidity ratios across every component of the circuit when the mass flow rate of the dry air is known. Aeraulic analysis and psychrometric analysis are therefore coupled problems. As opposed to electrical or hydraulic networks, the HVAC networks have two flows: dry air and moisture. Therefore, it is not possible to apply the methods already existing for electric or hydraulic network to the psychrometric analysis of HVAC networks.

The classical tool for representing the humid air processes in the HVAC systems coupled to buildings is the graphical representation on the psychrometric chart. Most practitioners rely on psychometric software and use the graphical representation on a psychrometric chart to show the state points and the connecting lines that model the air conditioning processes (Gatley 2004). The individual processes are presented in almost any primer on air-conditioning (Gatley and Perez-Ganlindo 2009) and implemented in software applications (e.g. Hands Down Software 2007, Munters 2015). By using these methods and tools, the psychometric analysis of HVAC networks, as defined above, can be conducted empirically for very simple networks. The present paper proposes a methodology for the psychrometric analysis of HVAC networks. The paper starts by presenting the linearization of the saturation curve and the decoupling of the sensible and latent heat in order to obtain linear models for the basic psychometric processes. Then, by coupling the models of the basic processes, we obtain a set of linear equations. The methodology is illustrated on a non-trivial HVAC system: air-conditioning of two thermal zones with balanced ventilation and heat recovery.

LINEARIZATION OF SATURATION CURVE

Moist air is a mix of dry air and water vapour. The humidity ratio (or the moisture content) is the ratio of the mass of water vapour, m_w, to the mass of dry air, m_{da}:

$$w = \frac{m_w}{m_{da}} = \frac{M_w}{M_{da}} \frac{n_w}{n_{da}} \tag{1}$$

where $M_w = 18\,\text{kg/kmol}$ and $M_{da} = 28.96\,45\,\text{kg/kmol}$ are the molar masses of water vapour and dry air, respectively, $n_w = m_w/M_w$ and $n_{da} = m_{da}/M_{da}$ are the molar fraction of water vapour and of dry air, respectively.
In the moist air, the vapour temperature is equal to the dry air temperature. By using the ideal gas law, it results that the humidity content on the saturation curve, $\varphi = 100\%$, has a non-linear dependence on temperature:

$$w_s = f(\theta) = \frac{M_w}{M_{da}} \frac{p_{ws}}{p - p_{ws}}$$
$$= 0.621\,547 \frac{p_{ws}}{p - p_{ws}} \tag{2}$$

where p [Pa] is the barometric pressure and p_{ws} [Pa] is the water vapour saturation pressure. A simple empirical expression for the water vapour saturation pressure is (Tetens 1930):

$$p_{ws} = 610.78 \exp\left(\frac{17.2694\,\theta}{\theta + 283.3}\right) \text{[Pa]} \tag{3}$$

where θ [°C] is the moist air temperature.

The equation (2) may be linearized for a given temperature, θ_s^0:

$$w_s = w_s^0 + f'_{\theta_s^0}(\theta_s - \theta_s^0) \tag{4}$$

where $f'_{\theta_s^0} \equiv f(\theta_s^0)$ is the value of the derivative of function (2) for θ_s^0, in which p_{ws} is given by equation (3),

$$f'_{\theta_s^0} \equiv \left.\frac{df(\theta)}{d\theta}\right|_{\theta = \theta_s^0} = \frac{d}{d\theta}\left(\frac{M_w}{M_{da}} \frac{p_{ws}}{p - p_{ws}}\right)$$
$$= \frac{M_w}{M_{da}} p \frac{2.51354 \cdot 10^{-6} e^{\frac{17.2694\theta_s^0}{\theta_s^0 + 283.3}}}{(\theta_s^0 + 283.3)^2 \left(p - 610.78\, e^{\frac{17.2694\theta_s^0}{\theta_s^0 + 283.3}}\right)}$$

DECOUPLING THE SENSIBLE AND THE LATENT HEAT

When sensible and latent heat is transferred to the humid air, its state, characterized by the dry bulb temperature and the humidity ratio, changes. For example, when the mass flow rate of air evolves from state 1, having the initial enthalpy rate

$$\dot{H}_1 = \dot{m}_{da}(c_{da}\theta_1 + w_1(l_w + c_w\theta_1)) \tag{5}$$

where
$\dot{H}_I$ is the enthalpy rate of the air in state 1 [kW];
$\dot{m}_{da}$ -mass flow rate of dry air [kg/s];
c_{da} -specific heat capacity of the dry air [kJ/kg K];
w_1 -humidity ratio of moist air in the state 1 [kg/kg$_{da}$];
l_w -specific latent heat for vaporization [kJ/kg];
c_w -specific heat capacity of the vapours [kJ/kg K];
θ_1 -temperature of the air in the state 1 [°C],

