1

Introduction

ETYMOLOGY AND HISTORY

The Word Hysteresis

The word "hysteresis" is of Greek origin, spelled as $V \sigma \tau \varepsilon \rho \varepsilon \omega$ in Greek alphabet, meaning to "lag behind". The dictionary meaning of the word is given as n. phenomenon whereby changes in an effect lag behind changes in its cause. Or a retardation of the effect when the forces acting upon a body are changed (as if from viscosity or internal friction); *esp.* lagging in the values of resulting magnetisation in a magnetic material (as iron) due to a changing magnetising force. **-hys-ter-et-ic** *adj.*

Whilst commonly used in relation to magnetic effects in ferromagnetic materials, hysteresis as defined above can be found in physics, chemistry, engineering, biology and economics and so on; for example, in the deformation of rubber bands and shapememory alloys and many other natural phenomena. In natural systems it is often associated with irreversible thermodynamic change such as phase transitions and with internal friction; dissipation is a common side effect.

Hysteresis can be a *dynamic lag* between an input and an output, known as *rate-dependent* hysteresis; phenomena such as the magnetic hysteresis loops are mainly rate-independent.

Hysteresis in Magnetic Materials

The phenomenon of hysteresis was first associated with magnetic materials by Warburg[1]*, and simultaneously by (Sir James Alfred) Ewing[2], about the year 1879. Until then, the use of 'iron' – the commonest example of magnetic material in those days in electric machines and equipment – was based on the assumption of single-valued relationship between magnetising force and induced magnetism[3] and perhaps the importance of residual magnetism was not fully realised. Ewing showed that the induced magnetism lagged behind the applied magnetomotive force (mmf) and named this property the "hysteresis effect"[4]. Cyclic variation of the mmf resulted in a closed loop – the hysteresis loop – and once again this was demonstrated on a theoretical basis by Ewing[5] with the help of various models using small permanent magnets pivoted on needle supports.

^{*}numbers in square brackets show references, inclusive or otherwise, listed at end.

6 The Hysteresis Machines

A typical hysteresis loop pertaining to a 'hard' or permanent magnet, describing its 'key' parts or sections, is shown in Fig.A1.1. In many cases, an inherent accessory of a (major) hysteresis loop is a simple minor loop, called a recoil loop, as drawn in Fig.A1.2¹.



Fig.A1.1 : Hysteresis loop of a permanent magnet



Fig.A1.2 : Demagnetisation curve and a minor loop

Referring to Fig.A1.1, the (initial) part OA represents the "virgin" magnetisation curve of the material till saturation sets in (point A)². At this point if the magnetising force is

¹As shown in Fig.A1.2, a recoil loop identified as "major or "minor" ensues if, say during demagnetisation of a given material such as section B_r c in the figure, the magnetising force is reversed in the 'positive' direction to some value and reversed again to revert to the original B,H point. This is discussed further in Appendix I.

²It is well-known that the curve is not smooth, but is characterised by increase of magnetisation with applied magnetising force in 'small', random steps, identified as "Barkhausen jumps". Discovered by German physicist Heinrich Barkhausen in 1919, it is caused by rapid changes of size of magnetic domains (similarly magnetically oriented atoms in ferromagnetic materials).

removed, the magnet is left magnetised with a *remanence* or "residual magnetism", the intercept OB_r , theoretically sustained indefinitely. If the magnet were to be demagnetised to reduce the remanence to zero, the magnetising force must be reversed, section OB', being defined as coercive force (or "coercivity" of the material). A further increase of the magnetising force in the reverse direction would result in magnetisation of the magnet with reverse polarity, up to saturation, shown by point C. If so desired, the process of demagnetisation and magnetisation to original condition would trace the path depicted by C E A. The complete cycle is identified as a hysteresis loop and is *material* dependent³. This is illustrated qualitatively in Fig.A1.3 for three samples of magnetic materials.



Fig.A1.3 : Hysteresis loops for 'hard' and 'soft' magnetic materials

PHYSICAL INTERPRETATION

Hysteresis loss

The physical meaning and importance of the hysteresis loop was again explained by Warburg[1]. It was established by further research[6],[7] that the *area* of the hysteresis loop represented the *energy* 'lost' as heat in taking a material through one complete cycle of magnetisation⁴. The energy is expressed in joules.

$$loss = V \times \int H dB$$

the integral representing the area of the loop.

³Clearly, 'soft' magnetic materials such as sheet steel that are commonly employed in electric machines would exhibit only feeble hysteretic property, or a very 'narrow' hysteresis loop.

