CHAPTER 1

# Simple Stresses and Strains

## 1.1 INTRODUCTION

Strength of materials is a branch of applied mechanics that deals with the behavior of solid bodies subjected to various types of loading. The main objective of this subject is to determine the stresses, strains and deflections produced by loads. If these quantities can be found for all values of load up to failure load, then we will have a complete picture of the mechanical behavior of the body.

In general a body can be subjected to three different types of loads, known as tensile, compressive and shear loads. Tensile load is force acting on a member which tends to elongate the member.

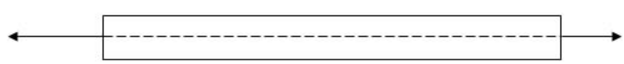


**Fig.1.1** Tensile Force

Compressive load is force acting on a member which tends to decrease the length of the member.

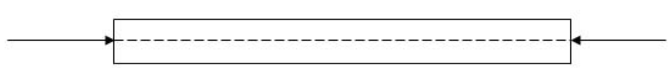


**Fig.1.2** Compressive Force

Tensile and Compressive loads are known as *direct (or) normal (or) axial loads*. Since these loads are acting normal to the surface and always passes through the centroid.

If the forces acting parallel to the surface (or) forces acting on a piece of material tend to slide one layer of the material over the next layer, then the forces are known as *shear loads*.

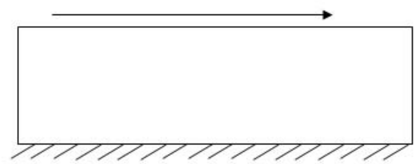


**Fig.1.3** Shear Force

## 1.2 NORMAL STRESS AND STRAIN

The fundamental concepts of stress and strain can be illustrated by considering a prismatic bar, loaded by axial forces P at the ends as shown in the figure. A prismatic bar is straight structural member having constant cross section throughout its length. In this example, the axial forces produces an uniform stretching of the bar, hence the bar is said to be in tension.

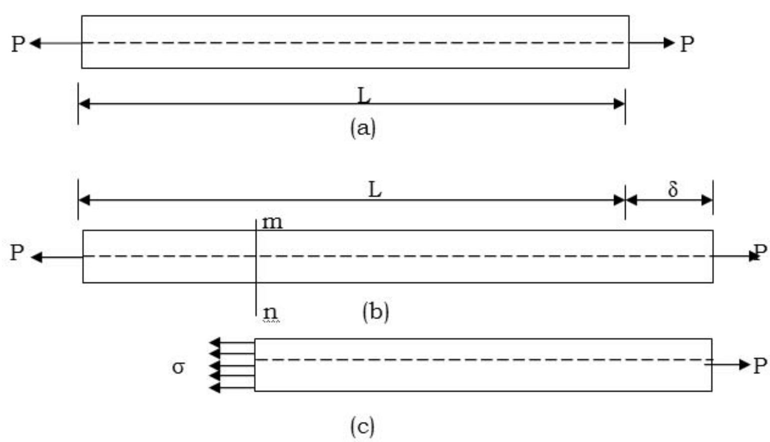



**Fig.1.4** Concept of Normal Stress

To investigate the internal stresses produced in the bar by axial forces, we make an imaginary cut at section m – n. This section is taken perpendicular to the longitudinal axis of the bar; hence it is known as cross section. We now isolate the part of the bar to the right of the cut as a free body. The tensile force P acting at the right hand end of the free body, at the other end the forces representing the action of removed part of the bar upon the part that remains. These forces are continuously distributed over the cross section. The intensity of force is called the stress. In other words, *the internal resistance force per unit area offered* *by the body against the externally applied force is known as stress,* usually denoted with a Greek letter 'σ' (Sigma).

$$\sigma = \frac{\text{Force}}{\text{Cross Sectional Area}} = \frac{P}{A} \qquad .....(1.1)$$

From the figure it is evident that this resultant must be equal in magnitude and opposite to the applied, hence stress can also be defined as force per unit area, since the bar is stretched by the applied load P. So, these stresses are known as tensile stresses. If the load is reversed in direction causing the member to compress, we obtain compressive stresses.

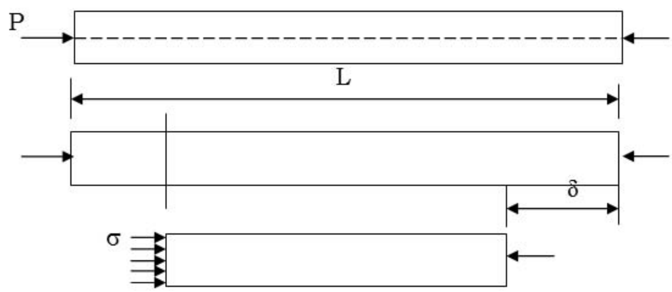



**Fig.1.5** Compressive stress

The stress σ acts in a direction perpendicular to the cut surface, it is referred to as normal stress. Thus normal stresses may be tensile or compressive stresses. In general tensile stresses will be denoted with '+' sign and compressive stresses with '–' sign.

In S.I Units the stress units are $N/m^2$ (or) Pascal (Pa).

$1\ N/m^2 = 1Pa$. (or) $1\ N/mm^2 = 1MPa = 1\times 10^6$ Pa.

(or) $1\ GPa = 1\times 10^9\ Pa = 10^3\ N/mm^2 = 10^3$ MPa

Since 1 Pa is very small quantity, MPa (or) GPa is normally used units for stress.

An axially loaded member undergoes change in length, becoming larger when the bar in tension and shorter when it is in compression. The total change in length is denoted with Greek letter 'δ'. Now, the normal strain or simply strain will be defined as change in length of the bar per unit length representing with Greek letter 'ε'.

$$\varepsilon = \frac{\text{Change in length}}{\text{Original length}} = \frac{\delta}{L} \qquad .....(1.2)$$

If the bar is in tension, the strain is called positive strain, if it is in compression, the strain is called compressive strain. Tensile strain will be taken as Positive and Compressive strain as negative.

## 1.3 SHEAR STRESS AND SHEAR STRAIN

If the forces acting parallel to the surface (or) forces acting on a piece of material tend to slide one layer of the material over the next layer, then the forces are known as shear loads. The stresses due to these loads are known as shear stresses. i.e. a shearing stress may produce whenever the applied loads cause one section of a body tend to slide past its adjacent section.

Shearing stress differs from both tensile and compressive stresses in that it is caused by forces acting along (or) parallel to the area of resisting the forces, where as tensile (or) compressive stresses are caused by forces perpendicular to the areas on which they act. For this reason tensile and compressive stresses are called *normal stresses* whereas shearing stress may called a *tangential stress.*

Consider a block shown in Fig. 1.6 subjected to a shear force V acting parallel to the surface.

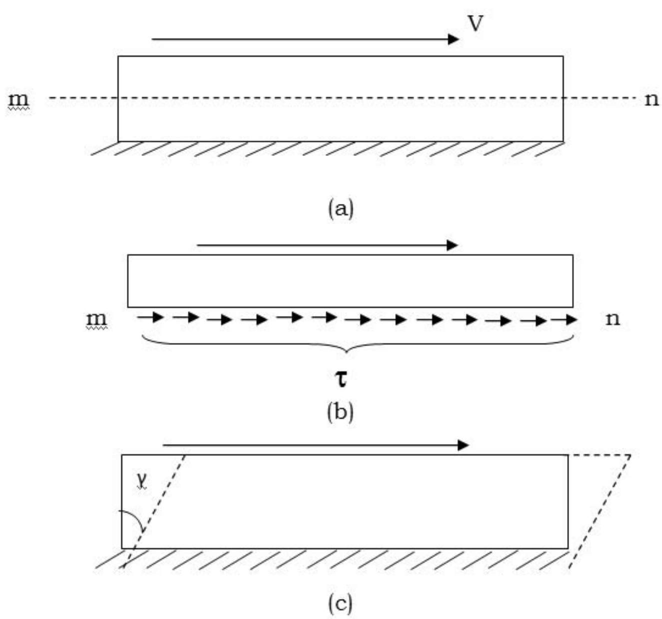



**Fig.1.6** Shear Stress and strain

If V is the force applied and A is the area being sheared, then the intensity of shear stress denoted with a Greek letter 'τ' is given by

$$\tau = \frac{\text{Shear Force}}{\text{Shear Area}} = \frac{V}{A} \qquad .....(1.3)$$

The units for the shear stress are the same as that of normal stresses. That is Pa (or) MPa (or) GPa.

The shear force produces change in the shape of the element. The angle 'γ' (Greek Letter GAMMA) is a measure of the distortion, or change in the shape of the element is called the shear strain. The shear forces (or) shear stresses have no tendency to change the lengths of the sides of the element. *The shear strain means only change in the angle or simply angular defomation.*

Fig. 1.7 shows some of the practical examples for shear stress.

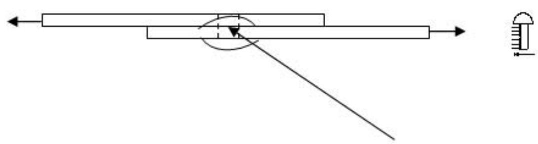


**Fig.1.7 (a)** Rivet in Single Shear


**Fig.1.7 (b)** Rivet in Double Shear

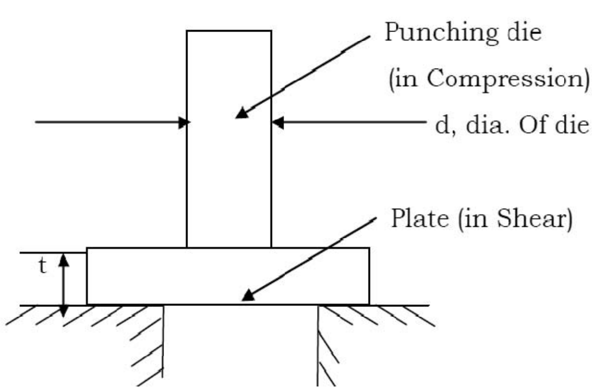



**Fig.1.7 (c)** Punching plate is in single shear

In Fig.17(a), since the rivet is in single shear, the shear area is $\frac{\pi}{4}d^2$, where 'd' is the diameter of the rivet. But the rivet in Fig. 17 (b) is in double shear so shear area is $\left(2.\frac{\pi}{4}d^2\right)$. Similarly one can imagine the bolts in single or double shear. In Fig.17(c), the punching die is subjected to compressive force, but plate is subjected to shear force and the shear area is $\pi dt$. Where 'd' is the diameter of the die punch and 't' is the thickness of the plate.

## 1.4 POISSON'S RATIO

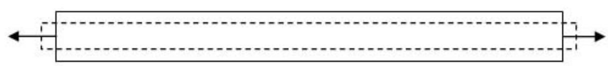


**Fig.1.8** Length increases, width decreases

When a prismatic bar loaded as shown in the figure 1.8, the length will be increased and the longitudinal strain (or normal stain or axial strain or simply strain) can be calculated from the equation 1.2. But, it can be readily visualized (dashed line in Fig. 1.8) that the lateral dimensions (which are not parallel to the load) are decreasing. The stain due to change in lateral dimensions are known as lateral strain. The ratio between lateral stain to longitudinal strain is known as Poisson's ratio denoted with a Greek letter 'μ'.

$$\text{Poisson's ratio } \mu = -\frac{\text{Lateral Strain}}{\text{Longitudinal Strain}} \qquad .....(1.4)$$

For a bar in tension, the lateral strain is in negative sign (decrease in width), longitudinal strain is in positive sign (elongation). Similarly in compression longitudinal strain is negative (decrease in length) but lateral strain is positive (increase in width). Therefore Poisson's ratio for most of the materials is positive. For many metals the Poisson's ratio is in the range of 0.25 to 0.35. However the cork has zero Poisson's ratio and rubber has a value of 0.5.

The lateral strain is proportional to longitudinal strain if the material is homogeneous and isotropic. A material is homogeneous if it has the same composition throughout the body; hence elastic properties are same at every point in the body. Isotropic materials have the same elastic properties in all directions.

## 1.5 STRESS STRAIN DIAGRAMS

We can broadly classify the available materials into 2 different types, viz Ductile and Brittle materials. Ductile materials undergo large strains before failure. Examples are Mild steel, Aluminum etc; Brittle materials are failing at relatively low values of strains. Examples are Concrete, Stone, Cast Iron, Glass etc.

The mechanical properties of materials used in engineering are determined by tests performed on small specimen of the material. The most common material test is Tension test, in which tensile loads are applied. The ends of the specimen are enlarged, where they fit in the grips so that failure will occur in the central uniform region, where the stress is easy to calculate. The Universal Testing Machine (UTM) shown in the figure 1.9 is a standard machine used to perform this test.

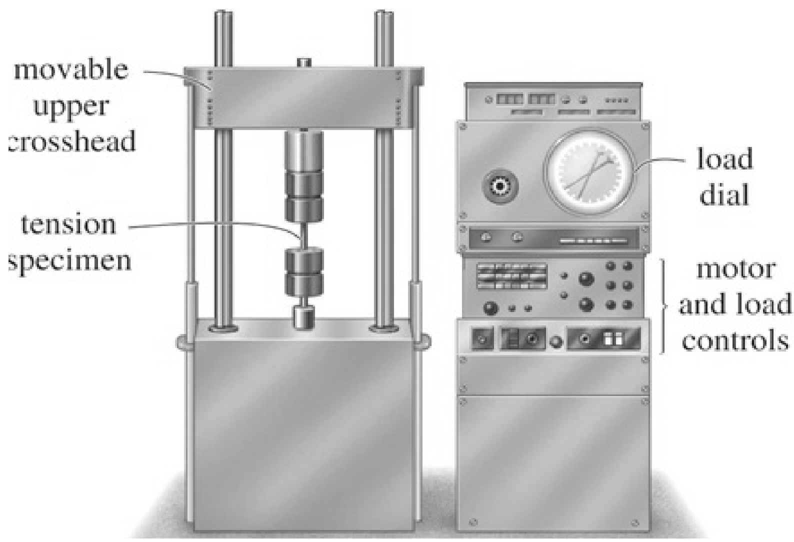



**Fig.1.9** Universal Testing Machine

### STRESS-STRAIN DIAGRAM FOR DUCTILE MATERIAL

The standard specimen has a diameter of 12.7 mm and a gage length of 50.8 mm between the gage marks, which are the points where the extensometer arms attached the specimen. As the specimen is pulled, the elongation over the gage length is measured along with the load.

The axial stress 'σ' in the test specimen is calculated by dividing the load with the cross sectional area. The average strain can be calculated by dividing the elongation with initial length. Fig. 1.10 gives a typical stress-strain diagram (not to scale) for mild steel. Strains are plotted on x – axis and stress are plotted on y – axis.

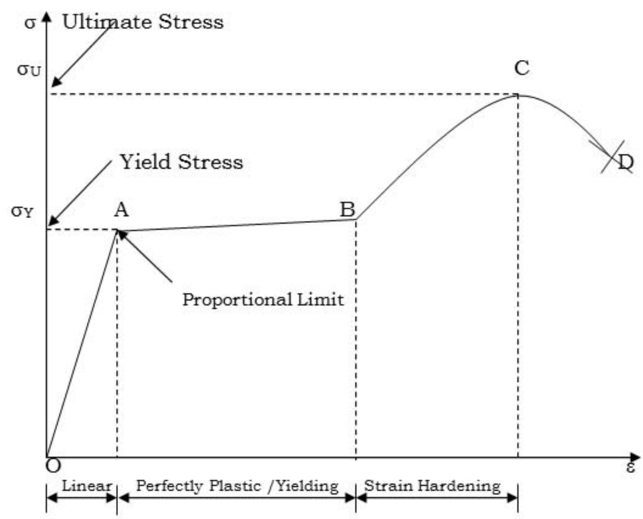



**Fig.1.10** Stress – Strain diagram for Mild steel

The diagram begins with a straight line from O to A. In this region, the stress and strain are directly proportional, and the behavior of the material is said to be linear. Beyond the Point A, the linear relationship between stress and strain does not exist.

From Point A onwards the curve is almost straight line. That is considerable elongation occurs, with no noticeable increase in the tensile force. This phenomenon is known as *Yielding of the material.* The stress corresponding to this is known as *Yield stress*. In this region A to B the material becomes perfectly plastic.

Further increase in the load, the material undergo strain hardening region B to C. In this material undergo changes in atomic and crystalline structure. The stress-strain diagram has a positive slope in this region B to C. The load reaches maximum load at C and corresponding stress is known as *Ultimate Stress.* Further stretching of the bar actually accompanied by a reduction in the load, and finally fracture occurs at point. D.

Certain ductile materials like Aluminum and its alloys will give the stress strain diagram as shown in the Fig.1.11. They do not have clearly definable yield point. They exhibit gradual transition from liner to non linear region. Aluminum and its alloys do not have obvious yield point, but it goes large strains before failure. The yield stress may be determined using *Offset method*. A line is drawn on the stress-strain diagram parallel to initial straight line but is offset by a constant distance of 0.2% (0.002). The intersection of the offset line with stress-strain curve is the yield point.

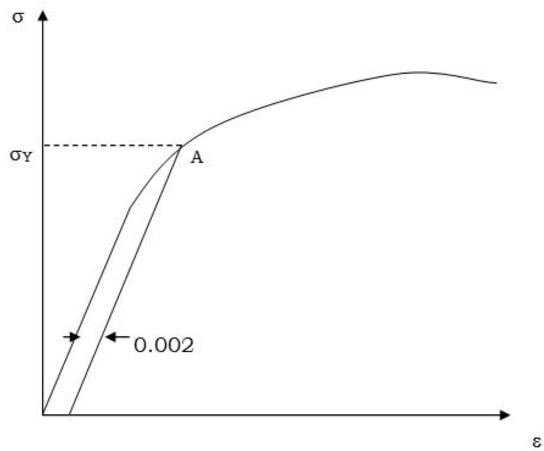



**Fig.1.11** Offset Method

In general, ultimate stress is considered for designing of brittle materials and yield stress is considered for ductile materials.

## STRESS – STRAIN DIAGRAM FOR BRITTLE MATERIALS

Glass is nearly Ideal brittle material. It exhibits almost no ductility whatsoever. The stress-strain diagram for Brittle material in tension is almost a straight line and failure occurs before any yielding takes place.

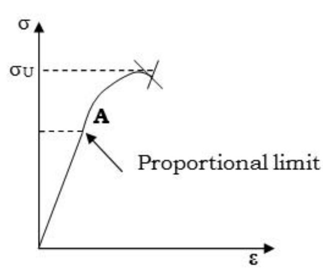



**Fig.1.12** Stress – strain diagram for brittle material

## 1.6 HOOKE'S LAW

In the stress-strain diagram, up to the point A, the material is in proportional limit. In this proportional limit the stress is directly proportional to the strain.

$$\sigma \propto \varepsilon \qquad \text{....(a)}$$

Or

$$\sigma = E\varepsilon \qquad \text{.....(1.5)}$$

where E is constant of property of the material known as Modulus of elasticity (or) young's modulus of elasticity. Similar equation can be written for shear stress and shear strain. The units for young's modulus is Pascal (same as the stress units).