to the state 2, having the enthalpy rate

$$\dot{H}_2 = \dot{m}_{da}[c_{da}\theta_2 + w_2(l_w + c_w\theta_2)] \tag{6}$$

where

$\dot{H}_2$ is the enthalpy rate of the air in state 2 [kW];
w_2 -humidity ratio (or the moisture content) in state 2 [kg/kg$_{da}$];
θ_2 -temperature of the air in state 2 [°C] ,

the total variation of the enthalpy rate is

$$\dot{H}_2 - \dot{H}_1 = \dot{m}_{da}[(h_2 - h_A) + (h_A - h_1)] \qquad (7)$$

The change of sensible heat in the process is

$$\Delta \dot{H}_s = \dot{m}_{da}[c_{da}(\theta_2 - \theta_1) + c_w(w_2\theta_2 \\ - w_1\theta_1)] \qquad (8)$$

where $\dot{m}_{da}c_{da}(\theta_2 - \theta_1)$ is the sensible heat of the dry air and $\dot{m}_{da}c_w(w_2\theta_2 - w_1\theta_1)$ is the sensible heat of the moisture. The change in the latent heat during the process is

$$\Delta \dot{H}_l = \dot{m}_{da}l_w(w_2 - w_1) \qquad (9)$$

In the range of variation of moist air temperature and humidity ratio encountered in air conditioning systems, the error introduced in the evaluation of enthalpy variation by neglecting the sensible heat of water vapour is less than 5.5% in relative value or 2.5 kJ/kg in absolute value (Ghiaus 2014). Neglecting the sensible heat of the moisture is a key assumption that allows us to decouple the sensible and the latent heat. The specific air enthalpy of a state is then

$$\begin{cases} h_s = c_{da}\theta \\ h_l = l_w w \end{cases} \qquad (10)$$

where h_s is the sensible specific enthalpy and h_l is the latent specific enthalpy. The enthalpy rate is then

$$\begin{cases} \dot{H}_s = \dot{m}_{da}c_{da}\theta \\ \dot{H}_l = \dot{m}_{da} \, l_w w \end{cases} \qquad (11)$$

Using equation (11), the transformation of the moist air can be decomposed in sensible heat and latent heat exchange:

$$\begin{cases} \Delta \dot{H}_s = \dot{m}_{da}c_{da}\theta_2 - \dot{m}_{da}c_{da}\theta_1 \\ \Delta \dot{H}_l = \dot{m}_{da}l_w w_2 - \dot{m}_{da}l_w w_1 \end{cases} \qquad (12)$$

where $\Delta \dot{H}_s$ is the variation of the sensible enthalpy rate, $1 \to A$, and $\Delta \dot{H}_l$ is the variation of the latent enthalpy rate, $A \to 2$.

LINEARIZED STEADY-STATE MODELS OF TYPICAL AIR CONDITIONING PROCESSES

If the following hypothesis are accepted:
• sensible heat of the water vapor, $c_w w\theta$, is neglected,

• specific heat capacity of the dry air, c_{da} , and latent heat for vaporization, l_w, are constant,
• mass flow rate of the dry air, $\dot{m}_{da}$, is constant,

then the models of the psychrometric processes in the air-conditioned space and the air handling unit (AHU) become linear equations in the two-dimensional vector space defined by the dry temperature, θ, and the humidity ratio, w. The causal dependence (i.e. the input – output relation) can be expressed from the energy balance equations (Ghiaus 2013). With the assumptions mentioned above, this causal relation is linear.

Air-conditioned space
Let us consider an air-conditioned space with the sensible heat gains,

$$\dot{Q}_s = UA(\theta_o - \theta_2) + \dot{m}_{inf}c(\theta_1 - \theta_2) \\ + \dot{Q}_{aux,s} \qquad (13)$$

where
$UA(\theta_o - \theta_2)$ are the heat gains through the envelope, UA is the global thermal conductance, θ_o is the outdoor temperature;
$\dot{m}_{inf}c(\theta_o - \theta_2)$ are the heat gains by air infiltration, $\dot{m}_{inf}$ is the mass flow rate of infiltration air,
$\dot{Q}_{aux,s}$ are the auxiliary sensible heat gains from sources (solar, occupants, electrical, etc.),

and the latent heat gains

$$\dot{Q}_l = \dot{m}_{inf}l(w_o - w_2) + \dot{Q}_{aux,l} \qquad (14)$$

where
$\dot{m}_{inf}l(w_o - w_2)$ are the latent heat gains through ventilation, w_o is the humidity content of the outdoor air,
$\dot{Q}_{aux,l}$ is the auxiliary latent heat gains from internal sources.