⁴Analytically, the total loss for a magnet of volume V during one complete cycle (or one hysteresis loop) would be

Alternatively, a permanent magnet will contain magnetic energy for any application, represented by the 'size' of hysteresis loop of the magnet material: the 'fatter' the loop, larger will be the energy content. In this respect, the hysteresis loop is akin to the "indicator diagram" of an internal combustion engine. It follows that the sub-area of the loop that would represent 'strength' of a magnet once magnetised and put to any use would be the area $B_rB'O$, Fig.A1.1; the larger the intercept OB, the remanence, and larger the coercivity, the stronger will be the magnet.

Steinmetz's Expression

 $Steinmetz^5$ suggested а frequently used empirical expression, based on his extensive laboratory experiments, to estimate hysteresis loss of a material that vields satisfactory results in most applications. The expression is given by 1 (

$$W_{hyst} = \eta B_{max}^{1.0} \times f$$

per unit volume of the material and where B_{max} is the maximum flux density pertaining to a given hysteresis loop; for example, denoted by Part A in Fig.A1.1 and f denotes the cycle of magnetisation. η is called "Steinmetz coefficient", being material dependent.

In this context, it is



Fig.A1.4 : The directional properties of magnetic flux density and magnetisation in a permanent magnet

important to observe how the intensity of magnetisation, M, the magnetising force, H, and resultant magnetic flux or flux density, B, are directed with respect to each other. This is shown qualitatively for a bar magnet in Fig.A1.4.

It is seen that the flux lines are *oppositely* directed to the direction of magnetising field within the magnet.

⁵Charles Proteus Steinmetz (1865-1923).

Experimental Determination of Hysteresis Loops

It often becomes necessary to obtain *actual* hysteresis loops, major as well as minor⁶, of a permanent magnet material by performing suitable experiment before its in an application. This is because the loops provided by the manufacturer may be somewhat at variance with the actual hysteresis loops. Further, many materials may be resorted to some degree of machining etc; for example, flexible or those in strip form being formed to, say, an annulus and may develop slightly different magnetic properties, esp. following a recommended heat treatment.

Experiments on permanent magnet materials to determine their magnetic properties, esp. obtaining hysteresis loops, are not simple and straightforward and may often require rather tedious methods or procedures.

Use of hysteresigraphs

A commonly employed device to obtain hysteresis loops of a permanent magnet material using an appropriate sample is known as hysteresisgraph. A typical hysteresigraph, shown schematically in Fig.A1.5, consists of



Fig.A1.5 : Schematic of a hysteresigraph

(i) A "massive" electromagnet, each of its poles excited by direct current passing through a winding of large number of turns that can carry sufficiently high current; the poles of the electromagnet being usually of (truncated) conical shape, about 8 cm in diameter at ends. A built-in mechanism in the device allows the

 $^{^{6}}$ As discussed, a minor loop ensues if during demagnetisation of a magnet, the magnetising force is reversed - inadvertently or deliberately - in the 'positive' direction to 'some' value and reversed again to revert to the original B,H point. Referring to Fig.A1.2, the recoil loop b c is more identified as a "major" recoil loop, the tip b reaching (or in constant with) the "B" axis a minor recoil loop may be identified as c b', shown dotted.

poles to be moved up and down in a vertical direction to vary the airgap between them, and lock in that position.

- (ii) A control mechanism to vary the excitation current from zero to a very large value to produce a magnetising field of, say, 300 to 500 kA/m so as to drive the sample material well into saturation, or demagnetise it if required, and to reverse the direction of current to apply magnetising field in the reverse direction.
- (iii) Devices or means to measure magnetising field and corresponding flux density in the sample. The former is usually measured using a calibrated Hall probe, positioned appropriately in the vicinity of the sample within the airgap whilst the latter is invariably measured with the help of specially designed "B coils" (or search coils), embedded in one of the poles (usually the lower pole), or by winding a search coil around the sample.
- (iv) An X-Y plotter or recorder to which are fed the outputs from the Hall probe and "B coil", respectively, to the X and Y terminal pairs to plot the B-H curve followed by the hysteresis loops⁷.

The details of an *actual* hysteresigraph and results obtained from a sample of permanent magnet material and other means of plotting hysteresis loops (using a permagraph) are described in Appendix I.