The Hooke's law for shear stress is

$$\tau \propto \gamma \qquad \text{.....(b)}$$

Or

$$\tau = G\gamma \qquad \text{.....(1.6)}$$

where 'G' is also a constant property of the material, known as Shear Modulus of Elasticity (or) Modulus of Rigidity. The relation between E and G is given by the equation

$$G = \frac{E}{2(1+\mu)} \qquad \text{.....(1.7)}$$

where μ is the Poisson's ratio of the material.

Now, Let us go back to the equation 1.5 again, $\sigma = E\varepsilon$

From equations 1.1 and 1.2, $\frac{P}{A} = E\frac{\delta}{L}$

(Or)

Deflection $\delta = \frac{PL}{AE}$ ....(1.8)

The eq.(1.8) can be used to find the change in length (elongation or contraction) of a prismatic bar subjected to normal forces (tensile or compressive forces). When a sign convention is needed, in general, elongation is taken as positive and contraction/shortening as negative. From the above equation we can observe that the elongation of a bar due to axial loads is directly proportional to the load and length, inversely proportional to modulus of elasticity and the cross sectional area. The product EA is known as axial rigidity of the bar.

The stiffness of an axially loaded member is defined as the force required to produce a unit deflection; hence from the equation 1.8, the stiffness of the bar = $K = \frac{AE}{L}$. The reciprocal of the stiffness may be considered as flexibility. The flexibility f is defined as deflection of the bar due to a unit load; hence from the equation 1.8, the flexibility of the axially loaded bar is $f = \frac{L}{EA}$.

## 1.7 VOLUMETRIC STRAIN (DILATION)

If a bar in tension, the dimensions will change, therefore volume too. To find the change in volume, let us consider a bar subjected to a tensile load P, whose sides are given by a, b, c as shown in the Fig.(1.13). The dotted line indicates the deformed shape.

The load is acting along the x-direction. So, x-axis dimension (length a) is a longitudinal dimension whereas the other direction dimensions (width b, thickness c) are later dimensions. Due the tensile loading, the length, 'a' increases, width 'b' and thickness 'c' decreases. Let 'σ' is the stress and 'ε' is the strain (longitudinal). Hence, increase in length is aε. Let change in width is δb and change in thickness is δc. From the concept of Poisson's ratio $\mu = -\frac{\frac{\delta b}{b}}{\epsilon} = -\frac{\frac{\delta c}{c}}{\epsilon}$. Therefore the decrease in width is -με.b and decrease in thickness is –με.c.

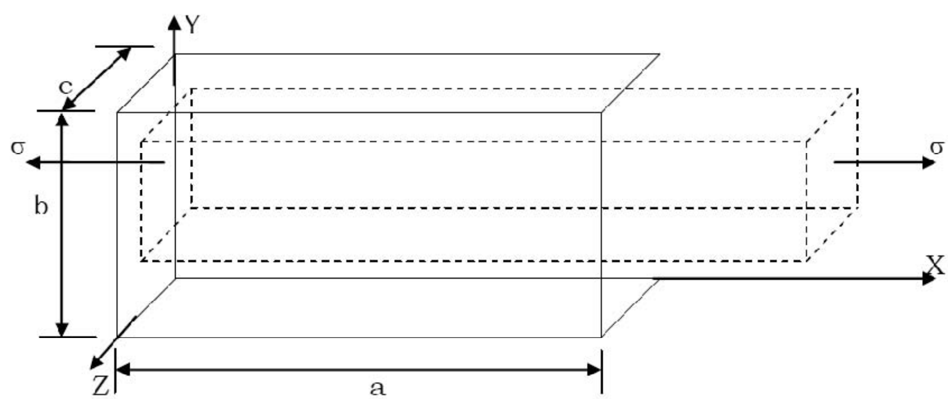



**Fig.1.13** Change in dimensions due to tensile loading

Final length = a + aε = a(1+ ε)

Final depth = b – με.b = b(1– με)

Final thickness = c – με.c = c(1 – με)

Initial Volume $V_i$ = abc

Final Volume $V_f$ = abc (1+ ε)(1– με)(1 – με)

= abc (1 + ε) (1 + $\mu^2\varepsilon^2$ – 2με) [neglect $\mu^2\varepsilon^2$]

= abc (1 + ε – 2 με – 2 $\mu\varepsilon^2$) [neglect $2\mu\varepsilon^2$]

= abc (1 + ε – 2 με)

Change volume $\Delta V = V_f - V_i$

= abc (1 + ε – 2με) – abc

= abcε (1 – 2μ)

Volumetric strain (or) Dilation

$$e = \frac{\Delta v}{v} = \varepsilon(1-2\mu) = \frac{\sigma}{E}(1-2\mu) \quad .....(1.9)$$

## 1.8 STATICALLY INDETERMINATE STRUCTURES

When a structural member or a machine element is in equilibrium, the three static equilibrium equations are used to find the axial forces in the members, reactions at the supports.

If one can able to find the axial forces in members and reactions by using static equilibrium equations, the system is said to be *statically determinate structure*. For many structures only the equations of static equilibrium not sufficient, for the calculation of axial forces and reactions. These structures are called *statically indeterminate structure*. To solve such problems, particularly in the strength of materials one should make use of the geometry of the deformations to obtain additional equations, known as compatibility equations.

From the Fig.1.14(a) one can easily identified that the reaction at the support $R_A = P_1 + P_2$. But from the Fig.1.14(b) we can write only one equation, $R_A + R_B = P_1 + P_2$. To solve this equation we require one more equation. So, it is a statically indeterminate structure. The compatibility equation for this problem is, from the geometry we can write that, net deflection δ = 0.

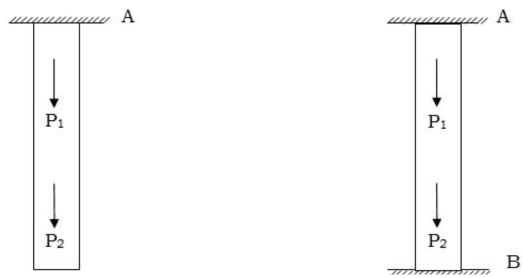



**Fig.1.14** (a) Statically determinate structure

(b) Statically indeterminate structure

## 1.9 THERMAL STRESSES

The dimensions of a material will undergo changes if change in temperature occurs. For example, if the temperature raises, the length will increase. If we allow the body for these changes, the stress induced is zero. But if we restrict these changes, stresses will be developed. If the body is restricted from the elongation compressive stresses will develop and if the body is not allowed to contract, tensile stresses will develop. For statically determinate structures the thermal stresses will be zero.

Consider a bar of length 'L' having coefficient of expansion 'α'. If its temperature increased (or decreased) by t $^{O}$C, the thermal strain and thermal stress (if restricted) in the body is given by

Thermal strain $\varepsilon_t = \alpha t$ ....(1.10)

Thermal strains $\sigma_t = E.\ \alpha.\ t$ .....(1.11)

If two different materials are joined together that is known as composite bar (one member in another member, but not one after the other). When a composite bar is subjected to change in temperature, different components try to change in length differently because of different coefficient of expansion values. Material which is having high coefficient of expansion tries to expand highly; if it is prevented to do so compressive load will be developed. Similarly materials having low coefficient of expansion will be subjected to tensile loads.

If no external load is applied, the compressive load in high coefficient of expansion material is equal to tensile load in the low coefficient of expansion material. The compatibility equation is net deflection in both the materials must be same. Net deflection can be calculated as deflection due to temperature change + deflection due to the induced loads due to prevention from deflection.

## Solved Examples

**1.1 A Prismatic bar with rectangular cross section (20 mm × 40 mm) and length is 2.8 m subjected to an axial tensile force of 70 kN. The measures elongation of the bar is 1.2 mm. Calculate stress and strain.**

**Sol: Given Data:**

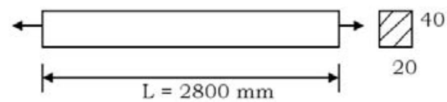


**Fig. P 1.1**

Length $l$ = 2800 mm**,** Width b = 20 mm,

Height h = 40 mm,

load P = 70 × $10^3$ N. change in length δ = 1.2 mm

Stress $\sigma = \frac{P}{A} = \frac{70\times10^3}{20\times40} = 87.5\frac{N}{mm^2} = 87.5 \text{ MPa}$

Strain $\varepsilon = \frac{\delta}{L} = \frac{1.2}{2800}$ = 4.286 × $10^{-4}$

**1.2A short hollow Cast Iron cylinder of wall thickness 10 mm is to carry a compressive a load of 600 kN. Determine the outside diameter of the cylinder if the stress 540 MN/m².**

**Sol: Given Data:** Wall thickness t = 10 mm, Load P = 600 × $10^3$ N. Stress σ = 540 N/mm².

Let outside diameter $d_0$ = d mm.

Inside diameter $d_1$ = (d – 20) mm

Cross sectional area A = $\frac{\pi}{4}\left(d^2-(d-20)^2\right)$ mm²

= $(10d - 100)\,\pi$ mm²

Stress $\sigma = 540 = \frac{P}{A} = \frac{600\times10^3}{(10d-100)\pi}$

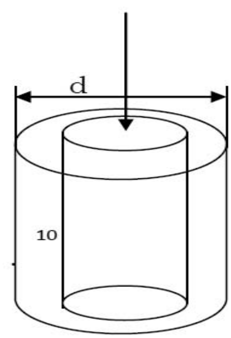


**Fig.P1.2**

On solving, the outside diameter d = 45.37 **mm**

**1.3 A horizontal bar CBD having a length of 2.4 m is supported and loaded as shown on the figure P1.3(a). The vertical member AB has cross sectional area of 550 mm². Determine the magnitude of load p so that it produces a normal stress of 40 MPa in member AB.**

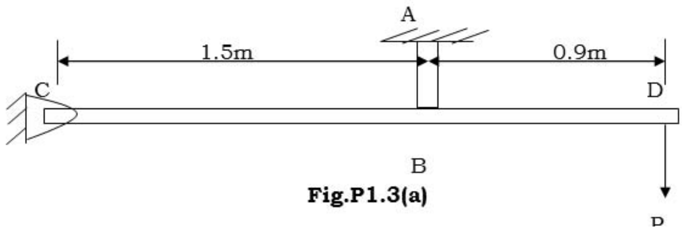



**Fig.P1.3(a)**

**Sol:** The Free body diagram is shown in the Fig.P1.3(b).

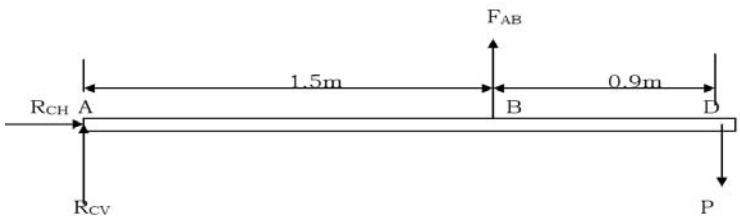



**Fig.P1.3(b)**

$\sum M_C = 0$

$\Rightarrow$ $F_{AB}\ (1.5) = P\ (2.4)$

$\Rightarrow$ $F_{AB} = 1.6P$

$$\Rightarrow \quad \sigma_{AB} = \frac{F_{AB}}{A}$$

$$40 = \frac{1.6P}{550}$$

$\Rightarrow$ **P = 13.75 kN**

**1.4 A strut and cable assembly ABC (Fig.P1.4(a)) supports a vertical load of P=15 kN. The cable has an effective area of 120 mm² and the strut has 250 mm². Calculate the normal stresses in strut and cable.**

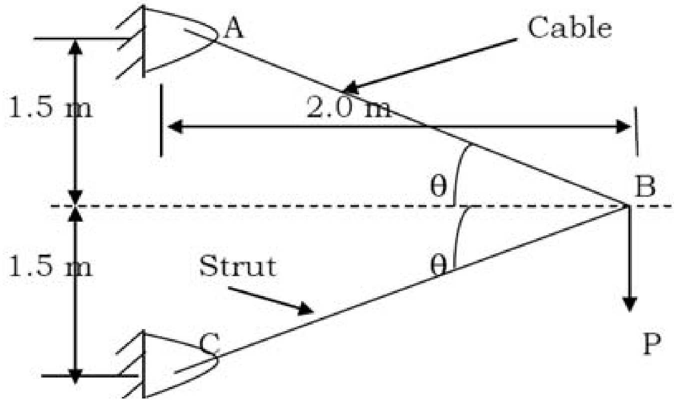



Fig.P1.4(a)

**Sol:** **Given Data**: $P = 15 \times 10^3$ N, $A_C = 120$ mm², $A_s = 250$ mm².

Let the force in cable is $F_C$ and the force in strut is $F_S$. The Free body diagram at B is shown in the Fig. P1.4(b).

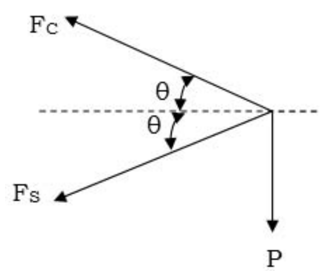



Fig. P.1.4(b)

From Fig. 1.4(a) $\tan\theta = \dfrac{1.5}{2}$

$\Rightarrow \quad \theta = 36.87^0.$

$\Sigma F_x = 0$

$\Rightarrow \quad F_C \cos\theta + F_S \cos\theta = 0$

$\Rightarrow \quad F_c = -F_S$ .....(1)

$\sum F_Y = 0$

$\Rightarrow \quad F_C \sin\theta - F_S \sin\theta = P$ .....(2)

On solving (1) and (2)

$F_C = -F_S = 12.55$ kN

So, Stress in strut = $\frac{12.55\times10^3}{250}$ = 50.02 MPa (Compression)

Stress in steel = $\frac{12.55\times10^3}{120}$ = 104.58 MPa (Tensile)

**1.5 A steel rod 1 m long and 13 mm in diameter carries a tensile load of 1.35 kN. The bar increases by a length of 0.5 mm when the load is applied. Determine the normal stress and strain in the bar.**

**Sol:** **Given Data:** length l = lm = 1000 mm, dia. d = 13 mm, load P = 1.35 × $10^3$ N, elongation δ = 0.5 mm.

Normal strain $\epsilon = \frac{\delta}{l} = \frac{0.5}{1000} = 5X10^{-4}$

Normal stress $\sigma = \frac{P}{A} = \frac{P}{(\frac{\pi}{4})d^2} = \frac{1350}{132.43}$ = 10.19 MPa

**1.6 Three pieces of wood each having a cross section of 2 cm × 2 cm are glued together and also to the foundation. If the middle member is subjected to a force of 30 kN, determine the average shear stress induced in each of the glued joint.**

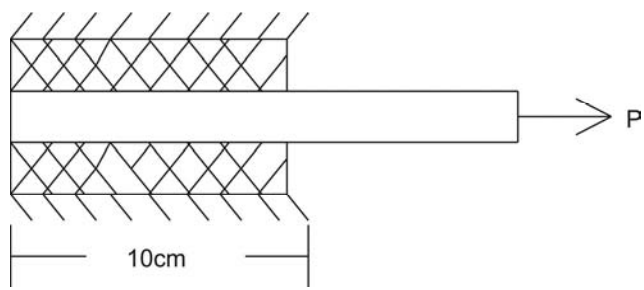



**Fig. P1.6**

**Sol:** **Given Data:** width b = 20 mm, thick t = 20 mm, length l = 100 mm, load P = 30,000 N.

The middle member is in double shear.

So, the shear area for the middle member = 2(100)(20) = 4000 $mm^2$.

The shear stress $\tau = \frac{P}{\text{Shear area}} = \frac{30000}{4000}$ = 7.5 MPa.

**1.7 Three pieces of wood are glued together on their planes of contact (see figure p 1.7). Each piece has cross section 50.8 × 101.6 mm and length 203.2 mm. A load of 10.68 kN is applied**

**to the top piece through a steel plate. Find the shear stress in the glued joints.**

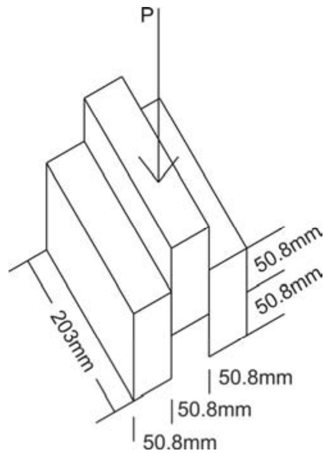



**Fig. P.1.7**

**Sol:** The glued joint is in double shear. So, the shear area = 2(50.8)(203.2) = 20,645.12 mm$^2$.

The shear stress $\tau = \dfrac{P}{\text{Shear area}} = \dfrac{10680}{20645.12} = 0.52\,\text{MPa}.$

**1.8 A punch with a diameter of 20 mm is used to punch a hole in aluminum plate thickness of 4 mm. If the ultimate shear stress for the aluminium is 275 MPa, what force P is required to punch the hole. What is the stress in the Punch.**

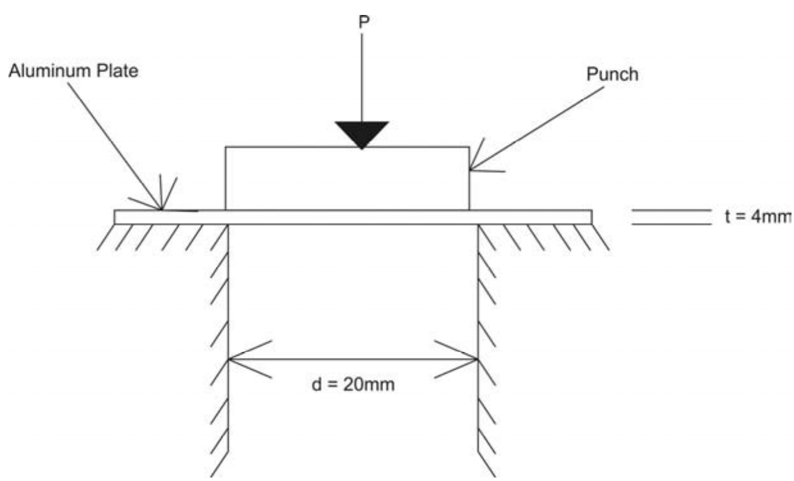



**Fig. P. 1.8**

**Sol:** **Given Data:** Punch dia. d = 20 mm, Plate thickness t = 4 mm, Shear stress $\tau$ = 275 MPa.

Since the Aluminum plate is in shear,

The shear area $\pi dt = 251.2\ mm^2$

The Shear stress $\tau = \frac{P}{\pi dt} = \frac{P}{251.2}$

$\Rightarrow \qquad 275 = = \frac{P}{251.2}$

$\Rightarrow \quad P = 69.08$ kN

The Punch is in compression.