The supplied moist air (characterized by dry bulb temperature θ_1 and humidity ratio w_1) needs to compensate the sensible and latent loads of the air-conditioned space. The energy balance in steady state is

$$\begin{cases} \dot{m}c \, \theta_1 - \dot{m}c \, \theta_2 + UA(\theta_o - \theta_2) \\ \quad + \dot{m}_{inf}c(\theta_o - \theta_2) + \dot{Q}_{aux,s} = 0 \\ \dot{m}l \, w_1 - \dot{m}l \, w_2 \\ \quad + \dot{m}_{inf}l(w_o - w_2) + \dot{Q}_{aux,l} = 0 \end{cases} \qquad (15)$$

where $\dot{m}$ is the air flow rate of dry air supplied to the air-conditioned space.
Considering that the airflow mass rates of outdoor air by ventilation and infiltration, $\dot{m}_{inf}$, and the global thermal conductance, US, are constant parameters, then the system of equations (15) represents a causal linear relation with
• output: the state of the outlet air, (θ_2, w_2);

Table 1 Linear model of the heating and humidification
of a two-zone air-conditioned space shown in Figure 2

N°	Equations	Symbol	Process
1.	$-\varepsilon_1 \dot{m}c\theta_1 + \dot{Q}_x + l\dot{m}_{wx} = -\varepsilon_1 \dot{m}c\theta_o$	XH	heating
2.	$-\varepsilon_1 \dot{m}lw_1 = -\varepsilon_1 \dot{m}lw_o$		
3.	$(1-b)\varepsilon_1 \dot{m}c\theta_{10} - (1-b)\varepsilon_1 \dot{m}c\theta_{11} - \dot{Q}_x = 0$	XC	cooling and dehumidification
4.	$(1-b)\varepsilon_1 \dot{m}lw_{10} - (1-b)\varepsilon_1 \dot{m}lw_{11} - h_w \dot{m}_{wx} = 0$		
5.	$w_{11} - f'_{\theta^0_{11}}\theta_{11} = w^0_{11} - f'_{\theta^0_{11}}\theta^0_{11}$		
6.	$b\varepsilon_1 \dot{m}c\theta_{10} + (1-b)\varepsilon_1 \dot{m}c\theta_{11} - \varepsilon_1 \dot{m}c\theta_{12} = 0$	XM	mixing
7.	$b\varepsilon_1 \dot{m}lw_{10} + (1-b)\varepsilon_1 \dot{m}lw_{11} - \varepsilon_1 \dot{m}lw_{12} = 0$		
8.	$UA\theta_1 - UA\theta_{10} - UA\theta_{11} + 2Q_x = UA\theta_o$		heat transfer between exhaust and fresh air
9.	$\varepsilon_1 \dot{m}c\theta_1 - \dot{m}c\theta_2 + (1-\varepsilon_1)\dot{m}c\theta_{10} = 0$	M1	adiabatic mixing
10.	$\varepsilon_1 \dot{m}lw_1 - \dot{m}lw_2 + (1-\varepsilon_1)\dot{m}lw_{10} = 0$		
11.	$\dot{m}c\theta_2 - \dot{m}c\theta_3 = -\dot{Q}_{sH}$	H	heating
12.	$\dot{m}lw_2 - \dot{m}lw_3 = 0$		
13.	$c\theta_3 + lw_3 - c\theta_4 - lw_4 = 0$	W	adiabatic humidification
14.	$w_4 - f'_{\theta^0_4}\theta_4 = w^0_4 - f'_{\theta^0_4}\theta^0_4$		
15.	$(1-\varepsilon_w)\dot{m}c\theta_3 + \varepsilon_w \dot{m}c\theta_4 - \dot{m}c\theta_5 = 0$	MW	mixing
16.	$(1-\varepsilon_w)\dot{m}lw_3 + \varepsilon_w \dot{m}lw_4 - \dot{m}lw_5 = 0$		
17.	$\varepsilon_2 \dot{m}c\theta_5 - \varepsilon_2 \dot{m}c\theta_6 = -\dot{Q}_{sH1}$	H1	heating
18.	$\varepsilon_2 \dot{m}lw_5 + \varepsilon_2 \dot{m}lw_6 = 0$		
19.	$\varepsilon_2 \dot{m}c\theta_6 - (\varepsilon_2 \dot{m}c + U_1A_1 + \dot{m}_{inf1}c)\theta_7 = -\dot{Q}_{aux,sS1} - (U_1A_1 + \dot{m}_{inf1}c)\theta_o$	S1	air-conditioned space
20.	$\varepsilon_2 \dot{m}lw_6 - (\varepsilon_2 \dot{m}l + \dot{m}_{inf1}l)w_7 = -\dot{Q}_{aux,lS1} - \dot{m}_{inf1}lw_o$		
21.	$(1-\varepsilon_2)\dot{m}c\theta_5 - (1-\varepsilon_2)\dot{m}c\theta_8 = -\dot{Q}_{sH2}$	H2	heating
22.	$(1-\varepsilon_2)\dot{m}lw_5 + (1-\varepsilon_2)\dot{m}lw_8 = 0$		
23.	$(1-\varepsilon_2)\dot{m}c\theta_8 - [(1-\varepsilon_2)\dot{m}c + U_2A_2 + \dot{m}_{inf2}c]\theta_9$ $= -\dot{Q}_{aux,sS2} - (U_1A_1 + \dot{m}_{inf1}c)\theta_o$	S2	air-conditioned space
24.	$(1-\varepsilon_2)\dot{m}lw_8 - [(1-\varepsilon_2)\dot{m}l + \dot{m}_{inf1}l]w_9 = -\dot{Q}_{aux,lS2} - \dot{m}_{inf2}lw_o$		
25.	$\varepsilon_2 \dot{m}c\theta_7 + (1-\varepsilon_2)\dot{m}c\theta_9 - \dot{m}c\theta_{10} = 0$	M2	mixing
26.	$\varepsilon_2 \dot{m}lw_7 + (1-\varepsilon_2)\dot{m}lw_9 - \dot{m}lw_{10} = 0$		