Alternating and Rotational Hysteresis

In practice, two types of hysteresis are known to occur in magnetic materials: alternating and rotational. The foregoing discussion applies to alternating hysteresis in which the cycle of magnetisation consists of increasing the magnetising force in one direction to a maximum value, usually to result in saturation of the material, reducing it to zero, followed by a similar variation in opposite direction, as shown in Fig.A1.1. In contrast, rotational hysteresis results from the rotation of a magnetic material, say a disc, through one cycle in an applied magnetising force of *constant* magnitude. This was predicted by Swinburn in 1890[8] and confirmed by Baily in 1894[9]. In most discussions and applications, it is the alternating hysteresis that is more relevant and takes precedence.

PERMANENT MAGNETS

Modern permanent magnet devices require the use of materials possessing large coercive forces; in turn this usually requires the presence of magneto-crystalline anisotropies. This is achieved these days by the use of rare-earth-based materials that possess sufficiently large anisotropies where this property originates from a combination of the crystal-field interaction of the 4f electrons with electrostatic charge of the surrounding ions and the relatively strong spin-orbit interaction of the 4f electrons. More than a decade ago high-performance permanent magnets were based on the use of alloys containing samarian (Sm) and cobalt (Co); such as SmCo₅. Recently, an even more powerful permanent

⁷The modern means to record the B-H curve and hysteresis loops may entail use of A-D converters and digital recording.

magnet material has been discovered which is based primarily on the ternary intermetallic compound $Nd_2Fe_{14}B$.

From the time of their historic developments decades ago, some of the commonly employed materials and their characteristics are summarised below.

Material	Composition	Characteristics
Ceramic, also known as ferrite	iron oxide and barium or strontium carbonate	energy up to 23,800 J/m ³ brittle, low cost, high coercive force, high resistance to corrosion
Alnico V	aluminium, nickel and cobalt	energy up to 37,400 J/m ³ high cost, high corrosion resistance high mechanical strength, high temperature stability, low coercive force, good resistance to demagnetisation
Samarium Cobalt	samarium and cobalt (rare earth magnetic material)	energy up to 102,000-136,000 J/m ³ can work up to 300°C high cost, high corrosion resistance, high temperature stability, high coercive force, low mechanical strength-brittle
Neodymium- iron-boron	neodymium, iron and Boron (another rare earth material)	energy up to 340,000 J/m ³ much higher cost, high coercive magnetic force, low mechanical strength-brittle, moderate temperature stability, low corrosion resistance
Injection moulded	resin and magnetic material powder	moderate energy product high cost, low temperature stability, moderate coercive force, high corrosion resistance highlyshapeable – can be manufactured in complex shapes owing to process of injection moulding
Flexible	same as injection moulded	available in strip and sheet form low energy product, low cost, high corrosion resistance, low stability, moderate coercive force

Table A1.1: Permanent magnet materials

An alloy introduced in 1970s offering many special features is known as "vicalloy".

Vicalloy⁸

Vicalloy is one of the family of cobalt-iron-vanadium based permanent magnet alloys. The alloy primarily consists of vanadium (10%), iron (about 37.4%) and cobalt (52%); the rest being manganese, carbon, silicone and a few more elements in very small quantities. The alloy is available commercially in the form of

- round bars of diameter 0.1 mm (wires) to 100 mm and random length
- strips or sheet of thickness 0.1 to 2.5 mm and width from 5.5 to 180 mm

⁸Used exclusively and extensively in the experimental hysteresis machine, discussed later in Part C.

Its development in commercially usable form is due to Nesbitt[10]⁹ who also held the original patent.

The material in strip form can be machined or bent in any shape; for example, an annulus, *before heat treatment*. Following the recommended heat treatment¹⁰, the material becomes "glass hard" and cannot be machined or formed further.

Vicalloy has been used variously in hysteresis machines¹¹.

Magnetic properties of vicalloy

- residual magnetism, B_r: 0.9 T
- coercivity, $H_c: 0.3 \times 10^5 \text{ A/m}$
- energy product: $8,000 \text{ J/m}^3$
- resistivity: $0.75 \ \mu\Omega \text{-m}^{12}$

Full magnetic characteristics of vicalloy are considered in Appendix I.

Applications of Permanent Magnets

Permanent magnets find extensive applications typically in*

- DC motors, esp. permanent magnet type
- Synchronous motors
- Stepper motors
- A variety of hysteresis machines
- Moving coil actuators
- Holding force actuators
- Magnetic suspensions
- Sensors
- 'Steady field' providers

*The list is suggestive, not exhaustive.

⁹See, for example, E. A. Nesbitt: Vicalloy – a workable alloy for permanent magnets, Trans AIMME, Vol.166, 1946, pp 416-25.