The Stress in Punch $\sigma_P = \frac{P}{\left(\frac{\pi}{4}\right)d^2} = \frac{69080}{314.16} = 220$ MPa

**1.9 A hole is to be punched out of a plate having a shear strength of 40 GPa. The compressive stress in the punch is 50 GPa. Find**

**(i) Thickness of the plate if the punch diameter is 2.5 mm**

**(ii) Diameter of the punch if the plate thickness is 0.25 mm**

**Sol:** **Given data:** Compressive stress in the punch $\sigma = 50 \times 10^3$ MPa,

Shear strength of the Plate $\tau = 40 \times 10^3$ MPa.

Let d = diameter of the punch, P = load and t = thickness of the plate

Compressive stress in the punch $= \frac{\text{Load}}{\text{Area of the punch}} = \frac{P}{\frac{\pi}{4}(d)^2}$

$\Rightarrow \qquad 50 \times 10^3 = \frac{P}{\frac{\pi}{4}(d)^2}$

So, Load $\qquad P = 39.27 \times 10^3\, d^2$ .....(1)

Shear strength of the Plate = $= \frac{\text{Load}}{\text{Shear Area}} = \frac{P}{\pi dt}$

$\Rightarrow \qquad 40 \times 10^3 = \frac{P}{\pi dt}$

So, Load $\qquad P = 125.67 \times 10^3\, dt$ .....(2)

From (1) and (2)

$39.27 \times 10^3 d^2 = 125.67 \times 10^3\, d\, t$

$\Rightarrow\ d = 3.2\, t$

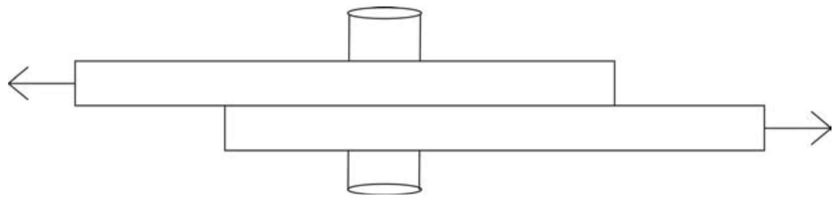
(i) Punch diameter d = 2.5 mm

So, Thickness t = 2.5/3.2 = 0.7813 mm

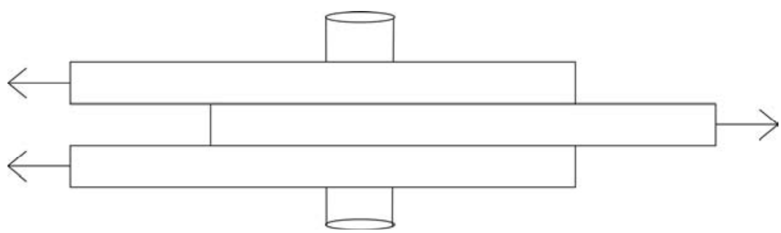
(ii) Thickness t = 0.25 mm

So, Punch diameter d = 3.2 × 0.25 = 0.8 mm

**1.10 Find the shear stress in the rivets for the following cases, take load = 1000 N and diameter of rivet = 15 mm.**



**Case (a)**



**Case (b)**

**Fig. P.1.10**

**Sol:** **Given data:** P = 1000 N, d = 15 mm

In Case (a) the rivet is in single shear only. So the shear area is

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(15)^2 = 176.72 \text{ mm}^2$$

Shear Stress $\tau = \frac{P}{A} = \frac{1000}{176.72} = 5.66$ MPa

In Case (b) the rivet is in double shear. So the shear area is

$$A = 2.\frac{\pi}{4} d^2 = 2.\frac{\pi}{4}(15)^2 = 353.44 \text{ mm}^2$$

Shear Stress $\tau = \frac{P}{A} = \frac{1000}{353.44} = 2.83$ MPa

**1.11 A tensile test is performed on a brass specimen of 10 mm diameter using a gage length of 50 mm. when applying a load P = 25 kN, it is observed that the distance between the gage marks increases by 0.152 mm. Calculate the modulus of elasticity of the brass**

**Sol:** **Given data:** Dia. Of the specimen d = 10 mm, gage length l = 50 mm, change in length d = 0.152 mm.

Normal Stress $\sigma = \frac{\text{Load}}{\text{Area}} = \frac{P}{A} = \frac{25000}{\frac{\pi}{4}(10)^2} = 318.31 \frac{N}{mm^2}$

Normal Strain $\varepsilon = \frac{\text{Change in length}}{\text{Original length}} = \frac{0.152}{50} = 3.04 \times 10^{-3}$

Modulus of Elasticity

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{318.31}{3.04 \times 10^{-3}} = 104.71 \times 10^3 \frac{N}{mm^2}.$$

= **104.71 GPa**

**1.12 A sample of Aluminium alloy is tested in tension. The load is increased until a strain of 0.0075 is reached. The corresponding stress in the material is 443 MPa. The load is then removed, and a permanent strain of 0.0013 is found to be present. What is the modulus of elasticity for the aluminum?**

**Sol:** **Given Data:** Strain $\varepsilon$ = 0.0075, stress $\sigma$ = 443 MPa, permanent strain $\varepsilon_p$ = 0.0013.

Strain in the elastic limit = $\varepsilon - \varepsilon_p$

= 0.0075-0.0013

= 0.0062

Modulus of elasticity $E = \frac{\text{Stress}}{\text{Strain}} = \frac{443}{0.0062}$

= $71.45 \times 10^3$ MPa

= **71.45 GPa.**

**1.13 A round bar of diameter 38.1 mm is loaded in tension by an axial force P. The change in diameter is measured as 0.07874 mm. Assuming E = 2.756 GPa and Poisson's ratio is 0.4, find the axial force P in the bar.**

**Sol:** **Given Data:** Diameter d = 38.1 mm, change in diameter $\delta d = -0.07874$ mm (let diameter decreases), $E = 2.756 \times 10^3$ MPa, $\mu = 0.4$

$$\text{Stress } \sigma = \frac{P}{A} = \frac{P}{\frac{\pi}{4}d^2} = \frac{P}{\frac{\pi}{4}(38.1)^2} = 8.77 \times 10^{-4} P$$

$$\text{Axial Strain } \sigma = \frac{\sigma}{E} = \frac{8.77\times10^{-4}P}{2.756\times10^3} = 3.182 \times 10^{-7} P$$

$$\text{Poisson's ratio } \mu = \frac{-\text{Lateral Strain}}{\text{Axial Strain}}$$

$\therefore$ Lateral Strain = $-\mu$ × Axial strain

$= -0.4 \times 3.182 \times 10^{-7}P$

$= -1.2728 \times 10^{-7}P$ .....(1)

$$\text{But, Lateral strain} = \frac{\text{change in diameter}}{\text{diameter}} = \frac{\delta d}{d}$$

$$= -\frac{0.07874}{38.1} = -2.067\times10^{-3} \quad .....(2)$$

From (1) and (2)

$-1.2728 \times 10^{-7}P = -2.067 \times 10^{-3}$

The axial load P = 16.24 kN.

**1.14 Two bars, one of aluminum and one of steel, are subjected to tensile forces that produce normal stress 165.36 MPa in both bars. What are the lateral strains in aluminum and steel bars respectively. If E = 73 GPa and Poisson's ratio = 0.33 for aluminum and E = 2.1 × 10³ MPa and Poisson's ratio = 0.3 for steel.**

**Sol:** **Given Data:** Stress $\sigma$ = 165.36 MPa, Young's modulus for aluminum $E_a = 73 \times 10^3$ MPa, for steel $E_s = 2.1 \times 10^5$ MPa, Poisson's ratio for aluminum $\mu_a = 0.33$ and for steel, $\mu_s = 0.3$

For Aluminium:

$$\text{Axial strain} = \frac{\text{Stress}}{\text{YoungsModulus}} = \frac{\sigma}{E} = \frac{165.36}{73\times10^3}$$

$= 2.27 \times 10^{-3}$

Lateral Strain = –Poissison's ratio × Axial Strain

$= -0.33 \times 2.27 \times 10^{-3}$

Lateral Strain = $-7.491 \times 10^{-4}$

For Steel: Axial strain = $\dfrac{\text{Stress}}{\text{Youngs Modulus}} = \dfrac{\sigma}{E} = \dfrac{165.36}{2.1 \times 10^3}$

$= 7.87 \times 10^{-4}$

Lateral Strain = –Poission's ratio × Axial Strain

$= -0.3 \times 7.87 \times 10^{-4}$

Lateral Strain = $-2.361 \times 10^{-4}$

**1.15 A compression member constructed from steel pipe (E = 200 GPa, μ = 0.3) has an outside diameter of 90 mm and a cross sectional area of 1580mm². What axial force will cause the outside diameter to increase by 0.0094 mm.**

**Sol:** **Given data:** $E = 200 \times 10^3$ MPa, $\mu = 0.3$, d = 90 mm, A = 1580 mm². $\delta d = 0.0094$ mm.

Stress $\sigma = \dfrac{\text{Load}}{\text{Area}} = \dfrac{P}{1580} = 6.33 \times 10^{-4}\ P$

Axial strain $\epsilon = \dfrac{\text{Stress}}{\text{Young's Modulus}} = \dfrac{6.33 \times 10^{-4} P}{200 \times 10^3}$

$= 3.165 \times 10^{-9} P$

Lateral Strain = –Poission's ratio × Axial Strain

$= 0.3 \times 3.165 \times 10^{-9} P$

$= 9.495 \times 10^{-10} P$ .....(1)

But, Lateral strain = $\dfrac{\text{Change in diameter}}{\text{diameter}} = \dfrac{\delta d}{d} = \dfrac{0.0094}{90}$

$= 1.044 \times 10^{-4}$ .....(2)

From (1) and (2)

$9.495 \times 10^{-10} P = 1.044 \times 10^{-4}$

Axial Load P = 110 kN

**1.16 During testing of a concrete cylinder in compression, the original diameter of 152.4 mm, was increased by 0.01016 mm, and the original length of 304.8 mm was decreased by 0.1651 mm under the action of a compressive load P = 231.4 kN. Calculate the modulus of elasticity and Poisson's ratio.**

**Sol:** **Given Data:** d = 152.4 mm, $\delta d$ = 0.01016 mm, l = 304.8 mm, $\delta l$ = –0.1651 mm (length decreases), P = 231.4 × $10^3$N.

$$\text{Stress } \sigma = \frac{\text{Load}}{\text{Area}} = \frac{P}{\frac{\pi}{4}d^2} = \frac{231.4\times10^3}{\frac{\pi}{4}(152.4)^2} = 12.69 \text{ MPa}$$

$$\text{Axial Strain } \in = \frac{\text{Change in length}}{\text{length}} = \frac{-0.1651}{304.8}$$

$$= -5.42 \times 10^{-4}$$

$$\text{Lateral Strain } \in_1 = \frac{\text{Change in diameter}}{\text{diameter}} = \frac{0.01016}{152.4} = 6.67 \times 10^{-5}$$

$$\text{Modulus of Elasticity } E = \frac{\text{Stress}}{\text{Axial Strain}} = \frac{\sigma}{\in} = \frac{12.69}{5.42\times10^{-4}}$$

$$= 23.4 \times 10^3 \text{ MPa} = 23.41 \text{ GPa}$$

$$\text{Poisson's ratio } \mu = \frac{-\text{lateral strain}}{\text{axial strain}} = \frac{-6.67\times10^{-5}}{-5.42\times10^{-4}}$$

$$= 0.12$$

**1.17 A steel bar of length 2.5m with a square cross section 100 mm on each side is subjected to an axial tensile force of 130 kN (see figure). Assuming that E = 200 GPa and μ = 0.3 find (a) the elongation of the bar (b) the change in cross sectional dimensions (c) change in volume.**

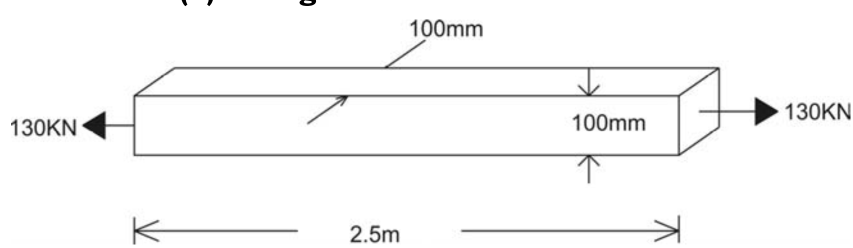



**Fig. P. 1.17**

**Sol:** **Given data:** l = 2.5 m, width b = 100 mm, height h = 100 mm, P = 130 × $10^3$ N, E = 200 × $10^3$ MPa, $\mu$ = 0.3

$$\text{Stress } \sigma = \frac{\text{Load}}{\text{Area}} = \frac{P}{A} = \frac{130\times10^3}{100\times100} = 13 \text{ MPa}$$

$$= 13 MPa$$

$$\text{Axial Strain } \in = \frac{\text{Stress}}{\text{Modulus of Elasticity}} = \frac{\sigma}{E} = \frac{13}{200\times10^{3}} = 6.5\times10^{-5}$$

$$= 6.5x10^{-5}$$

$$\text{But Axial Strain } \in = 6.5\times10^{-5} = \frac{\text{Change in length}}{\text{Length}} = \frac{\delta l}{l}$$

(a) Change in length $\delta l = \varepsilon . l = 6.5 \times 10^{-5} \times 2.5 = 0.1625$ mm

(b) Lateral strain $= \mu . \varepsilon = 0.3 \times 6.5 \times 10^{-5} = 1.95 \times 10^{-5}$

$$\text{But lateral strain } = \frac{\text{change in width}}{\text{width}} = \frac{\text{change in height}}{\text{height}}$$

Change in width = Change in height = $19.5 \times 10^{-5} \times 100$

= 0.0195 mm

(c) Volume $V = 2500 \times 100 \times 100 = 25 \times 10^{6}$ mm$^3$

Change in volume $\delta V = v.\varepsilon(1 - 2\mu)$

$= 25 \times 10^{6} \times 6.5 \times 10^{-5}(1 - 0.6)$

$= 650$ mm$^3$

**1.18 A steel pipe of length 1.83 m, outside diameter 114.3 mm and wall thickness 7.62 mm is subjected to an axial compressive load of 178 kN. Assuming E = 206.7 GPa, μ = 0.3, find (a) Shortening of the pipe (b) Increase in outside diameter (c) Increase in thickness**

**Sol:** **Given data:** l =1830m, d =114.3 mm, t = 7.62 mm, inside diameter $d_i = d - 2t = 99.06$ mm P = 178 kN, E = $206.7 \times 10^{3}$ MPa, $\mu = 0.3$

$$\text{Stress } \sigma = \frac{\text{Load}}{\text{Area}} = \frac{P}{A} = \frac{178\times10^{3}}{\frac{\pi}{4}\left((d)^2 - (d_i)^2\right)}$$

= 69.69MPa

$$\text{Strain } \in = \frac{\sigma}{E} = \frac{69.69}{206.7\times10^{3}}$$

$= 3.37 \times 10^{-4}$

But Strain $\in = 3.37\times10^{-4} = \dfrac{\text{Change in length}}{\text{length}}$

(a) Change in length $\delta l = 3.37 \times 10^{-4} \times 1830 = 0.617$mm (decrease)

(b) Lateral strain = $\mu.\varepsilon = 0.3 \times 3.37 \times 10^{-4} = 1.011 \times 10^{-4}$

But Lateral strain =

$$= \frac{\text{Change in outer dia}}{\text{outer dia}} = \frac{\text{change in thickness}}{\text{Thickness}}$$

Change in outer diameter = $1.011 \times 10^{-4} \times 114.3$

= 0.011 mm (increases)

(c) Change in thickness = $1.011 \times 10^{-4} \times 7.62 = 7.7 \times 10^{-4}$ mm

**1.19 A 25.4 mm diameter steel rod, E = 206.7 Gpa must carry a load in tension of 133.5 kN. if the initial length of the rod is 552.45 mm, what is its final length.**

**Sol:** **Given data:** d = 25.4 mm, E = $206.7 \times 10^3$ MPa, P = $133.5 \times 10^3$, $l$ = 552.45 mm.

$$\text{Change in length } \delta = \frac{PL}{AE} = \frac{133.5\times10^3\times552.45}{\frac{\pi}{4}(25.4)^2\times206.7\times10^3}$$

= 0.704 mm (elongation)

Final length = l + δ

= **553.154mm**

**1.20 An 8 m long round bar made of Aluminum (E = 70 GPa), carries a load of 720 kN. What is the minimum required diameter of the bar if the maximum allowable elongation is 10 mm?**

**Sol:** **Given data:** l = 8000 mm, E = $70 \times 10^3$ MPa, P = $720 \times 10^3$ N, δ = 10 mm.

$$\text{Change in length } \delta = \frac{PL}{AE} = \frac{720\times10^3\times8000}{\frac{\pi}{4}(d)^2\times70\times10^3} = 10$$

$$\frac{\pi}{4}(d)^2 = 8228.57$$

⇒ Diameter of the rod d = 102.36 mm

**1.21 A concrete pedestal of circular cross section has an upper part of diameter 0.5 m and height 0.5 m, and a lower part of diameter 1.0 m and height 1.2 m. It is subjected to loads**

**$P_1$ = 7 MN and $P_2$ = 18 MN as shown in the figure P 1.21. Assuming E = 25 GPa, calculate the deflections and stresses in top and bottom pedestals.**

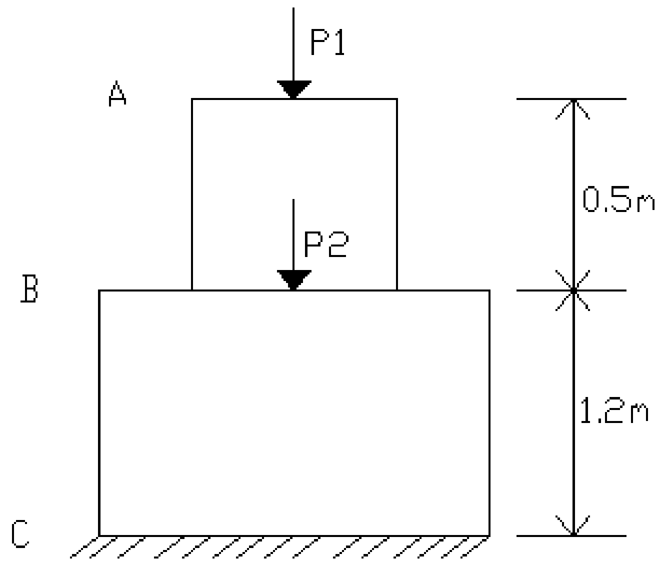



Fig. P.1.21

**Sol:** **Given data:** $d_1$ = 500 mm, $h_1$ = 500 mm, $d_2$ = 1000 mm, $h_2$ = 1200 mm, $E = 25 \times 10^3$ MPa, $P_1 = 7 \times 10^6$N, $P_2 = 18 \times 10^6$ N.