- inputs: the state of the inlet air, (θ_1, w_1), the state of the outdoor air, (θ_o, w_o), and the sensible and latent heat gains from internal sources, $\dot{Q}_{aux,s}$ and $\dot{Q}_{aux,l}$;
- constant parameters: the mass flow rate of supplied air, $\dot{m}$, the mass flow rate of infiltration air, $\dot{m}_{inf}$, and the global thermal conductance, UA.

Air-handling unit (AHU)

The air-handling unit controls the temperature and the humidity of the moist air to be supplied to the air-conditioned space. The state of the outlet air depends on the state of the input air and on the heat and moisture algebraically added to the airflow.

Adiabatic mixing

Adiabatic mixing of two air-streams implies no sensible or latent heat exchange between the air

volume and its environment. The sensible and latent heat balance equations in steady-state are:

$$\begin{cases} \varepsilon\dot{m}c\theta_1 + (1-\varepsilon)\dot{m}c\theta_2 - \dot{m}c\theta_3 = 0 \\ \varepsilon\dot{m}lw_1 + (1-\varepsilon)\dot{m}lw_2 - \dot{m}lw_3 = 0 \end{cases} \quad (16)$$

where ε is the mixing coefficient. Equations (16) represent a causal linear relation with
- outputs: the state of the outlet air, (θ_3, w_3);
- inputs: the states of the inlet air, (θ_1, w_1) and (θ_2, w_2);
- constant parameters: the dry air mass flow rate, $\dot{m}$, and the mixing coefficient, ε.

Heating or cooling without dehumidification

In heating or cooling without dehumidification, heat is added or removed without changing the humidity

content of the air. The sensible and latent heat balance in steady-state are

$$\begin{cases} \dot{m}c\theta_1 - \dot{m}c\theta_2 + \dot{Q}_{sH} = 0 \\ \dot{m}lw_1 - \dot{m}lw_2 = 0 \end{cases} \qquad (17)$$

Equations (17) represent a causal linear relation with:
- outputs: the state of the outlet air, (θ_2, w_2);
- inputs: the state of the inlet air, (θ_1, w_1), and the sensible heat supplied, $\dot{Q}_{sH}$;
- constant parameter: the dry air mass flow rate, $\dot{m}$.