 $^{^{10}\}text{Heating to}~600^{\circ}\text{C} \pm 2^{\circ}\text{C}$ in an inert or vacuum oven, followed by gradual cooling to room temperature.

¹¹For example, vicalloy has been in use in hysteresis motors both in solid and laminated form (for higher frequency applications).

¹²Although the energy product of vicalloy is low in comparison to, say, ALNICO V and far lower than those of rare earth materials, it was chosen for hysteresis machine (Part C) on account of its availability in strip for, ease of machining (including drilling of holes for search coils etc.) prior to heat treatment and high resistivity to minimise eddy-current loss in the annulus.

Hysteresis Loop Approximations

The implicit property of any magnetic material, esp. permanent magnets, results in highly non-linear relationship in magnetising force 'H', and corresponding induced flux density, B, more significantly in saturated condition of the material. The relationship cannot be expressed by a single mathematical equation or form. This assumes a more complicated form when considering closed hysteresis *loops*, following multi-valued relationship between H and B, esp. when required to be modelled in analytical treatment of operation and performance of machines and devices incorporating permanent magnet materials. This has led to various simplifications or approximations of hysteresis loops whilst modelling various machines. Some of these are qualitatively illustrated in Fig.A1.6.



(c) Hysteresis loop as a parallelogram

Fig.A1.6 : Hysteresis loop approximations

An approximation that has been in use to effect some distinct advantages is the "inclined ellipse".

Inclined ellipse

This approximation has been widely used [11], [12], [13], [14], [15] corresponding to an *assumed sinusoidal variation of flux density* and considering only *fundamental* component of applied mmf. The elliptic hysteresis loop, inclined to the mmf axis, permits inclusion of the angle of hysteretic advance consistently in the analysis based on the solution of Laplace's equation[15], resulting in a relatively simple analysis. Whilst the elliptic hysteresis loop approximation may retain nearly the entire area of the actual hysteresis loop, and hence the energy product, and may be easy to model analytically, a source of inaccuracy may arise from the assumption of a similar variation for higher-order space harmonics in the applied mmf and the subsequent superposition of, say power loss, in the analysis. Since the harmonics rotate at different speeds relative to the fundamental, these do not necessarily form closed loops of elliptical shape and the loops may not be repeatable.

The basis of assuming an elliptic hysteresis loop in hysteresis machines to account for *spatial* hysteresis is to express the sinusoidal "B" wave being shifted in space by a *constant* angle ' γ '. For any point of the rotor, there are thus two components of H and the cycle of magnetisation is elliptic. Thus,

$$\begin{bmatrix} B_{r} \\ B_{\theta} \end{bmatrix} = \mu \times \begin{bmatrix} \cos \gamma & \sin \gamma \\ -\sin \gamma & \cos \gamma \end{bmatrix} \begin{bmatrix} H_{r} \\ H_{\theta} \end{bmatrix}$$
$$B = \sqrt{B_{r}^{2} + B_{\theta}^{2}} = \mu \sqrt{(H_{r}^{2} + H_{\theta}^{2})}$$

with

The assumption of constant γ and μ allows the hysteresis to be accounted for, and an analysis using Laplace's equation would be applicable.

Parallelogram approximation

This was first used by Copeland and Slemon [16], [17], later modified to a rectangular shape to represent the steeply rising part and saturation stage of the B-H curve. Beyond the 'knee' of the curve, the saturation condition of the material is simply regarded as the "airgap line" with unity relative permeability. The advantages claimed of this approximation were analytic simplification and the feasibility of deriving an equivalent circuit for the machine using the particular material. However, an obvious limitation would be that the flux distribution would contain large number of harmonics even when the applied mmf would be assumed to be sinusoidal.

Note that both approximation shown in Fig.A1.4(b) and (c) pertain to the above discussion.

Other Approximations

At low magnetising forces, hysteresis loops formed by displaced parabolas may justify close resemblance to the actual loops and provide an accurate means to estimate alternating hysteresis loss analytically[18]. A displaced rectangular hyperbola shape may seem to be more appropriate approximation for loops having B_m , H_m point near 'knee' of the B-H curve.

Machines Incorporating Hysteretic Materials

There are a variety of machines incorporating permanent magnetic material, almost invariably being a part of the rotor. The characteristic aspect of these machines is to exploit the hysteresis loss in the material to result in useful power or torque in the machine; in some cases the torque may be a braking torque.

Some of these machines such as

- the Hysteresis Motor
- the Hysteresis Coupling
- the Hysteresis Brake
- the Hysteresis Clutch
- the Hysteresis-reluctance Motor

are described later in Part B.