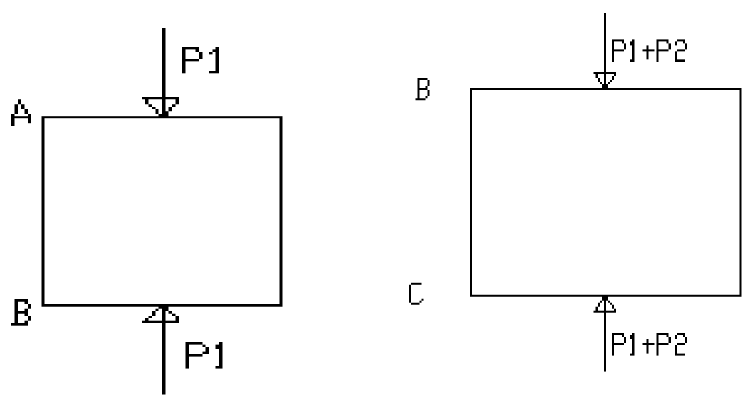



Fig. P.1.21 (a)

The free body diagram is shown in the Fig p 1.21 (a).

For Portion AB: (Top portion)

$P = P_1 = 7 \times 10^6$N, $l = l_1 = 500$ mm, $d = d_1 = 500$ mm

Stress $\sigma = \frac{P}{A} = \frac{P}{\frac{\pi}{4}(d)^2} = \frac{7\times10^6}{\frac{\pi}{4}(500)^2}$ = 35.65 MPa

Deflection $\delta = \frac{PL}{AE} = \frac{7\times10^6\times500}{\frac{\pi}{4}(500)^2\times25\times10^3} = 0.713$

For Portion BC: (Bottom portion)

$P = P_1 + P_2 = 25 \times 10^6$ N, $l = l_2 = 1200$ mm, $d = d_2 = 1000$ m

Stress $\sigma = \frac{P}{A} = \frac{P}{\frac{\pi}{4}(d)^2} = \frac{25\times10^6}{\frac{\pi}{4}(1000)^2} = 31.83$ MPa

Deflection $\delta = \frac{PL}{AE} = \frac{25\times10^6\times1200}{\frac{\pi}{4}(1000)^2\times25\times10^3}$ = 1.53 mm

Net deflection of the pedestal = 0.713 + 1.53

**= 2.243 mm**

**1.22 A Prismatic bar ABCD is subjected to loads $P_1$, $P_2$, and $P_3$ as shown in the Fig.P.122 the bar is made of steel with modulus of elasticity 200 GPa and cross sectional area is 225 mm². Determine the stress in each portion and the net deflection in the bar due to loads $P_1$, $P_2$, and $P_3$**

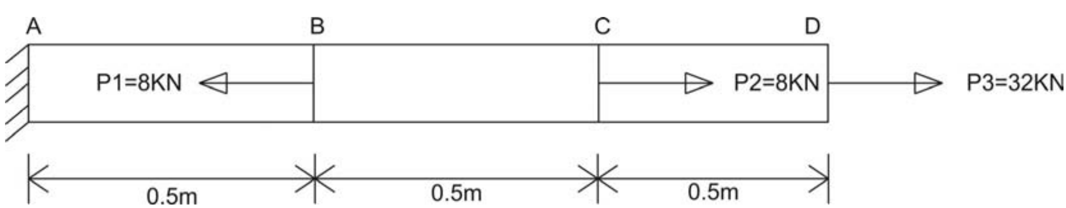



**Fig P. 1.22**

**Sol:** The Free body diagrams of portions CD, BC, AB are shown in the Fig P.1.22(a).

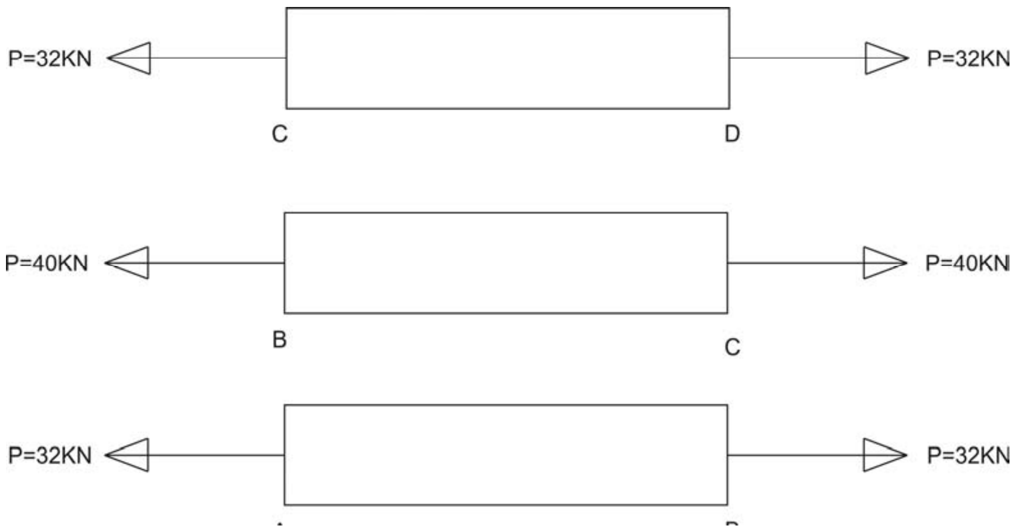



Fig P.1.22(a)

For the portion CD:

P = 32 × $10^3$ N, l = 500 mm, A = 225 $mm^2$, E = 200 × $10^3$ Pa.

$$\text{Stress } \sigma = \frac{P}{A} = \frac{32\times10^3}{225} = 142.22 \text{ MPa}$$

$$\text{Deflection } \delta_1 = \frac{PL}{AE} = \frac{32\times10^3\times500}{225\times200\times10^3} = 0.356 \text{ mm}$$

For the portion BC:

P = 40 × $10^3$ N, l = 500 mm, A = 225 $mm^2$, E = 200 × $10^3$ Pa.

$$\text{Stress } \sigma = \frac{P}{A} = \frac{40\times10^3}{225} = \mathbf{177.78MPa}$$

$$\text{Deflection } \delta_2 = \frac{PL}{AE} = \frac{40\times10^3\times500}{225\times200\times10^3} = 0.444 \text{ mm}$$

For the portion AB:

P = 32 × $10^3$N, l = 500 mm, A = 225 $mm^2$, E = 200 × $10^3$Pa.

$$\text{Stress } \sigma = \frac{P}{A} = \frac{32\times10^3}{225} = 142.22 \text{ MPa}$$

$$\text{Deflection } \delta_3 = \frac{PL}{AE} = \frac{32\times10^3\times500}{225\times200\times10^3} = 0.356\text{mm}$$

The Net deflection $\delta = \delta_1 + \delta_2 + \delta_3$

$= 0.356 + 0.444 + 0.356$

$= 1.156$ mm

**1.23 A steel bar (E = 200 GPa) is supported and loaded as shown in the figure P.1.23. The cross sectional area of the bar is 250 mm². Determine the force $P_1$ so that the net deflection in the bar is zero.**

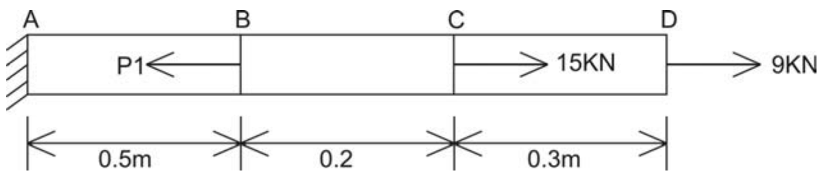



Fig.P.1.23

**Sol:** The Free body diagrams of portions CD, BC, AB are shown in the fig P.1.23(a).

For the portion CD:

$P = 9 \times 10^3$ N, $l = 300$ mm, $A = 250$ mm², $E = 200 \times 10^3$ Pa.

Deflection $\delta_1 = \dfrac{PL}{AE} = \dfrac{9\times10^3\times300}{250\times200\times10^3}$

$= 0.054$mm

For the portion BC:

$P = 24 \times 10^3$ N, $l = 200$ mm, $A = 250$ mm², $E = 200 \times 10^3$Pa.

Deflection $\delta_2 = \dfrac{PL}{AE} = \dfrac{24\times10^3\times200}{250\times200\times10^3} = 0.096$ mm

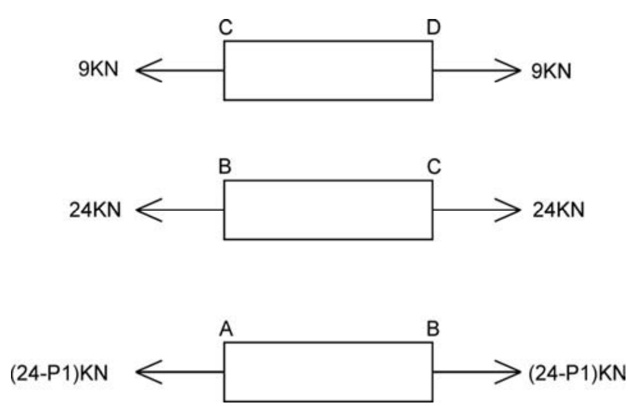



Fig. P.1.23(a)

For the portion AB:

P = (24 – $P_1$) ×$10^3$ N, l = 500 mm, A = 250 $mm^2$, E = 200 × $10^3$Pa.

Deflection

$$\delta_3 = \frac{PL}{AE} = \frac{(24-P_1)\times10^3\times500}{250\times200\times10^3} = 0.01(24-P_1)$$

$$= 0.01(24-P_1)mm$$

The Net deflection $\delta = \delta_1 + \delta_2 + \delta_3$

$0 = 0.054 + 0.096 + 0.01(24-P_1)$

$\mathbf{P_1 = 39\ kN}$

Stress in CD $\sigma = \frac{P}{A} = \frac{9\times10^3}{250}$ = 36 MPa

Stress in BC $\sigma = \frac{P}{A} = \frac{24\times10^3}{250}$ = 96 MPa

Stress in AB $\sigma = \frac{P}{A} = \frac{(24-39)\times10^3}{250}$ = – 60 MPa (Compressive)

**1.24 Figure P.1.24 shows a bar consisting of 3 lengths. Find the stresses in 3 parts and total extension of the bar. Take E = 2 × $10^5$ MPa.**

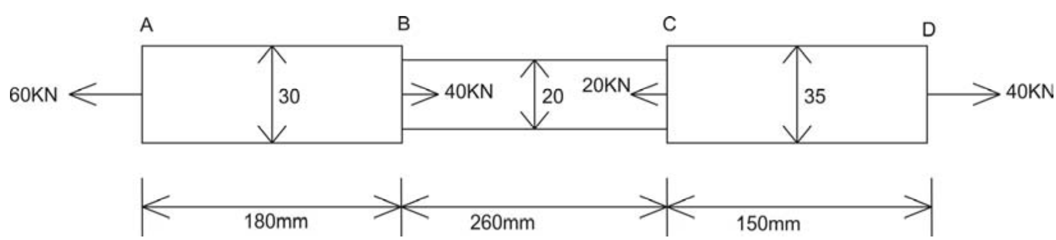



**Fig. P.1.24**

**Sol:** The Free body diagrams of portions CD, BC, AB are shown in the fig.P.1.24(a)

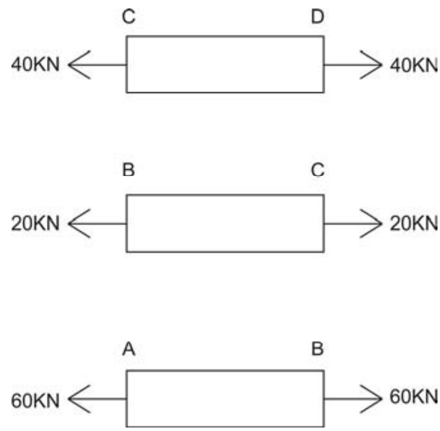



**Fig.P.1.24(a)**

For the portion CD:

P = 40 × $10^3$ N, l =150 mm, d = 35 mm, E = 2 × $10^5$ Pa.

$$\text{Area A} = \frac{\pi}{4}d^2 = 962.11\ \text{mm}^2$$

$$\text{Stress}\ \sigma = \frac{P}{A} = \frac{40\times10^3}{962.11}$$

$$= 41.58\ \text{MPa}$$

$$\text{Deflection}\ \delta_1 = \frac{PL}{AE} = \frac{40\times10^3\times150}{962.11\times2\times10^5}$$

$$= 0.0312\text{mm}$$

For the portion BC:

P = 20 × $10^3$N, l = 260 mm, d = 20 mm, E = 2 × $10^5$ Pa.

$$\text{Area A} = \frac{\pi}{4}d^2 = 314.16\ \text{mm}^2$$

$$\text{Stress}\ \sigma = \frac{P}{A} = \frac{20\times10^3}{314.16}$$

$$= 63.66\ \text{MPa}$$

$$\text{Deflection}\ \delta_2 = \frac{PL}{AE} = \frac{20\times10^3\times260}{314.16\times2\times10^5}\frac{\pi}{4} = 0.0828$$

For the portion AB:

$P = 60 \times 10^3$ N, $l = 180$ mm, $d = 30$ mm, $E = 200 \times 10^3$Pa.

$$\text{Area } A = \frac{\pi}{4}d^2 = 706.86 \text{ mm}^2$$

$$\text{Stress } \sigma = \frac{P}{A} = \frac{60\times10^3}{706.86}$$

$$= 84.88 \text{ MPa}$$

$$\text{Deflection } \delta_3 = \frac{PL}{AE} = \frac{60\times10^3\times180}{706.86\times2\times10^5} = 0.0764$$

$$= \mathbf{0.0764mm}$$

The Net deflection $\delta = \delta_1 + \delta_2 + \delta_3$

= 0.0312 + 0.0828 + 0.0764

**= 0.1904 mm**

**1.25 Find the maximum values of P for the bar shown in the Fig.P.1.25 that will not exceed an overall deformation of 3mm (or) the following stresses: Steel 140 MPa, Bronze 120 MPa and Aluminium 80 MPa. Take $E_{St}$ = 200 GPa, $E_{Br}$ = 83 GPa, $E_{Al}$ = 70 GPa. $A_{st}$ = 480 mm², $A_{Br}$ = 650 mm² and $A_{Al}$ = 320 mm².**

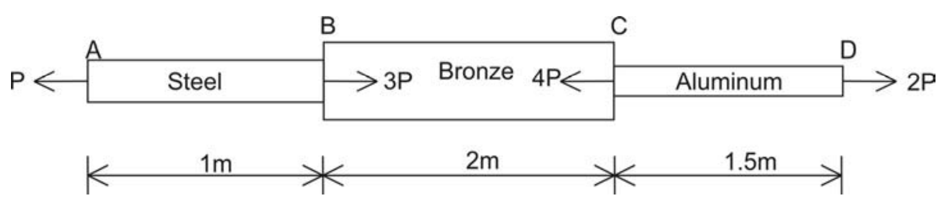



**Fig.P.1.25**

**Sol:** The Free body diagrams of portions CD, BC, AB are shown in the fig.P.1.25(a)

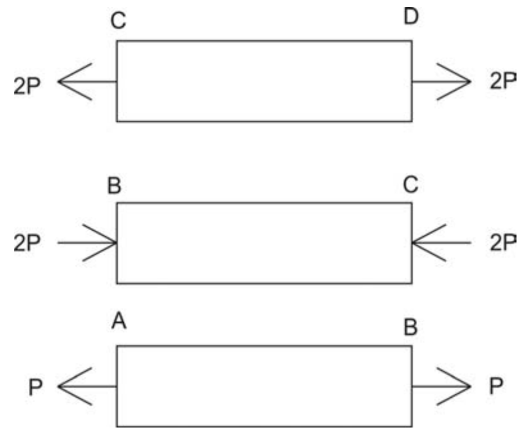



**Fig.P.1.25(a)**

By Considering Deflection:

For the portion CD (Aluminum):

P = 2P, l =1500 mm, A = 320 mm$^2$, E = 70 × 10$^3$ MPa. σ = 80 MPa

$$\text{Deflection } \delta_1 = \frac{PL}{AE} = \frac{2P \times 1500}{320 \times 70 \times 10^3}$$

$$= 1.34P \times 10^{-4} \text{mm}$$

For the portion BC (Bronze):

P = –2P, l = 2000 mm, A = 650 mm$^2$, E = 83 × 10$^3$ MPa, σ =120 MPa.

$$\text{Deflection } \delta_2 = \frac{PL}{AE} = \frac{-2P \times 2000}{650 \times 83 \times 10^3}$$

$$= -7.414 \text{ P} \times 10^{-5} \text{ mm}$$

For the portion AB (Steel):

P = P, l = 10000 mm, A = 480 mm$^2$, E = 200 × 10$^3$ MPa, σ = 140 MPa.

$$\text{Deflection } \delta_3 = \frac{PL}{AE} = \frac{P \times 1000}{480 \times 200 \times 10^3}$$

$$= 1.042P \times 10^{-5} \text{ mm}$$

The Net deflection $\delta = \delta_1 + \delta_2 + \delta_3$

$3 = 1.34P \times 10^{-4} - 7.414P \times 10^{-5} + 1.042P \times 10^{-5}$

$\Rightarrow$ P = 42.69 kN

By Considering Stress:

For the portion CD (Aluminum):

$P = 2P$, $l = 1500$ mm, $A = 320\ mm^2$, $E = 70 \times 10^3$ MPa. $\sigma = 80$ MPa

$$\text{Stress } \sigma = \frac{P}{A} = \frac{2P}{320}$$

$$80 = 6.25 \times 10^{-3}\ P\ \text{MPa}$$

$$\Rightarrow \quad P = 12.8\ \text{kN}$$

For the portion BC (Bronze):

$P = -2P$, $l = 2000$ mm, $A = 650\ mm^2$, $E = 83 \times 10^3$ MPa, $\sigma = 120$ MPa.

$$\text{Stress } \sigma = 120 = \frac{P}{A} = \frac{2P}{650}$$

$$= 3.076P \times 10^{-3}\ \text{MPa}$$

$$\Rightarrow \quad P = 39\ \text{kN}$$

For the portion AB (Steel):

$P = P$, $l = 10000$ mm, $A = 480\ mm^2$, $E = 200 \times 10^3$ MPa, $\sigma = 140$ MPa.

$$\text{Stress } \sigma = 140 = \frac{P}{A} = \frac{P}{480}$$

$$= 2.08P \times 10^{-3}\ \text{MPa}$$

$$\Rightarrow \quad P = 67.2\ \text{kN}$$

The Maximum possible value of **P = 12.8kN**

**1.26 Determine an expression for the deflection of a uniformly tapered circular cross section bar subjected to an axial load P. The diameter at the big end is $d_1$ and the diameter at the small end is $d_2$.**

**Sol:** Since the area is not uniform throughout the length, let us find a generalized equation for the deflection of a small strip which can be integrated over a length of 'L'.