Cooling and dehumidification

The process of cooling and dehumidification occurs when the inlet air is cooled to a temperature below the dew point. The temperature of the moist air exiting the cooling coil is higher than the temperature of the cooling coil fins. It may be considered that a fraction of the moist air, $\beta\dot{m}$, bypasses the cooling coil exiting at unchanged conditions, $[\theta_1 \quad w_1]^T$, and the rest of the air mass flow rate, $(1-\beta)\dot{m}$, is cooled to the dew temperature and then evolves on the saturation curve to reach a temperature equal to the surface temperature of the coil, θ_s, exiting at the state $[\theta_s \quad w_s]^T$. The outlet air may be considered as a mix of the by-passing air and of the air cooled at the temperature of the cooling coil

$$\begin{cases} \beta\dot{m}c\theta_1 + (1-\beta)\dot{m}c\theta_s - \dot{m}c\theta_2 = 0 \\ \beta\dot{m}lw_1 + (1-\beta)\dot{m}lw_s - \dot{m}lw_2 = 0 \end{cases} \qquad (18)$$

The state of the moist-air exiting the cooling coil results from the intersection of the line that represents the heat balance (sensible and latent),

$$\begin{cases} (1-\beta)\dot{m}c\theta_1 - (1-\beta)\dot{m}c\theta_s - \dot{Q}_s = 0 \\ (1-\beta)\dot{m}lw_1 + (1-\beta)\dot{m}lw_s - l\dot{m}_w = 0 \end{cases} \qquad (19)$$

and the saturation curve given by equation (2) or its linearization given by equation (4).
The set of equations (19) together with the equation (4) form a causal model with:
- outputs: the state of the cooled air, (θ_2, w_2), and the mass flow rate of condensed vapor, $\dot{m}_w$.
- state variables: the state of saturated air, (θ_s, w_s).
- inputs: the state of the inlet air, (θ_1, w_1), the sensible heat transferred by the heat exchanger, $\dot{Q}_s$.
- constant parameters: the dry air mass flow rate, $\dot{m}$, the by-pass factor, β.

Humidification by water vapour injection

Air may be humidified by injecting steam into the air stream. If the sensible heating of the air by the steam is neglected, and if the humidified air is not saturated, then the heat balance is

$$\begin{cases} \dot{m}c\theta_1 - \dot{m}c\theta_2 = 0 \\ \dot{m}lw_1 - \dot{m}lw_2 + l m_w = 0 \end{cases} \qquad (20)$$

The system of equations (20) represents a causal model with:
- outputs: the state of the humidified air, (θ_2, w_2);
- inputs: the state of the inlet air, (θ_1, w_1), and the mass flow rate of water vapor, $\dot{m}_w$;
- constant parameter: the dry air mass flow rate, $\dot{m}$.

Adiabatic injection of water in moist air

Alternatively, moist air may be humidified by injecting water into the air stream. Practically, if the water temperature is between 10°C and 20°C, the mixing process is adiabatic. We may consider that the mass exchange between the water droplets and the air takes place in the boundary layer of the droplets. The air flow may be divided in a mass flow rate, $(1-\varepsilon)\dot{m}$, which bypasses the droplets and exits at the same state as the inlet air, $[\theta_1 \quad w_1]^T$, and a mass flow rate, $\varepsilon\dot{m}$, which is completely saturated, exiting at $[\theta_s \quad w_s]^T$. Then the outlet air is a mixture of these two air streams:

$$\begin{cases} (1-\varepsilon)\dot{m}c\theta_1 + \varepsilon\dot{m}c\theta_s - \dot{m}c\theta_2 = 0 \\ (1-\varepsilon)\dot{m}lw_1 + \varepsilon\dot{m}lw_s - \dot{m}lw_2 = 0 \end{cases} \qquad (21)$$

Since the transformation is adiabatic, the points 1, 2 and s have the same specific enthalpy,

$$c\theta_1 + lw_1 = c\theta_2 + lw_2 = c\theta_s + lw_s \qquad (22)$$

The state at the outlet of the cooling coil, s, is at the intersection of the adiabatic line given by equation (22) and the saturation curve given by equation (2), or its linearization given by equation (4).

The system of equations (21), the equation (22) and the equation (4) represent a linearized model of the adiabatic humidification of the moist-air with:
- outputs: the state of the humidified air, (θ_2, w_2), and the mass flow rate of water injected, $\dot{m}_w$;
- state variables: the state of saturated air, (θ_s, w_s);
- inputs: the state of the inlet air, (θ_1, w_1);
- constant parameters: the dry air mass flow rate, $\dot{m}$, and the effectiveness, ε.

The mass flow rate of water injected, $\dot{m}_w$, is an output of the process, not an input. It depends of the state of the inlet air and on the contact factor.
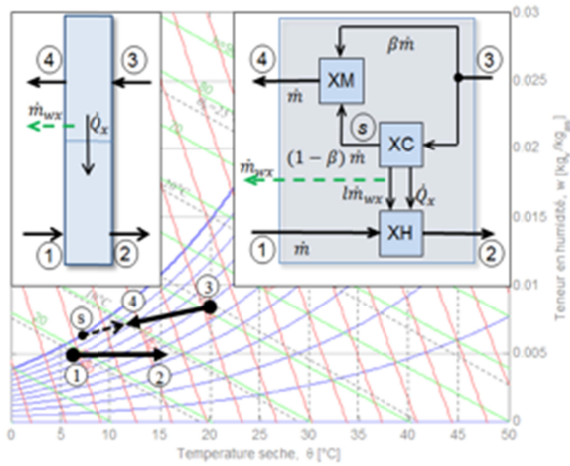
Figure 1 Heat recovery during heating season

Heat recovery

Maintaining the indoor air quality requires the supply of fresh air from outdoors and the rejection of the same quantity of air from the conditioned space. The temperature of the exhaust air is usually between 15°C and 30°C and its relative humidity around 50%. The sensible and the latent heat contained by the exhaust air may be recovered by transferring it to the inlet air (which has the conditions of the outdoor air, Figure 1). The majority of energy recovery heat exchangers do not transfer latent heat (with the exception of the rotary heat exchangers). In winter, the inlet air is heated; the process is similar to air heating and can be modelled by a sensible heat exchanger (XH in Figure 1). The exhaust air is cooled like in a dry cooling coil, or it is cooled and dehumidified like in a wet cooling coil.

The energy recovery heat exchangers are characterized by their effectiveness, *E*, which is the ratio between the actual and the maximum possible heat transfer rates. The manufactures give the values of the effectiveness for different airflow rates. The effectiveness is a function of the global heat transfer conductance of the heat exchanger, *UA*. For a large class of counter-current heat exchangers used in HVAC systems, the heat capacity rates (i.e. the product of the mass flow rate and the specific heat) of the hot air and the cold air are almost equal. In this case, the effectiveness of the counter-current heat exchanger is (Incropera et al. 2006):

$$E = \frac{NTU}{1 + NTU} \tag{23}$$

where the number of transfer units, *NTU*, is

$$NTU = \frac{UA}{\dot{m}c} \tag{24}$$

For a given heat exchanger, the global heat transfer
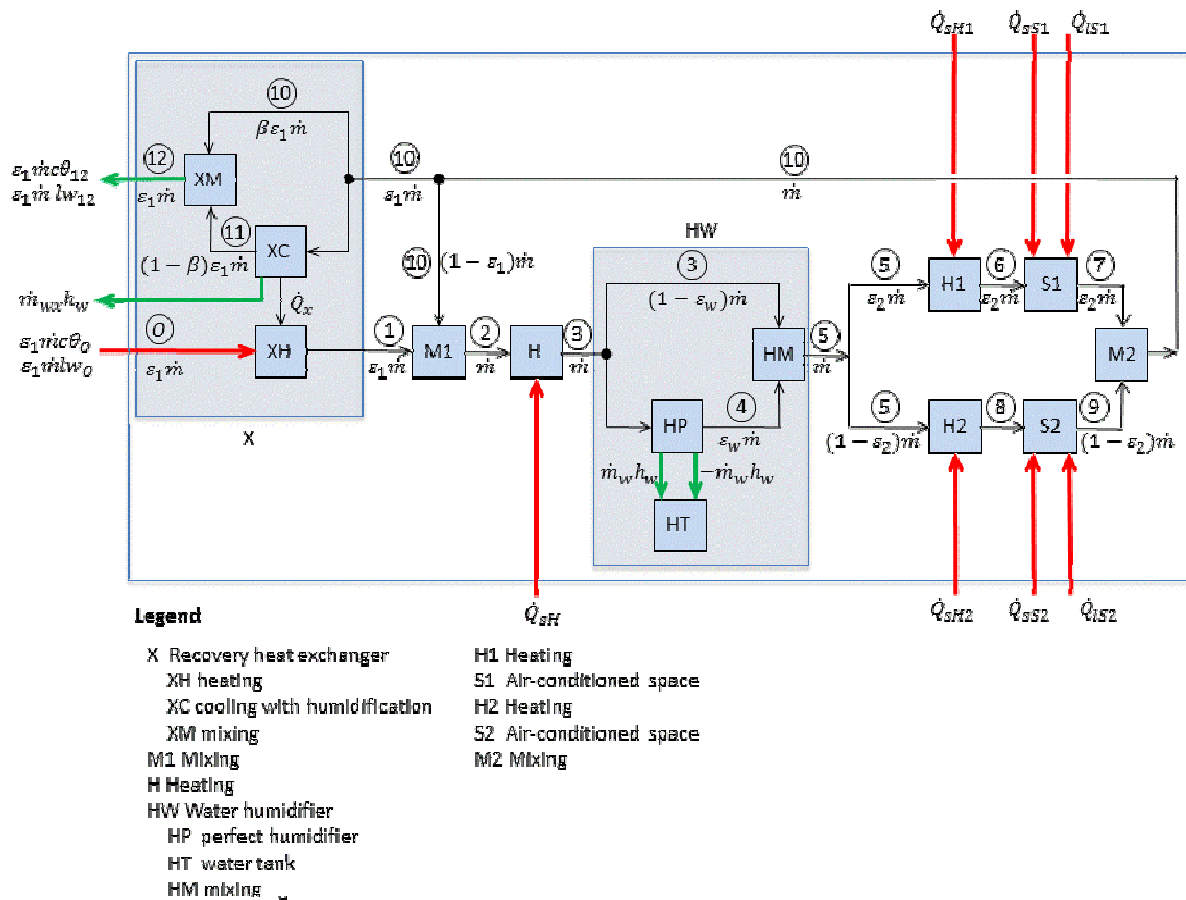
Figure 2 Heating and humidification of a two-zone air-conditioning space with air recycling and heat recovery