Consider a small strip of length dx, taken at a distance of x from the big end. $d_1$ and $d_2$ are the diameters of the big and small ends respectively. The diameter at the small strip can be written as

$$dx = \left[d_1 - \frac{(d_1 - d_2)x}{L}\right]$$

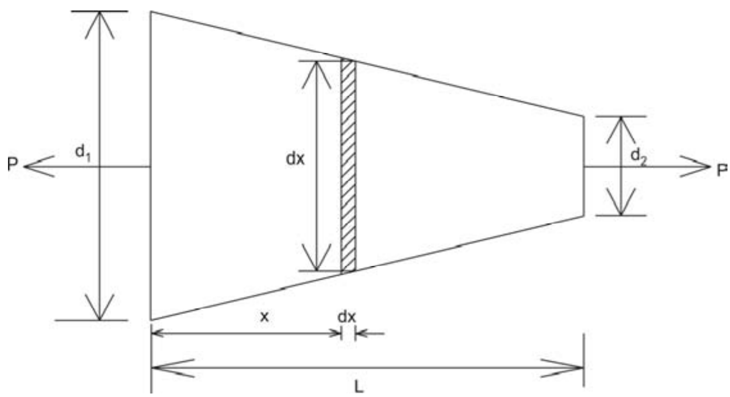



**Fig. P.1.26**

Area of the small strip = $\frac{\pi}{4}\left[d_1 - \frac{(d_1 - d_2)x}{L}\right]^2$

$=\frac{\pi}{4}[d_1 - \frac{(d_1-d_2)x}{L}]^2$

Deflection of the small strip d δ = $= \frac{PL}{AE} = \frac{P.dx}{\frac{\pi}{4}\left[d_1 \frac{(d_1 - d_2)x}{L}\right]^2 .E}$

Now, the total deflection is $\delta = \int_0^L \frac{P.dx}{\frac{\pi}{4}\left[d_1 \frac{(d_1 - d_2)x}{L}\right]^2 .E}$

$$= \quad \frac{4P}{\pi E}\int_0^L \frac{dx}{[d_1 - \frac{(d_1-d_2)x}{L}]^2}.$$

Let $d_1 - \frac{(d_1 - d_2)x}{L} = t$

$$-\frac{(d_1 - d_2)dx}{L} = dt$$

$$dx = \frac{-dt.L}{(d_1 - d_2)}$$

where t is varying from $d_1$ to $d_2$

$$\delta = -\frac{4P}{\pi E}\int_{d_1}^{d_2}\frac{L.dt}{t^2.(d_1-d_2)}$$

$$= \frac{4PL}{\pi E(d_1-d_1)}\left[\frac{-1}{t}\right]_{d_1}^{d_2}$$

$$= -\frac{4PL}{\pi E(d_1-d_2)}\left[\frac{1}{d_1}-\frac{1}{d_2}\right]$$

$$= \frac{4PL}{\pi E d_1 d_2}$$

**1.27. A flat bar of rectangular cross section of constant thickness t and width is varying from $b_1$ to $b_2$ is subjected to an axial load of P. Find the elongation.**

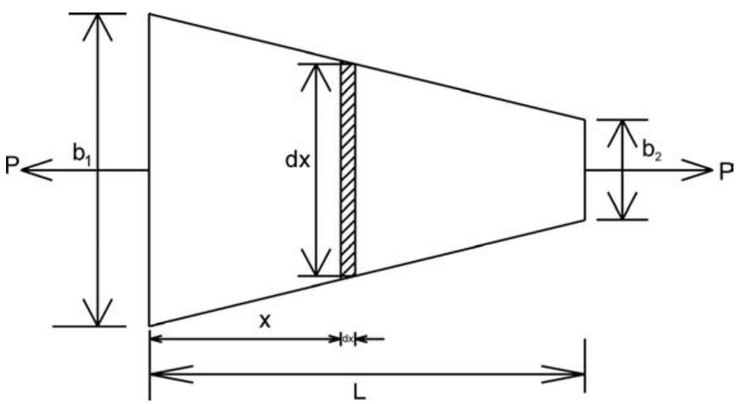



**Fig. P.1.27**

**Sol:** Consider a small strip of length dx, taken at a distance of x from the big end. $b_1$ and $b_2$ are the widths at the big and small ends respectively. The width at the small strip $= \left[b_1 - \frac{(b_1-b_2)x}{L}\right]$

Area of the strip $= \left[b_1 - \frac{(b_1-b_2)x}{L}\right]t$

Deflection of the small strip $d\delta = \frac{PL}{AE} = \frac{P.dx}{\left[b_1 - \frac{(b_1-b_2)x}{L}\right]t.E}$

Net deflection $\delta = \int_0^L \frac{P.dx}{\left[b_1 - \frac{(b_1-b_2)x}{L}\right]t.E}$

Let $b_1 - \frac{(b_1-b_2)x}{L} = k$

$$-\frac{(b_1-b_2)dx}{L} = dk$$

$$dx = \frac{-dk.L}{(b_1-b_2)}$$

where k is varying from $b_1$ to $b_2$

$$\delta = -\frac{P}{tE}\int_{b_1}^{b_2} \frac{L.dk}{t.(b_1-b_2)}$$

$$= \frac{PL}{Et(b_1-b_2)}\left[\log\frac{b_1}{b_2}\right]$$

$$= \frac{PL}{Et(b_1-b_2)}\,[\log t]_{b_1}^{b_2}$$

**1.28 A tension bar is found to be uniformly taper of circular cross section varying from (D–a)cm to (D+a)cm. Prove that error involved in using the mean diameter to calculate E is $\left(\frac{10a}{D}\right)^2$ percent.**

**Sol:** **Given data:** $d_1 = (D + a)$cm, $d_2 = (D - a)$cm.

The deflection of a uniformly tapered circular cross section member is $\delta = \frac{4PL}{\pi E d_1 d_2}$

$$\delta = \frac{4PL}{\pi E(D-a)(D+a)}$$

$$\delta = \frac{4PL}{\pi E(D^2-a^2)}$$

$$E = \frac{4PL}{\pi\delta(D^2-a^2)}$$

If we consider mean dia. $d = \frac{d_1+d_2}{2}$

$$= \frac{D-a+D+a}{2} = D$$

Deflection $\delta = \delta = \dfrac{PL}{\frac{\pi}{4}D^2E^1} = \dfrac{4PL}{\pi D^2E^1}$

$$E^1 = \frac{4PL}{\pi D^2\delta}$$

Percentage error $= \dfrac{E - E^1}{E} \times 100 = \dfrac{\dfrac{4PL}{\pi\delta(D^2 - a^2)} - \dfrac{4PL}{\pi D^2\delta}}{\dfrac{4PL}{\pi\delta(D^2 - a^2)}} \times 100$

$$= \frac{\dfrac{1}{\left(D^2 - a^2\right)} - \dfrac{1}{D^2}}{\dfrac{1}{D^2 - a^2}} \times 100$$

$$= \frac{a^2}{d^2} \times 100$$

$$= \left[\frac{10a}{d}\right]^2$$

**1.29. Determine the total elongation of the bar due to its own weight.**

**Sol:** Let the weight of the bar is W is acting at the centroid as shown in the Fig.P.1.29(a). The free body diagrams for the portions AB and BC are shown in the Figures. P.1.29 (b) and (c)

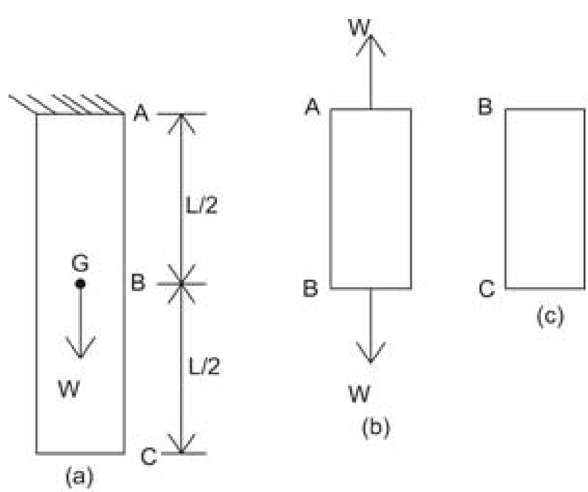



**Fig. P.1.29**

For the portion BC

P = 0, l = l/2, A = A mm², E = E MPa.

Deflection $\delta_1 = \frac{PL}{AE} = 0$

For the portion AB

P = W, l = l/2, A = A mm², E = E MPa.

Deflection $\delta_2 = \frac{PL}{AE} = \frac{WL}{2AE}$ mm

The Net deflection $\delta = \delta_1 + \delta_2$

$$= \frac{WL}{2AE} \text{ mm}$$

It may be observed that the total extension produced by the self weight of the bar is equal to that produced by a load of half of its weight applied at the lower end.

**1.30. Derive an expression for the elongation of a conical bar of circular cross section under the action of self weight W. Length of the bar is L and Young's modulus of elasticity of the material is E.**

**Sol:** Let the weight of the bar is W is acting at the centroid as shown in the Fig.P.1.30(a). The centroid is at a distance of h/3 from big end for a conical shape bar. The free body diagrams for the portions AB and BC are shown in the Fig. P.1.30(b) and (c). The diameter at the small end (fig. (b) ) can be found from the equation

$$d_x = d - \frac{d}{h}.x \text{ (since the diameter is uniformly varying).}$$

By substituting x = h/3 in the above equation we get the diameter at the small end (Fig.(b)) = 2d/3.

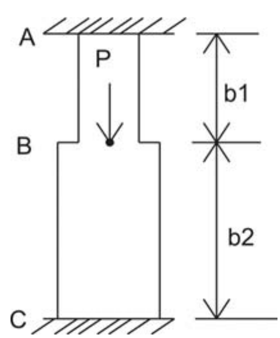



**Fig.P.1.30**

For the portion BC

P = 0, l = 2h/3, A = A mm², E = E MPa, $d_1 = \frac{2d}{3}$, $d_2 = 0$

Deflection $\delta_1 = \frac{4PL}{\pi E d_1 d_2} = 0mm$ (See the example 1.26)

For the portion AB

P = W, $l = \frac{h}{3}$, A = A mm², E = E MPa, $d_1 = d$, $d_2 = \frac{2d}{3}$

$$\text{Deflection } \delta_2 = \frac{4WL}{\pi E d_1 d_2} = \frac{4w\frac{h}{3}}{\pi Ed.\frac{2d}{3}} = \frac{2wh}{\pi Ed^2} = \frac{2wh}{4.\pi E\frac{d^2}{4}} = \frac{wh}{2EA}$$

The Net deflection $\delta = \delta_1 + \delta_2$

$$= 0 + \frac{wh}{2EA} mm$$

$$= \frac{wh}{2EA} mm$$

**1.31. A Steel bar AB having two different cross sectional areas $A_1$ and $A_2$ is held between rigid supports and loaded at C by a force P as shown in the Fig.P.1.31 Determine the reactions $R_A$ and $R_B$ at the supports.**

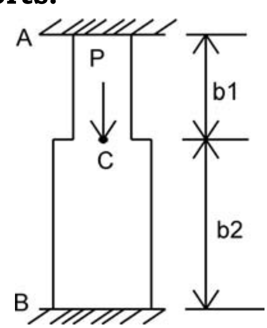



**Fig.P.1.31**

**Sol:** Let the reactions at A and B are $R_A$ and $R_B$. The Free body diagram with assumed directions of $R_A$ and $R_B$ is shown in the Fig.P.1.31(a)

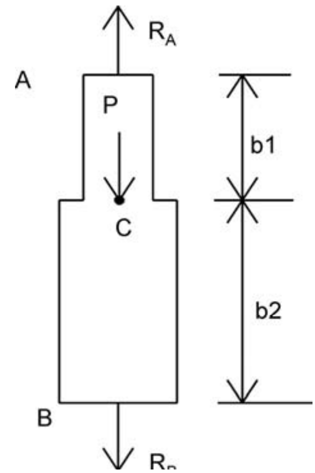



**Fig.1.31 (a)**

From equilibrium equation $\sum F_Y = 0$

We can write $R_A - R_B = P$ .....(1)

The free body diagrams of the Portions AB and BC are shown in the Fig.P.1.31(b)

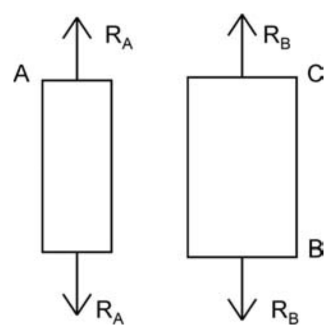



**Fig.P.1.31(b)**

For the Portion AB:

$P = R_A$, $A = A_1$, $L = b_1$ and E = E (Let)

Deflection in AB $\delta_{AB} = \dfrac{PL}{AE} = \dfrac{R_A b_1}{A_1 E}$

For the Portion BC:

$P = R_B$, A = A2, $L = b_1$ and E = E (Let)

Deflection in AB $\delta_{BC} = \dfrac{PL}{AE} = \dfrac{R_B b_2}{A_2 E}$

Net deflection is $\delta = \delta_{AB} + \delta_{BC}$

But from geometry of the figure we can say that net deflection $\delta$ = 0 (Compatibility)

So, $\delta_{AB} + \delta_{BC} = 0$

$$\frac{R_A b_1}{A_1 E} + \frac{R_B b_2}{A_2 E} = 0$$

$R_A b_1 A_2 + R_B b_2 A_1 = 0$ .....(2)

On solving (1) and (2) we can get $R_A = \dfrac{PA_1 b_2}{b_1 A_2 + b_2 A_1}$ and

$$R_B = \frac{PA_2 b_1}{b_1 A_2 + b_2 A_1}$$

**1.32. A Square column of reinforced concrete is compressed by an axial force P as shown in the Fig.P.1.32. What fraction of the load will be carried by the concrete if the total cross sectional area of the steel bars is one-tenth of the cross sectional area of the concrete and the modulus of elasticity of the steel is ten times that of concrete.**

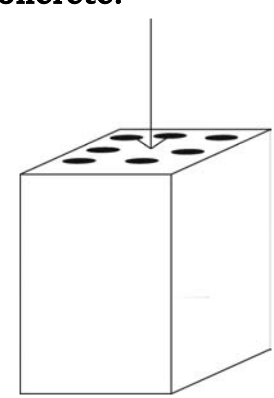


**Fig.P.1.32**

**Sol:** Let the load shared by the steel bars = $P_S$

The load shared by the concrete = $P_C$

Modulus of elasticity of steel = $E_S$

Modulus of elasticity of concrete = $E_C$

Cross sectional Area of the Steel = $A_S$

Cross sectional Area of the Concrete = $A_C$

Given that $E_S = 10E_C$ and $A_S = 0.1A_C$

The total P should be shared by steel rods and concrete.

So, $P_S + P_C = P$ .....(1)

The compatibility in this problem is deflection in the steel = deflection in the concrete

$\delta_S = \delta_C$

$$\frac{P_S L}{A_S E_S} = \frac{P_C L}{A_C E_C}$$

$$\frac{P_S}{0.1 A_C 10 E_C} = \frac{P_C}{A_C E_C}$$

$P_S = P_C$ .....(2)

On solving (1) and (2) we get $P_S = P_C = \dfrac{P}{2}$

i.e., half of the load will be taken by steel rods and concrete.

**1.33. A Square column is formed by a 25 mm thick metal casing is filled with concrete as shown in the Fig.P.1.33 The casing has modulus of elasticity is 84 GPa and the concrete core has modulus of elasticity is 14 Gpa. Find the maximum permissible load P if the allowable stresses in metal and concrete are 42MPa and 5.6Mpa**

**Sol:** **Given Data:** Area of Concrete $A_c = 200 \times 200 = 40000 mm^2$, Area of metal $A_m = 250 \times 250 - 200 \times 200 = 22500 mm^2$. $E_m = 84 \times 10^3$ MPa and $E_c = 14 \times 10^3$ MPa. $\sigma_{um} = 42$MPa. $\sigma_{uc} = 5.6$MPa

Let the load taken by metal = $P_m$

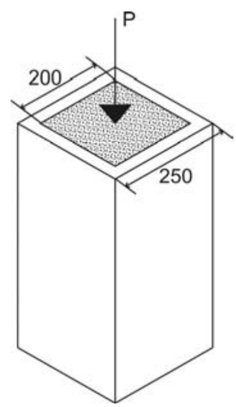



Fig.P.1.33

And load taken by the concrete = $P_c$

$P_m + P_c = P$ .....(1)

The compatibility equation is deflection in metal = deflection in concrete

$\delta_m = \delta_c$

$$\frac{P_m L}{A_m E_m} = \frac{P_c L}{A_c E_c}$$

$$\frac{P_m L}{A_m E_m} = \frac{P_c L}{A_c E_c}$$

$$\frac{P_m}{22500 \times 84 \times 10^3} = \frac{P_c}{40000 \times 14 \times 10^3} \quad \ldots\ldots (2)$$

On solving equations (1) and (2) we get

$P_m = 0.77P$

$P_c = 0.23P$

Now, Stress in metal $\sigma_m = \frac{P_m}{A_m} = \frac{0.77P}{22500}$

$42 = 3.42 \times 10^{-5}P$

Or, $P = 1.23 \times 10^6$N

Similarly stress in concrete $\sigma_m = \frac{P_c}{c} = \frac{0.23P}{40000}$

$5.6 = 5.75 \times 10^{-6}P$

Or, $P = 973.91 \times 10^3$N

The Maximum permissible load = **973.91** kN

**1.34. Find the reactions at the supports for the structure shown in the Fig.P.1.34 and also find the stress at the middle portion of the structure. if the cross sectional area near the ends $A_1$ = 400mm², cross sectional area in central region $A_2$ = 600mm², P = 24kN and b = 2a.**

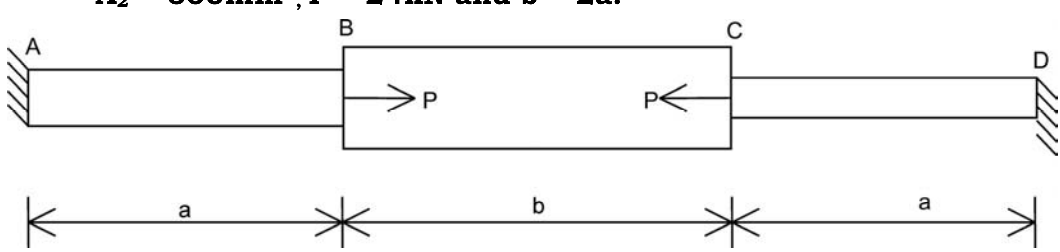



Fig.P.1.34

**Sol: Given data:** $A_1$ = 400mm², $A_2$ = 600mm², P = 24kN, b = 2a.

Let the reactions at the supports are $R_A$ and $R_D$. the free body diagram with assumed directions of $R_A$ and $R_D$ is shown in the Fig.P.1.34 (a)