conductance, UA, may be obtained from Equations (23) and (24). Its value is used to express the heat transfer between the two airflows:

$$\dot{Q}_x = UA(\bar{\theta}_{34} - \bar{\theta}_{12}) \qquad (25)$$

where $\bar{\theta}_{34}$ is the logarithmic mean of the temperature of the exhaust air and $\bar{\theta}_{12}$ is the logarithmic mean of the fresh air. A linear approximation of the logarithmic mean is the arithmetic mean. Then, the linear model of the recovery heat exchanger (Figure 1) is composed of:

-the model of the heating side of the heat exchanger (XH),
$$\begin{cases} \dot{m}c\theta_1 - \dot{m}c\theta_2 + \dot{Q}_x + l\dot{m}_{wx} = 0 \\ \dot{m}lw_1 - \dot{m}lw_2 = 0 \end{cases} \qquad (26)$$

-the model of the cooling and dehumidification side of heat exchanger (XC),

$$\begin{cases} (1-\beta)\dot{m}c\theta_3 - (1-\beta)\dot{m}c\theta_s - \dot{Q}_x = 0 \\ (1-\beta)\dot{m}lw_3 - (1-\beta)\varepsilon_1\dot{m}lw_s - l\dot{m}_{wx} = 0 \\ w_s - f'_{\theta_s^0}\,\theta_s = w_s^0 - f'_{\theta_s^0}\,\theta_s^0 \end{cases} \;(27)$$

-the linear model of the heat flow rate exchanged between the two airflows, considering the arithmetic mean of the temperature differences,

$$\dot{Q}_x = UA\left(\frac{\theta_3 + \theta_4}{2} - \frac{\theta_1 + \theta_2}{2}\right) \qquad (28)$$

The set of equations (26), (27) and (28) form a linearized model of the energy recovery heat exchanger with:
- outputs: the state of the fresh air, $(\theta_2,\ w_2)$, the state of the exhaust air, $(\theta_4,\ w_4)$, and the mass flow rate of condensed water, $\dot{m}_w$;
- state variables: the saturation state, $(\theta_s,\ w_s)$, and the heat flow rate exchanged between the fresh air and the exhaust air, $\dot{Q}_x$;
- inputs: the state of the outdoor air, $(\theta_1,\ w_1)$, and the state of the return air, $(\theta_3,\ w_3)$;
- constant parameters: the dry air mass flow rate, $\dot{m}$, and the by-pass factor, β.

CAUSAL MODELS OF THE MOIST-AIR TRANSFORMATIONS

The equations used for modelling the psychrometric processes are energy balance equations that do not reveal the physical causality. They may be easily rearranged so that the computational causality is the same as the physical causality. The equations (13) - (28) together with the equation (4) allows us to model a HVAC system by a system of linear

equations **if the inputs and the outputs of the computational model are chosen to respect the physical causality**. For some of the elements of the HVAC system, the inputs are the output of another element; they will be considered states and will be the unknowns of the system of equations. The outputs of the mathematical model, that are the same with the outputs of the physical system, are obtained from the state variables.