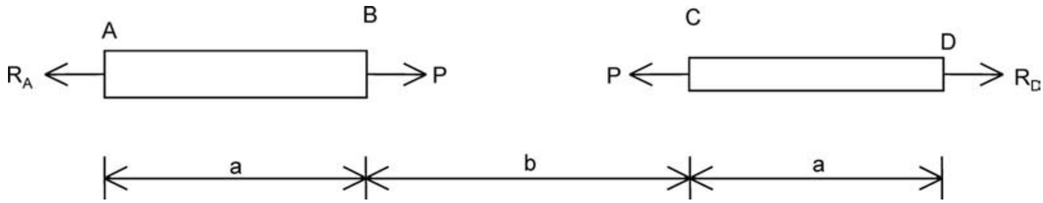



**Fig.P.1.34 (a)**

From equilibrium equation $\Sigma F_X = 0$

We can write $R_A + P = R_B + P$

$R_A = R_B$ .....(1)

The free body diagrams for the portions AB, BC and CD are shown in Fig.P.1.34(b)

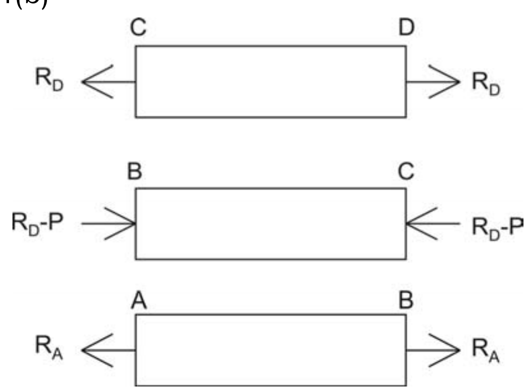



**Fig.P.1.34(b)**

For the Portion AB:

$P = R_A$, $A = 400$ $L = a$ and $E = E$ (Let)

Deflection in AB $\delta_{AB} = \dfrac{PL}{AE} = \dfrac{R_A a}{400E}$

For the Portion BC:

$P = R_A - P$, $A = 600$, $L = b = 2a$ and $E = E$ (Let)

Deflection in AB $\delta_{BC} = \dfrac{PL}{AE} = \dfrac{(R_A - P)2a}{600E}$

For the Portion CD:

$P = R_B$, $A = 400$ $L = a$ and $E = E$ (Let)

Deflection in AB $\delta_{CD} = \dfrac{PL}{AE} = \dfrac{R_B a}{400E}$

Net deflection is $\delta = \delta_{AB} + \delta_{BC} + \delta_{CD}$

But from geometry of the figure we can say net deflection $\delta = 0$ (Compatibility)

$$\frac{R_A a}{400E} + \frac{(R_A - P)2a}{600E} + \frac{R_B a}{400E} = 0$$

On solving the above equation we get,

$6R_A + 8(R_A - P) + 6R_B = 0$

Or $7R_A + 3R_B = 4P$.....(2)

On solving (1) and (2) we get

$R_A = 0.4P = 9.6kN$ and $R_B = 0.4P = 9.6kN$

The Stress in the middle region is = $\dfrac{R_A - P}{600} = \dfrac{14.4 \times 10^3}{600}$ = 24MPa (Compressive)

**1.35. A rigid block AB of weight W hangs on three equally spaced vertical wires, two of steel and one of Aluminum as shown in the fig.P.1.35 The wires also support a load P at the center. The diameter of the steel wires = 3.175mm and the diameter of the aluminum wire = 4.77mm. What load P can be supported if the allowable stress in steel wires is 137.8MPa and the aluminum wire is 32.68MPa. W = 356N, $E_s$ = $2.067 \times 10^{11}$Pa, $E_a$ = $6.89 \times 10^{10}$Pa.**

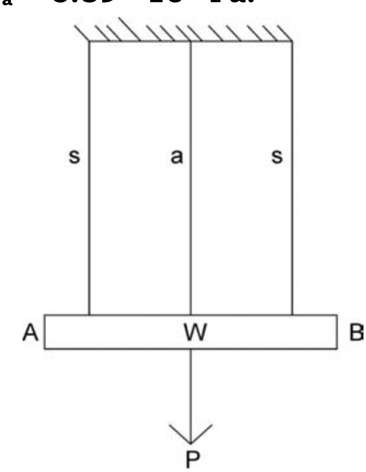



Fig.P.1.35

**Sol:** **Given data:** $d_s$ = 3.175mm, $d_a$ = 4.77mm, $A_s = \prod/4\times d^2_s$ = 7.92mm², $A_a = \frac{\pi}{4}\times d_a^2 = 17.88\text{mm}^2$, $\sigma_s$ = 137.8MPa, $\sigma_a$ = 82.68MPa, W = 356N, $E_s = 2.067\times10^{11}$Pa, $E_a = 6.89\times10^{10}$Pa.

Let the loads in steel and aluminum wires are $P_s$ and $P_a$

From equilibrium equation $\Sigma F_Y = 0$

We can write $2P_s + P_a = W + P = 356 + P$ .....(1)

The compatibility equation is

Deflection in steel = deflection in aluminum

$\delta_s = \delta_a$

$$\frac{P_sL}{A_sE_s} = \frac{P_aL}{A_aE_a}$$

$$\frac{P_s}{7.92\times2.067\times10^{11}} = \frac{P_a}{17.88\times6.89\times10^{10}}$$

$P_s = 1.33P_a$ .....(2)

On solving (1) and (2) we get,

$P_s = 129.37 + 0.363P$

$P_a = 97.27 + 0.273P$

Now, Stress in steel $\sigma_s = 137.8 = \frac{P_s}{A_s} = \frac{129.37+0.363P}{7.92}$

P = 2650N

Similarly, Stress in aluminium $\sigma_a = 82.68 = \frac{P_a}{A_a} = \frac{97.27+0.273P}{17.88}$

P = 5059N

The maximum possible load = 2650N

**1.36. A steel bar as shown in fig.P.1.36 consists of two parts AB & BC having cross sectional areas of 4cm² and 5cm². End A is fixed rigidly and end C is at a distance of 1mm from the other rigid end. Determine the reactions produced by rigid supports if 100kN is applied at B. E = 200GPa.**

**Sol:** **Given data:** $A_1$ = 4cm² = 400mm², $A_2$ = 5cm² = 500mm². Let the reactions are $R_A$ and $R_C$

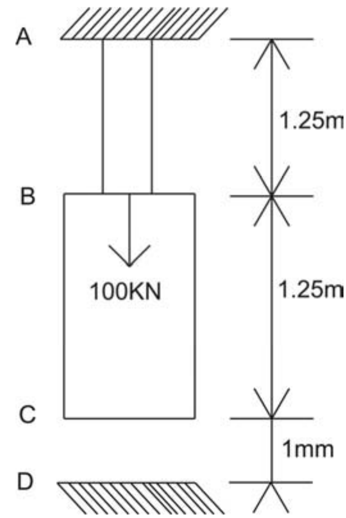


Fig.P.1.36

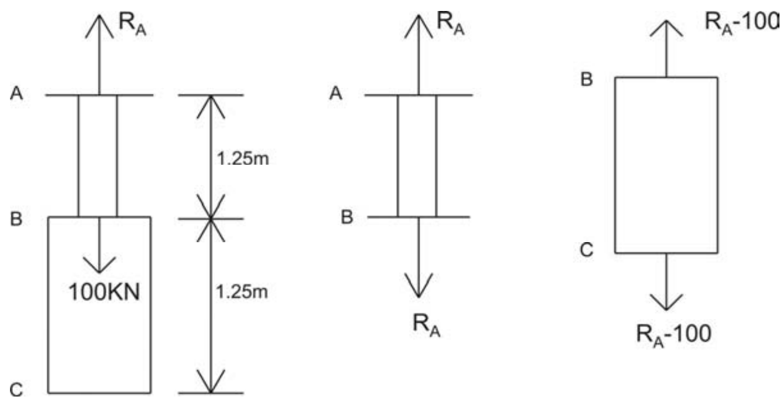


Fig.P.1.36(a)

Fig.P.1.36(a) shows the free body diagrams of portion AB and BC. From the equilibrium equation we can write

$R_A + R_C = 100x10^3$ .....(1)

Portion AB:

$P = R_A$, $l = 1250mm$, $A = 400mm^2$, $E = 200 \times 10^3 MPa$.

Deflection in AB $\delta_{AB} = \frac{PL}{AE} = \frac{R_A \times 1250}{400 \times 200 \times 10^3}$

$= 1.56 \times 10^{-5} R_A$

Portion BC:

$P = R_A - 100 \times 10^3$, $l = 1250mm$, $A = 500mm^2$, $E = 200 \times 10^3 MPa$.

Deflection in AB $\delta_{BC} = \dfrac{PL}{AE} = \dfrac{(R_A - 100\times10^3)\times1250}{500\times200\times10^3}$

$= 1.25 \times 10^{-5}(R_A - 100 \times 10^3)$

Net deflection $\delta = \delta_{AB} + \delta_{BC}$

Since the Permissible deflection is 1mm Compatibility equation is

$\delta = 1mm$

$1.56 \times 10^{-5}R_A + 1.25 \times 10^{-5}(R_A - 100 \times 10^3) = 1$

$2.81 \times 10^{-5} R_A = 2.25$

$\mathbf{R_A = 80kN}$

From (1) $\mathbf{R_C = 20kN}$

**1.37. A solid steel bar 50cm long, 7cm diameter is placed inside and Aluminum tube having 7.5cm inner diameter and 10cm outer diameter. The aluminum tube is 0.015cm longer than the steel bar. An axial load of 600Kn is applied to the bar as shown in the Fig.P.1.37. Find the stresses in steel and Aluminum. Young's modulus of elasticity for steel and aluminum is 200GPa and 70GPa respectively.**

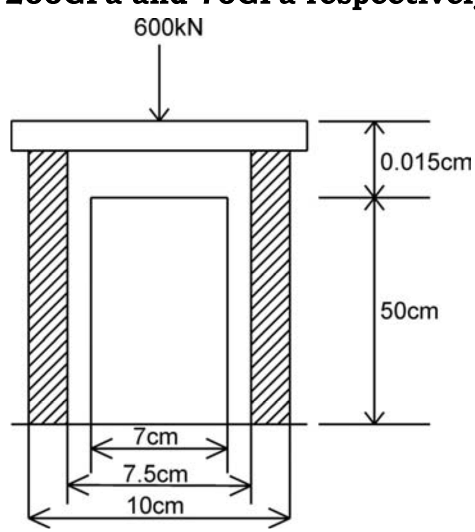



**Fig.P.1.37**

**Sol:** **Given Data:** $l_s$ = 500mm, $d_s$ = 70mm, $d_i$ = 75mm, $d_o$ = 100mm, $l_a$ = 500.15mm, P = 600 × $10^3$N, $E_s$ = 200 × $10^3$MPa, $E_a$ = 70 × $10^3$N.

Let the load in Aluminum = $P_a$

The Load in Steel = $P_s$

From the equilibrium equation we can write

$P_a + P_s = 600 \times 10^3 N$.....(1)

Area of Steel $A_s = \frac{\pi}{4} d_s^2 = 3848.45 mm^2$

Area of Aluminum $A_a = \frac{\pi}{4}\left(d_o^2 - d_i^2\right) = 3436.11 mm^2$

From the figure we can understood that the Aluminum has to compress 0.015cm (0.15mm) more than steel. So the compatibility equation is

$\delta_a = 0.15 + \delta_s$

$$\frac{P_a l_a}{A_a E_a} = 0.15 + \frac{P_s l_s}{A_s E_s}$$

$$\frac{P_a (500.15)}{3436.11 \times 70000} = 0.15 + \frac{P_s (500)}{3848.45 \times 200000}$$

$2.08 \times 10^{-6} P_a = 0.15 + 6.49 \times 10^{-7} P_s$ ......(2)

On solving (1) and (2)

We get, $P_a = 197.65 kN$ (Compressive)

$P_s = 402.35 kN$ (Compressive)

Stress in Aluminum $\sigma_s = \frac{P_s}{A_s} = \frac{402.35 \times 10^3}{3848.45}$

= **104.54MPa (Compressive)**

Stress in Aluminium $\sigma_s = \frac{P_a}{A_a} = \frac{197.65 \times 10^3}{3436.11}$

= **57.52MPa (Compressive)**

**1.38. The Aluminum rod as shown has a cross sectional area of 20cm² and a length of 25.004cm. The steel tube has also same cross sectional area and length is 25cm. what is the value of P for equal stresses in steel and Aluminum. Young's modulus of elasticity for steel and aluminum is 200GPa and 70GPa respectively.**

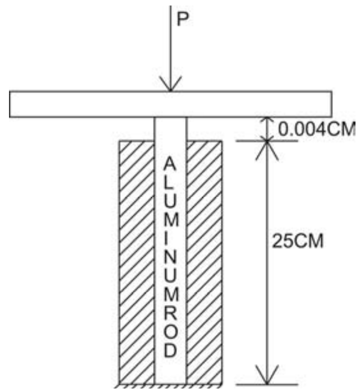



Fig.P.1.38

**Sol:** **Given Data:** $A_a$ = 2000mm$^2$, $l_a$ = 250.04mm, $A_s$ = 2000mm$^2$, $l_s$ = 250mm, $E_s$ = 200 × 10$^3$MPa, $E_a$ = 70 × 10$^3$N

Let the load in Aluminum = $P_a$

The Load in Steel = $P_s$

From the equilibrium equation we can write

$P_a + P_s = P$ .....(1)

Given that the stresses in both the members must be same. Since the area of the both the members are same we can say that the loads in Steel and Aluminum is same

$P_s = P_a$ .....(2)

On solving (1) and (2) we get $P_s = P_a = \frac{P}{2}$ .....(3)

From the figure we can understood that the Aluminum has to compress 0.004cm (0.04mm) more than steel. So the compatibility equation is

$\delta_a = 0.04 + \delta_s$

$$\frac{P_a(250.04)}{2000 \times 70000} = 0.04 + \frac{P_s(250)}{2000 \times 200000}$$

$1.786 \times 10^{-6} P_a = 0.04 + 6.25 \times 10^{-7} P_s$ .....(4)

On solving equations (3) and (4)

We get the load P = **68.9kN**

**1.39. The two vertical rods CD and EF are made of same material and have same cross sectional area. Find the loads in the rods.**

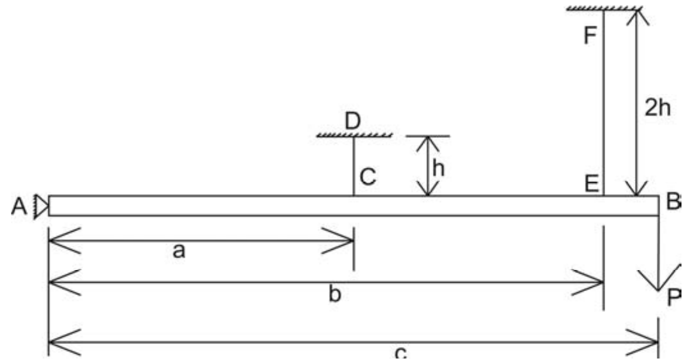



**Fig.P.1.39**

**Sol:** Let the loads in the rods CD and EF are $P_C$ and $P_E$

From the equilibrium equation $\Sigma M_A = 0$ we can write

$P_C(a) + P_E(b) = P.C$ .....(1)

The compatibility equation can be written from the deflection diagram.

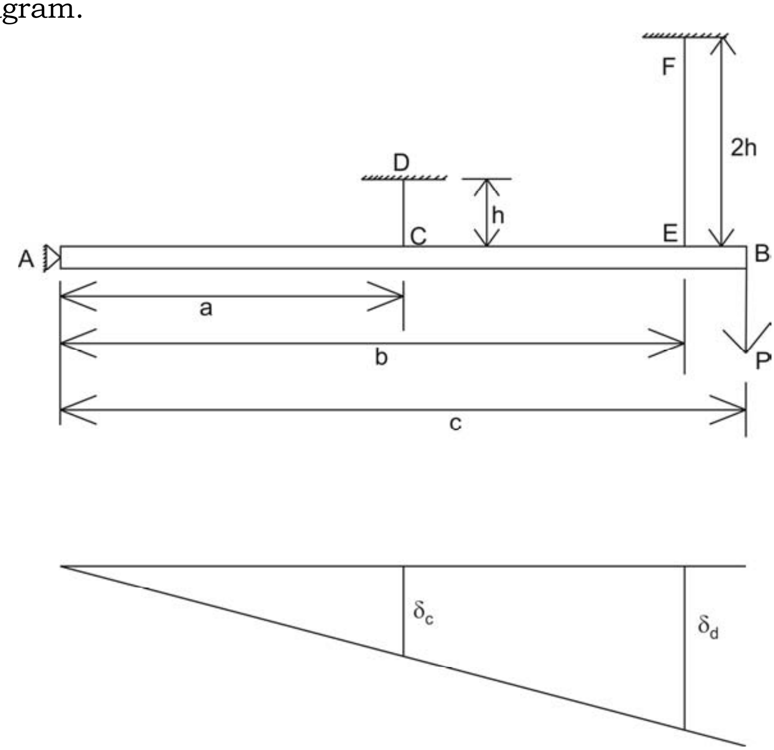



**Fig.P.1.39(a)**

From the diagram we can write,

$$\frac{\delta_c}{a} = \frac{\delta_d}{b}$$

$$\frac{\delta_c}{\delta_d} = \frac{a}{b}$$

$$\frac{\frac{P_c h}{AE}}{\frac{P_E .2h}{AE}} = \frac{a}{b}$$

$$P_c = \frac{2P_E a}{b} \qquad .....(2)$$

On solving (1) and (2)

$$\frac{2P_E a^2}{b} + P_E b = PC$$

$$P_E = \frac{PCb}{2a^2 + b^2}$$

$$P_C = \frac{2PCa}{2a^2 + b^2}$$

**1.40. A rigid bar AB supported by three wires as shown in the Fig.P.1.40. The outer bars are made of steel 2.5cm diameter and 50cm long each. The central bar is made of brass and is 75cm long and 2cm diameter. Calculate the forces in bars due to applied load of 150kN by assuming the bar AB is remains horizontal after the load applied. $E_s$ = 210GPa and $E_b$ = 105GPa.**

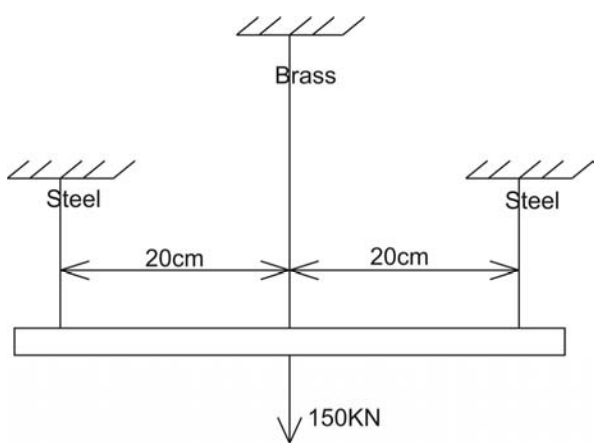



Fig.P.1.40

**Sol:** **Given Data:** $d_s$ = 25mm, $l_s$ = 500mm, $l_b$ = 750mm, $d_b$ = 20mm, P = 150 × $10^3$N, $E_s$ = 210 ×$10^3$ MPa, $E_b$ = 105 × $10^3$ MPa.

Area of the Steel rods $A_s = \frac{\pi}{4}(25)^2 = 490.87\text{mm}^2$

Area of the Brass rod $A_b = \frac{\pi}{4}(20)^2 = 314.16\text{mm}^2$

Let the loads in the steel bar and brass bar is $P_s$ and $P_b$. The free body diagram with assumed loads direction is shown in Fig.P.1.40(a)

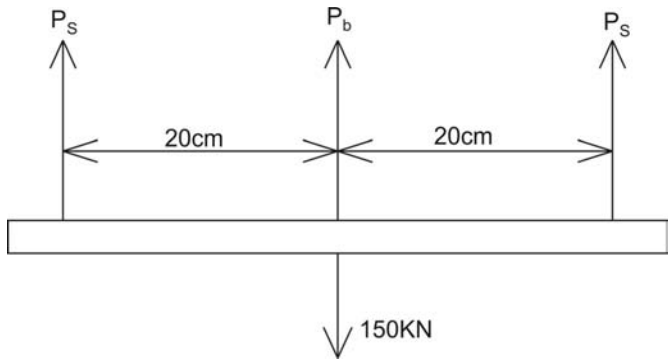



**Fig.P.1.40(a)**

From the Equilibrium equation we can write that

$2P_s + P_b = 150 \times 10^3\text{N}$ .....(1)

Since the bar is to be maintained in horizontal position, the compatibility equation is deflection in steel rod = deflection in brass rod.

$\delta_s = \delta_b$

$$\frac{P_s l_s}{A_s E_s} = \frac{P_b b}{A_b E_b}$$

$$\frac{P_s(500)}{490.87 \times 210 \times 10^3} = \frac{P_b(750)}{314.16 \times 105 \times 10^3}$$

$$\frac{P_s}{P_b} = 4.687 \quad .....(2)$$

On solving (1) and (2) we can get

**$P_s$ = 14.46kN**

**$P_b$ = 67.77kN**

**1.41. A steel rod 20mm diameter is passed through a brass tube 25mm internal diameter and 30mm external diameter. The tube is 80cm long and closed by nuts. The nuts are tightened until the compressive force in the tube is 5kN. Find the stresses in rod and tube if $E_s$ = 210GPa, $E_b$ = 80GPa.**

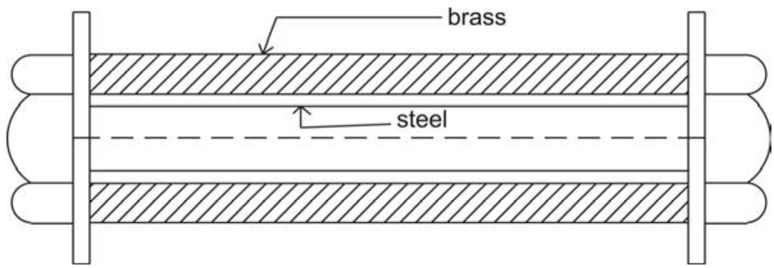



**Fig.P.1.41**

**Sol:** **Given data:** $d_s$ = 20mm, $d_o$ = 30mm, $d_i$ = 25mm, l = 80mm, $E_s$ = 210 × $10^3$ MPa, $E_b$ = 80 × $10^3$ MPa.

Area of Steel rod $= \frac{\pi}{4}(20)^2 = 314.16\text{mm}$

Area of Brass tube $= \frac{\pi}{4}\left(30^2 - 25^2\right) = 215.98\text{mm}^2$

Let the load in the steel rod is $P_s$ and load in the brass tube is $P_b$.

Total compressive load in the tube = 5 × $10^3$N. This load will be taken by steel rod and brass tube.

So, $P_s + P_b = -5 \times 10^3$N ......(1)

(– Indicates compressive force)

The compatibility equation is deflection in steel rod = deflection in brass tube.

$\delta_s = \delta_b$

$$\frac{P_s l_s}{A_s E_s} = \frac{P_b l_b}{A_b E_b}$$

$$\frac{P_s(80)}{314.16 \times 210 \times 10^3} = \frac{P_b(80)}{215.95 \times 80 \times 10^3}$$

$$\frac{P_s}{P_b} = 3.817 \quad .....(2)$$

On solving (1) and (2) we can get

$P_s$ = –3.96kN

$P_b = -1.03kN$

The stress in steel rod $\sigma_s = \frac{P_s}{A_s} = \frac{-3.96}{314.16} = -12.61MPa$

The stress in Brass tube $\sigma_b = \frac{P_b}{A_b} = \frac{-1.03}{215.95} = -4.77MPa$

**1.42. All three wires have same axial rigidity (EA). Find the value 'x' to maintain rigid body is always horizontal.**

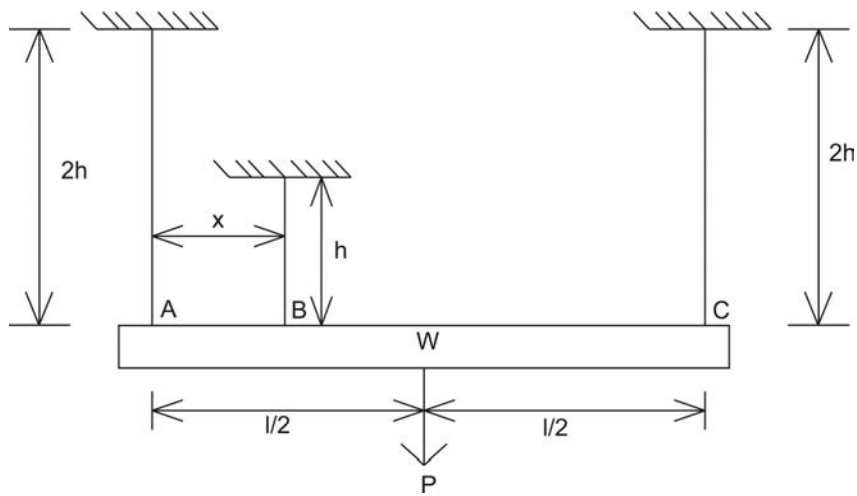



**Fig.P.1.42**

**Sol:** Let the loads in the wires are $P_A$, $P_B$ and $P_C$ (All the forces in upward direction).

From the equilibrium equation we can write

$\sum F_Y = 0$ we can write $P_A + P_B + P_C = P$ .....(1)

$\sum M_A = 0$ we can write $P_B.x + P_C.L = P.\frac{L}{2}$ .....(2)

Since the body is to be maintained din horizontal position, the compatibility equation we can write as

$\delta_A = \delta_B = \delta_C$

From the equation $\delta_A = \delta_B$; $\frac{P_A L_A}{EA} = \frac{P_B L_B}{EA}$

$$\frac{P_A(2h)}{EA} = \frac{P_B(h)}{EA}$$

$2P_A = P_B$ .....(3)

From the equation $\delta_B = \delta_C$

$$\frac{P_B L_B}{EA} = \frac{P_c L_c}{EA}$$

$$\frac{P_B(h)}{EA} = \frac{P_c(h)}{EA}$$

$P_B = P_C$ .....(4)

From equations (1), (3) and (4)

$P_A + 2P_A + 2P_A = P$

$$P_A = \frac{P}{5} \quad .....(5)$$

$$P_B = \frac{2P}{5} \quad .....(6)$$

From the equation (2) $\frac{2P}{5}x + \frac{25}{5}L = \frac{PL}{2}$

$$\frac{2P}{5}x + \frac{PL}{2} - \frac{2P}{5}L$$

$$\frac{2P}{5}x = \frac{PL}{10}$$

$$x = \frac{L}{4}$$

**1.43. A steel rod with cross sectional area of 161.25mm² is stretched between the two fixed points. The tensile force in the rod is 70ºC is 5340N. What will be the stress at 0ºC if E = 200GPa, α = 6.5 × 10⁻⁶/ºC.**

**Sol:** **Given data:** $A = 161.25mm^2$, $E = 200 \times 10^3 MPa$, $\alpha = 6.5 \times 10^{-6}/^oC$, $t_2 = 70^oC$, $F = 5340N$.

Let the initial (Stress free) temperature = $t_1$

Change in temperature $t = t_1 - 70$

Strain in the steel rod $\varepsilon = \alpha t$

$= 6.5 \times 10^{-6}.t$

Stress $\sigma = E.\varepsilon$

$= 200 \times 10^3 \times 6.5 \times 10^{-6}.t$

$= 1.3t$ MPa

Force in the material $F = \sigma.A$

$5340 = 1.3t(161.25)$

$t = 25.47$

$\Rightarrow t_1 - 70 = 25.47$

$\Rightarrow$ Initial temperature $t_1 = 95.47^0C$.

Now to find the stress at $0^oC$,

$t = 95.47 - 0 = = 95.47$

Stress at this temperature = E. ε

$= 200 \times 10^3 \times 6.5 \times 10^{-6} \times 95.47$

= **124.111MPa**

**1.44. A railway line is laid so that there is no stress in rails at 60°C. Calculate the stress in the rails at 20°C, if**

**(a) No allowance is made for contraction**

**(b) There is an allowance of 5mm for contraction of per rail. The rails are 30m long and E = 210GN/m², α = 0.000012per°C**

**Sol:** **Given data:** l = 30m, t = 60 – 20 = 40°C, E = 210 $\times 10^3 N/mm^2$.

(a) Strain $\varepsilon = \alpha t$

$= 0.000012(40)$

$= 0.00048$

Stress $\sigma = E.\ \varepsilon$

$= 210 \times 10^3 \times 0.00048$

= **100.8MPa.**

(b) Expansion of the rail $\Delta l = l\ \alpha t$

$= 30(0.000012)(40)$

$= 0.0144m$

$= 14.4mm$

5 mm expansion is permissible, so the amount of expansion prevented is = 14.4 – 5

= 14.4 – 5 = 9.4 mm.

Now, Strain $\varepsilon = \frac{\Delta l}{l} = \frac{9.4}{30000}$

$= 3.133 \times 10^{-4}$

Stress $\sigma = E.\ \varepsilon = 210 \times 10^3 \times 3.133 \times 10^{-4}$

= **65.793 MPa.**

**1.45. A rod is 2m long at a temperature of $10^0C$. Find the expansion of rod, if the temperature is raised to $80^0C$. If this expansion is prevented, find the stress. E = 100GPa, $\alpha = 12 \times 10^{-6}/^oC$.**

**Sol:** **Given data:** l = 2000mm, t = 80 – 10 = $70^oC$, E = 100GPa, $\alpha = 12 \times 10^{-6}/^oC$

Strain in the rod $\varepsilon = \alpha t$

$= 12 \times 10^{-6} (70)$

$= 840 \times 10^{-6}$

Stress $\sigma$ = E. $\varepsilon$

$= 100 \times 10^3 \times 840 \times 10^{-6}$

= **84MPa.**

**1.46. An Aluminum pipe has a length of 50m, steel pipe is 10mm longer than Aluminum Pipe at $18^oC$. At what temperature will the difference in lengths of the two pipes be 15mm. $\alpha_a = 23\times10^{-6}/^oC$. $\alpha_s = 12 \times10^{-6}/^oC$.**

**Sol:** **Given data:** $l_a$ = 50m, $l_s$ = 50.01mm, $t_1 = 18^{oC}$, $l_a - l_s$ = 0.015m, $\alpha_a = 23 \times 10^{-6}/^oC$. $\alpha_s = 12 \times 10^{-6}/^oC$

Let at the temperature $t_2$, the difference in lengths be 15mm. Due to change in temperature from $18^oC$ to $t_2$, the change in length in Aluminum pipe $\Delta l_a = l\,\alpha_a.t$

$= (50)( 23 \times 10^{-6})(t_2 - 18)$

$= 1.15 \times 10^{-3}(t_2 - 18)$

Final length of the Aluminum Pipe = $50 + 1.15 \times 10^{-3}(t_2 - 18)$

Similarly, The change in length in steel Pipe

$\Delta l_s = l\,\alpha_s.t$

$= (50.01)(12 \times 10^{-6})( t_2 - 18)$

$= 6 \times 10^{-4} (t_2 - 18)$

Final length of the Steel Pipe = $50.01 + 6 \times 10^{-4} (t_2 - 18)$

Given that difference in lengths must be 0.015m,

$50 + 1.15 \times 10^{-3}(t_2 - 18) - 50.01 - 6 \times 10^{-4} (t_2 - 18) = 0.015$

$5.5 \times 10^{-4}\, t_2 - 0.0199 = 0.015$

$\mathbf{t_2 = 63.45^oC}$

**1.47. A 15mm diameter steel rod passes through a copper tube of 50mm external diameter and 40mm internal diameter. The tube is closed with nuts at both the ends. If the temperature is raised by $60^oC$, calculate the stresses developed in copper**

**and steel. Take $E_s$ = 2.1 × $10^5$MPa, $E_c$ = 1.05 × $10^5$MPa, $\alpha_s$ = 12 × $10^{-6}$/°C, $\alpha_c$ = 17.5 × $10^{-6}$/°C.**

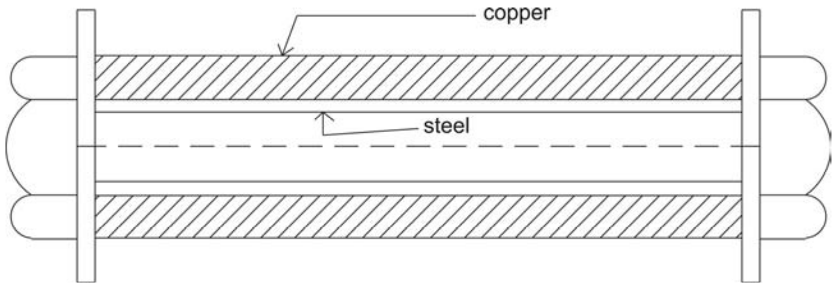



**Fig.P.1.47**

**Sol:** **Given data:** $d_s$ = 15mm, $d_o$ = 50mm, $d_i$ = 40mm, t = 60°C, $E_s$ = 2.1 × $10^5$MPa, $E_c$ = 1.05 × $10^5$MPa, $\alpha_s$ = 12 ×$10^{-6}$/°C, $\alpha_c$ = 17.5 ×$10^{-6}$/°C

Area of the Steel rod $A_s = \frac{\pi}{4} d_s^2 = 176.71\text{mm}^2$

Area of copper tube $A_c = \frac{\pi}{4}\left(d_o^2 - d_i^2\right) = 706.86\text{mm}^2$

Stresses due to loads:

Let the $P_c$ and $P_s$ are the loads developed in the Copper and steel. ($P_c$ = Ps)

Since $\alpha_c > \alpha_s$ Copper is in compression and steel is in tension

Due to the load $P_c$ deflection in copper

$$\delta_c = \frac{-P_c 1_c}{A_c E_c} = \frac{-P_c 1}{(706.86)(1.05\text{x}10^5)} = -1.347\text{x}10^{-8} P_c 1$$

Due to the load $P_s$ deflection in steel $\delta_s = \frac{P_s 1_c}{A_s E_s}$

$$= \frac{P_s 1}{(176.71)\left(2.1 \times 10^5\right)} = 2.695\text{x}10^{-8} P_S 1$$

Stresses due to thermal load:

Due to change in temperature deflection in copper $\delta_{tc}$ = l. $\alpha_c$.t

= l.(17.5 ×$10^{-6}$)60

= 1.05 × $10^{-3}$l

Due to change in temperature deflection in steel $\delta_{ts}$ = l. $\alpha_s$.t

= l.(12 × $10^{-6}$)60

= 7.2 × $10^{-4}$ l.

Net deflection in Copper = $1.05 \times 10^{-3} l - 1.347 \times 10^{-8} P_c l$

Net deflection in steel = $7.2 \times 10^{-4} l. + 2.695 \times 10^{-8} P_s l$

The compatibility equation is net deflection in both the members are same $1.05 \times 10^{-3} l - 1.347 \times 10^{-8} P_c l = 7.2 \times 10^{-4} l. + 2.695 \times 10^{-8} P_s l$

But $P_c = P_s$

On solving the above equation we get $P_c = P_s = 8164.3N$

Stress in Steel = $\frac{P_s}{A_s} = \frac{8164.28}{176.71}$ = 46.2MPa (Tension)

Stress in Copper = $\frac{P_c}{A_c} = \frac{8164.28}{706.86}$

**= 11.55MPa (Compression)**

**1.48. A composite bar is at a temperature of 38°C. What are the stress if the temperature decreases to 21°C. $E_s$ = 210GPa, $E_a$ = 74GPa, $\alpha_s = 11.7 \times 10^{-6}/°C$, $\alpha_a = 23.4 \times 10^{-6}/°C$.**

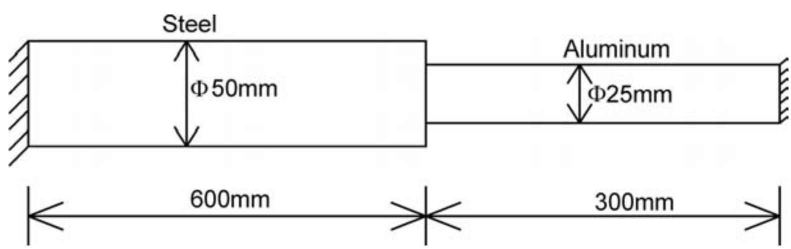



**Fig.P.1.48**

**Sol:** **Given data:** t = 38 – 21 = 17°C, $d_s$ = 50mm, $d_a$ = 25mm, $E_s$ = 210GPa, $E_a$ = 74GPa, $\alpha_s = 11.7 \times 10^{-6}/°C$, $\alpha_a = 23.4 \times 10^{-6}/°C$

Area of the Steel $A_s = \frac{\pi}{4} d_s^2 = 1963.5 mm^2$

Area of the Aluminium $A_a = \frac{\pi}{4} d_a^2 = 490.8 mm^2$

In this case, the members are not allowed to contract. The net deflection is zero. The net deflection is deflection due to temperature change and deflection to induced loads.