Algorithm
The simulation problem may be solved by using the following algorithm:
- take the values of the inputs of the model:
  - outdoor air state, (θ_o, w_o), delivered sensible and latent heat flow rates, $\dot{Q}_s, \dot{Q}_l$;
  - initial values of temperatures for the iterative calculation of the saturation points, θ_s^0;
- assign the physical constants: $p, M_w, M_{da}, R, c, l, h_w$;
- take the constant parameters of the model: dry air mass flow rates, $\dot{m}, \dot{m}_{inf}$, effectiveness and mixing coefficients, ε, by-pass factors, β, values of global thermal conductance, UA;
- form the matrix of coefficients **A** and the vector of inputs **b** of the system of linear equations **Ax = b**, with exception of the points that are on the saturation curve;
- determine iteratively the points on the saturation curve:

repeat
- calculate the humidity ratio for the temperature of saturation (equations (2) and (3));
- estimate the derivative of the humidity ratio for the temperature of saturation (equation (3));
- calculate the elements of the matrix **A** and the vector **b** corresponding to the linearized saturation curve (equation (4)) for air condition processes in which condensation may occur (adiabatic humidifier, cooling coil, energy recovery);
- solve the linear system, $\mathbf{x} = \mathbf{A^{-1}b}$;
- calculate the maximum difference between the saturation temperatures obtained in two successive iterations;
- update the values of the saturation temperatures with the values from the vector **x** found by solving the linear system;

until the difference between the saturation temperatures of two successive iterations is acceptably small (e. g. 0.01 °C);

- deliver the solution **x;**
- use heat and mass balances for the whole system to check if the results are correct.

Example: air-conditioning of two thermal zones with heat recovery balanced ventilation system

In order to clarify these concepts, let us consider a simple, yet not trivial, balanced ventilation system with heat recovery used to air-condition two spaces, S1 and S2. The outdoor air, having temperature θ_o and humidity ratio w_o, recovers sensible heat, $\dot{Q}_x$, from the exhaust air. The fresh air (state 1) is mixed with the return air (state 10). Then the mixed air (state 2) is heated (state 3), and humidified by adiabatic injection of water. The obtained airflow (state 5) is supplied to two spaces (states 6 and 8). The return air from the two air-conditioned spaces (states 7 and 9) is mixed to form the return air (state 10). A fraction of it is recycled and a fraction exhausted. An energy recovery heat exchanger preheats the fresh air by cooling and dehumidifying the return air (state 12); the exhaust air (state 12) is rejected into the atmosphere. The set of equations modelling this system is given in Table 1.

CONCLUSION

Psychrometric charts are essential tools for climate control engineering. The representation of air condition processes on the psychometric chart results in linear or linearized curves. If the sensible and latent heat are decoupled, by neglecting the sensible heat of the moisture (which introduces relative errors smaller than 5%), and if the non-linear processes are linearized for small variations around an operating point, then the psychrometric processes may be modelled by a set of linear equations. If the computational causality is the same with the physical causality of the individual processes, then the model representing the HVAC system coupled with the air conditioned spaces is given by a system of linear (or linearized) equations. The system of equations represents an algorithmic solution to the analysis of the HVAC circuits that may successfully replace the heuristic solutions that use the psychrometric chart. This computational method is especially suited for multizone systems that use heat recovery.

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REFERENCES

Bakshi, U.A, Bakshi A.V. 2008. Electrical Network Analysis and Synthesis, Technical Publications Pune, India

Clarke, J. 2001 Energy Simulation in Building Design. 2nd. Oxford: Butterworth-Heinemann, UK

Gatley, D. 2004. Psychrometric chart celebrates 100th Anniversary. ASHRAE Journal: 16-20.

Gatley, D. P. and J. A. Perez-Ganlindo. 2009. Chapter 1 Psychrometrics. ASHRAE Fundamentals:10-26. Atlanta, GE, USA

Ghiaus, C. 2013. Causality issue in the heat balance method for calculating the design heating and cooling load. Energy 50:292-301

Ghiaus, C. 2014. Linear algebra solution to psychrometric analysis of air-conditioning systems. Energy 74: 555-566

Hands Down Software 2007. Psychrometric Analysis. Professional Edition User Manual. Oklahoma City, OK, USA

Incropera, F. P., et al. 2006. Fundamentals of Heat and Mass Transfer 6th ed. John Wiley & Sons

Karnopp, D.C. et R.C. Rosenberg. 1975. System dynamics: a unified approach. New York: John Wiley & Sons

Kirby B. 2010. Hydraulic Circuit Design in Micro- and Nanoscale Fluid Mechanics, Cambridge University Press, UK

McKew, H. and Duda S.W.. 2008. "Chapter 4 Air handling and distribution." ASHRAE Handbook - HVAC Systems and Equipement: 38-55. ASHRAE, Atlanta, GE, USA

Munters 2015. PsychroApp. Färögatan, Sweden

Tetens, O. 1930. Uber einige meteorologische Begriffe. Zeitschrift fur Geophysik:297-309