Deflection due to change in temperature:

In Steel rod $\delta_{ts} = -l.\alpha_s.t$

$= -(600)(11.7 \times 10^{-6})(17)$

$= -0.1193mm$

In Aluminum rod $\delta_{tc} = -l.\ \alpha_c.t$

$= -(300)(23.4 \times 10^{-6})(17)$

$= -0.1193$mm

Net deflection due to temperature change is = –0.2386mm

Deflection due to loads:

Since the body is prevented from contraction, tension forces will be developed. Let P is the load in Steel and Aluminum.

Deflection in steel $\delta_s = \frac{Pl}{A_s E_s} = \frac{P(600)}{(1963.5)(210 \times 10^3)}$

$= 1.458 \times 10^{-6}P.$

Deflection in Aluminum $\delta_a = \frac{Pl}{A_a E_a} = \frac{P(300)}{(490.8)(74 \times 10^3)}$

$= 8.25 \times 10^{-6}P.$

Net deflection due to loads = $9.708 \times 10^{-6}P$.

Since net deflection is Zero

$9.708 \times 10^{-6}P - 0.2386 = 0$

$P = 24.57 \times 10^3$N.

Stress in Steel $\sigma_s = \frac{P_s}{A_s} = \frac{24.57 \times 10^3}{1963.5} = 12.52\text{MPa}$

$= \mathbf{12.51MPa}$

Stress in Aluminum $\sigma_a = \frac{P_a}{A_a} = \frac{24.57 \times 10^3}{490.8}$

$= 50$MPa

**1.49. Find the stresses in the bars if the temperature raised from –15°C to 85°C. $E_s = 2E_c$ = 200GPa, $\alpha_s = 12.5 \times 10^{-6}/°C$ and $\alpha_c = 16.5 \times 10^{-6}/°C$.**

**Sol:** **Given data:** t = 85–(–15)= 100°C, $E_s = 200 \times 10^3$MPa, $E_c = 100 \times 10^3$MPa, $\alpha_s = 12.5 \times 10^{-6}/°C$ $\alpha_c = 16.5 \times 10^{-6}/°C$.

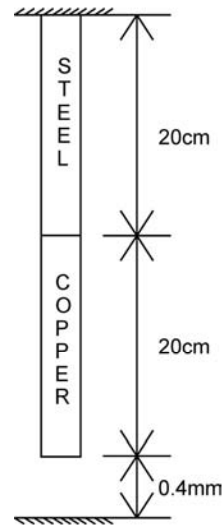


Fig.P.1.49

$l_s$= 200mm, $l_c$ = 200mm, $A_s = A_c$

The permitted net deflection is 0.4mm.

Deflection due to change in temperature:

In Steel rod $\delta_{ts} = l.\ \alpha_s.t$

= (200)(12.5 × $10^{-6}$)(100)

= 0.25mm

In Aluminum rod $\delta_{tc} = l.\ \alpha_c.t$

= (200)(16.5 × $10^{-6}$)(100)

= 0.33mm

Net deflection due to change in temperature

= 0.58mm

Deflection due to induced loads:

Let P is the load in Steel and Copper rods (Compressive load because of prevention from expansion).

Deflection in steel rod $\delta_s = -\frac{Pl_s}{A_s E_s} = -\frac{P(200)}{A(200 \times 10^3)}$

$$= -0.001\frac{P}{A}$$

Deflection in copper rod $\delta_c = -\frac{Pl_c}{A_c E_c} = -\frac{P(200)}{A(100 \times 10^3)}$

$$= -0.002\frac{P}{A}$$

Net deflection due to loads $= -0.003\frac{P}{A}$

Net deflection $= 0.58 - 0.003\frac{P}{A}$

The permissible deflection is 0.4mm

So we can writ e the compatibility equation is

$$0.58 - 0.003\frac{P}{A} = 0.4$$

On solving the above equation $\sigma = \frac{P}{A} = 60\text{MPa}$ which is same in Steel and copper.

**1.50. A bronze bar 3m long with cross sectional area of 320mm² is placed between two rigid walls as shown in Fig.P.1.50. At a temperature of – 20°C, the gap Δ= 2.5mm. Find the temperature at which the stress in the bar is 35MPa. α = 18 × $10^{-6}$/°C and E = 80GPa.**

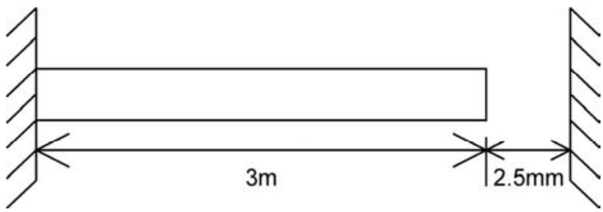



**Fig.P.1.50**

**Sol:** **Given data:** l = 3000m, A = 320mm², Δ = 2.5mm, α = 18 × $10^{-6}$/°C and E = 80 × $10^3$MPa. σ = 35MPa.

Let the temperature change = t.

Deflection of the bar due to

change in temperature = l αt

= 3000(18 × $10^{-6}$)t

= 0.054t

Deflection due to load $P = -\frac{PL}{AE} = -\frac{3000P}{A \times 80 \times 10^3}$

$$= -0.0375\frac{P}{A}$$

Net deflection = $0.054t - 0.0375\frac{P}{A}$

But permitted deflection is 2.5mm

So, we can write the compatibility equation as

$$0.054t - 0.0375\frac{P}{A} = 2.5$$

And we know that stress P/A = 35MPa

0.054t – 0.0375(35) = 2.5

t = 70.60°C

So, raise in the temperature = 70.60°C – 20°C

= **50.60°C**

**1.51. A body of weighing 30kN is carried by 3 equidistance vertical wires. The outer wires made of steel and inner one is made of brass. The cross sectional area is 3 cm$^2$. Find the stress if the temperature rises to 50$^o$C. $E_b$ = 93GPa, $E_s$ = 200GPa, $\alpha_b$ = 18.7 × $10^{-6}/^oC$, $\alpha_s$ = 11.7 × $10^{-6}/^oC$**

**Sol:** **Given Data:** W = 30 × $10^3$N, A = 300mm$^2$, t = 50$^0$C, $E_b$ = 93 × $10^3$ MPa, $E_s$=200 × $10^3$ MPa, $\alpha_b$ = 18.7 × $10^{-6}/^oC$, $\alpha_s$ = 11.7 × $10^{-6}/^oC$

Let $P_s$, $P_b$ are the loads in the steel and brass wires. Form the Fig.1.51(a) we can write the equilibrium equation

$\Sigma F_y = 0 \Rightarrow 2P_s + P_b = 30 \times 10^3$ .....(1)

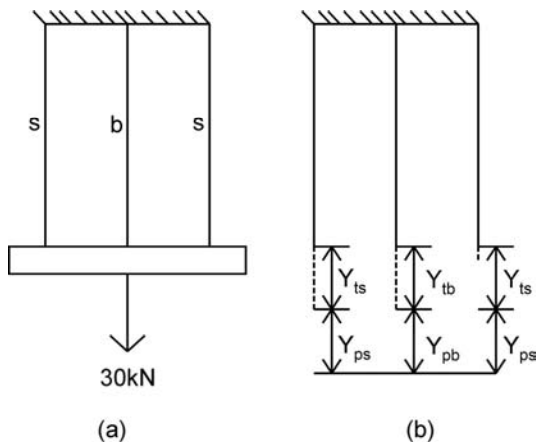



**Fig.P.1.51**

The compatibility equation can be written from the Figure P.1.51(b).

Let us assume that the deflection in the steel wires is $Y_{ts}$ due to change in temperature and due to the load it is $Y_{PS.}$ Similarly the deflection in brass wire due to change in temperature is $Y_{tb}$ and due to the load it is $Y_{Pb.}$ Since the block is remains horizontal due to symmetry, the compatibility equation is

Net deflection in steel = Net deflection in brass

$\delta_s = \delta_b$

But the net deflection is deflection due to temperature and deflection due to load.

$\Rightarrow \quad Y_{ts} + Y_{PS} = Y_{tb} + Y_{Pb}$

Deflection in steel wires due to temperature:

$Y_{ts} = l.\ \alpha_s t$

$= l(11.7 \times 10^{-6})(50)$

$= 5.85 \times 10^{-4} l$

Deflection due to load $P_s$

$$Y_{PS} = \frac{P_s l}{AE_s} = \frac{P_s l}{300\left(200 \times 10^3\right)}$$

$= 1.67 \times 10^{-8} P_s l.$

Net deflection in steel $\delta_s = 5.85 \times 10^{-4} l + 1.67 \times 10^{-8} P_s l$

Deflection in brass wires due to temperature:

$Y_{tb} = l.\ \alpha_s t$

$= l(18.7 \times 10^{-6})(50)$

$= 9.35 \times 10^{-4} l$

Deflection due to load $P_b$

$$Y_{Pb} = \frac{P_b l}{AE_b} = \frac{P_b l}{300\left(93 \times 10^3\right)}$$

$= 3.58 \times 10^{-8} P_b l.$

Net deflection in brass $\delta_b = 9.35 \times 10^{-4} l + 3.58 \times 10^{-8} P_b l$

From the compatibility equation

$5.85 \times 10^{-4} + 1.67 \times 10^{-8} P_s = 9.35 \times 10^{-4} + 3.58 \times 10^{-8} P_b$

$\Rightarrow 1.67 \times 10^{-8} P_s - 3.58 \times 10^{-8} P_b = 3.5 \times 10^{-4}$

$\Rightarrow 1.667 P_s - 3.58 P_b = 3.5 \times 10^4$ .....(2)

On solving (1) and (2) we get

$P_s = 16.13\text{kN}$

$P_b = -2.26\text{kN}$

$\therefore$ Stress in steel $\sigma_s = \frac{P_s}{A} = \frac{16.13\text{x}10^{10}}{300}$

= **53.77MPa**

$\therefore$ Stress in brass $\sigma_b = \frac{P_b}{A} = \frac{2.26\text{x}10^{10}}{300}$

= **– 7.53 MPa**

**1.52. Find the stresses in each material if the temperature raised from 60⁰C to 120⁰C. Area of the Aluminum rod is 2mm² and Steel is 3mm². If F = 50kN, $\alpha_s$=6.5 × $10^{-6}$/°C, $\alpha_a$=12.8 × $10^{-6}$/°C, $E_a$=10 × $10^6$MPa, $E_s$=29 × $10^6$MPa.**

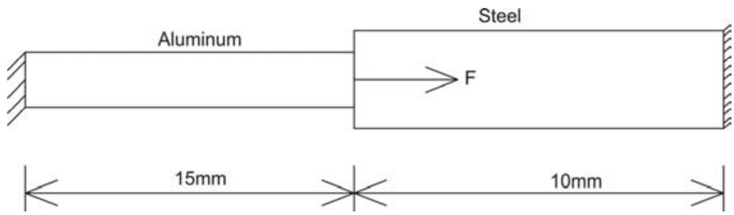



**Fig.P.1.52**

**Sol:** **Given data:** $t = 120 - 60 = 60^0C$, $l_a = 15mm$, $l_s = 10mm$, $A_a = 2mm^2$, $A_s = 3mm^2$, $F = 50kN$, $\alpha_s = 6.5 \times 10^{-6}/^oC$, $\alpha_a = 12.8 \times 10^{-6}/^oC$, $E_a = 10 \times 10^6Mpa$, $E_s = 29 \times 10^6Mpa$

In this problem stresses are to be calculate due to change in temperature, due to induced loads for preventing from expansion and due to applied load.

Stresses due to change in temperature:

Deflection in aluminium bar due to temperature change

$= l\alpha t$

$= 15(12.8 \times 10^{-6})60$

$= 0.01152$

Deflection in steel bar due to temperature change

$= l\alpha t$

$= 10(6.5 \times 10^{-6})60$

$= 0.0039$

Deflection due to temperature change=0.01152+0.0039 = 0.01542

Let P (compression) is the force induced in steel and Aluminum rods due to preventing these bars from expansion.

Deflection of aluminium rod due to load $= -\frac{Pl_s}{A_sE_s} = -\frac{15P}{2\times10\times10^6}$

$= -7.5 \times 10^{-7}P$

Deflection of steel rod due to load $= -\frac{Pl_s}{A_sE_s}$

$= -\frac{10P}{3\times29\times10^6}$

$= -1.15 \times 10^{-7}P$

Deflection due to induced loads $= -8.65 \times 10^{-7}P$

But the net deflection is zero

$0.01542 - 8.65 \times 10^{-7}P = 0$

$P = 17.83kN$.

Stress in the Aluminum rod= $\frac{P}{A_a}$ = –8915MPa

Stress in Steel rod = $\frac{P}{A_s}$ = –5943.3MPa

Stresses due to Applied Load F:

Let $R_A$ and $R_B$ are the reactions at the supports.

From the equilibrium equation $\sum F_x = 0$,

We can write $R_A + R_B = 50 \times 10^3$ .....(1)

The free body diagrams of Aluminum and Steel are shown in Fig.P.1.52(a)


**Fig.P.1.52(a)**

For the Steel rod:

$P = -R_B$, $A = 3mm^2$ $L = 10mm$, and $E = 29 \times 10^6 MPa$

Deflection in AB $\delta_{AB} = \frac{PL}{AE} = -\frac{R_B(10)}{3 \times 29 \times 10^6}$

$= -1.15 \times 10^{-7} R_B$

For the Aluminum rod:

$P = R_A$, $A = 2mm^2$, $L = 15mm$ and $E = 10 \times 10^6 MPa$.

Deflection in AB $\delta_{BC} = = \frac{PL}{AE} = \frac{R_A(15)}{2 \times 10 \times 10^6} = 7.5 \times 10^{-7} R_A$

But from geometry of the figure we can say net deflection $\delta = 0$ (Compatibility) $-1.15 \times 10^{-7} R_B + 7.5 \times 10^{-7} R_A = 0$ .....(2)

On solving (1) and (2) we can get

$R_A = 6640N$ and $R_B = 43350N$.

Stress in the Aluminium rod = $\frac{R_A}{A_a}$ = 3320MPa

Stress in steel rod= $-\frac{R_B}{A_s}$ = –14450MPa

Final stresses in Aluminium rod = –8915MPa + 3320MPa

= **5595MPa (Comp.)**

Final stresses in Steel rod = –5943.3MPa –14450MPa

= **20393MPa (Comp.)**

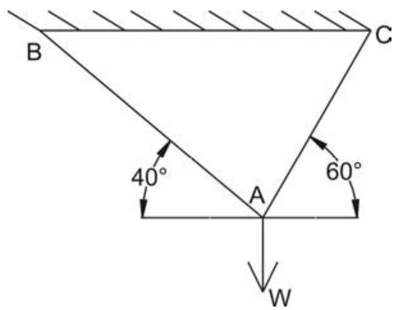
## PROBLEMS FOR PRACTICE

**1.1** A mild steel rod of 20mm diameter is subjected to an axial load of 50kN. Determine the tensile stress induced in the rod and the elongation if the unloaded length is 5m. E = 210GPa.

(**160MPa, 3.79mm**)

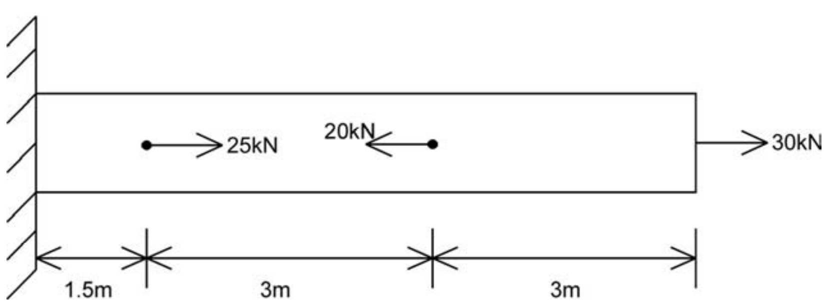
**1.2** Fig. shows a two member truss supporting a block of weight W. The cross sectional areas of the members are 800mm² for AB and 400mm² for AC. Determine the maximum safe value of W if the working stresses are 110MPa for AB and 120MPa for AC.

(**61.7kN**)




**Problem 1.2**

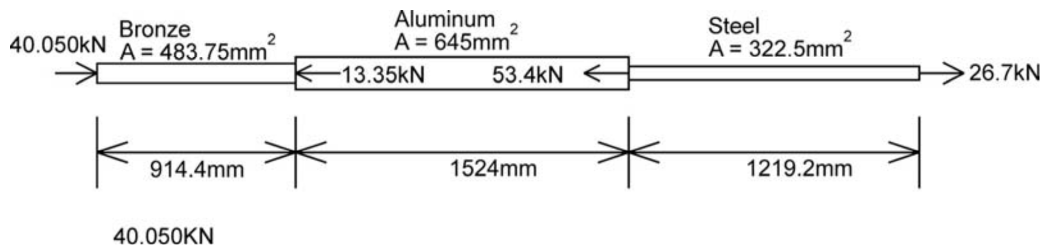
**1.3** The cross sectional area of the bar ABCD is 600mm². Determine the maximum normal stress in the bar. (**58.3MPa**)




**Problem 1.3**

**1.4** Axial loads are applied to the compound rod that is composed of an aluminum segment rigidly connected between steel and bronze segments. What is the stress in each material?

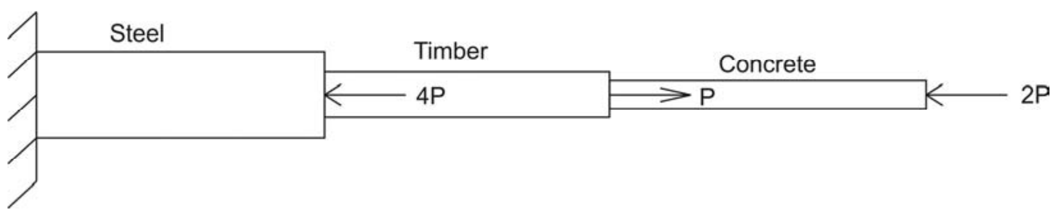
**($\sigma_{br}$ = - 82.68MPa, $\sigma_{Al}$ = - 41.34MPa, $\sigma_{st}$ = 82.68MPa)**




**Problem 1.4**

**1.5** Find the maximum allowable value of P for the column. The cross sectional area of the steel member is 500mm$^2$, Timber is 2000mm$^2$, Concrete is 8000mm$^2$. The permissible stress foe steel, timber and concrete are 120MPa, 12MPa and 16MPa respectively.

(**24KN**)

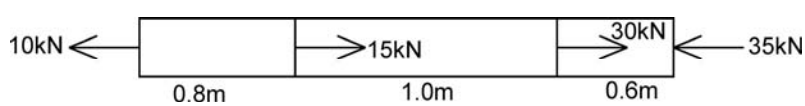



**Problem 1.5**

**1.6** A 5.08mm diameter steel wire, 3.048m long carries an axial load P. Find the maximum safe value of P if the allowable normal stress is 275.6MPa and the elongation of the wire is limited to 3.81mm. Use E = 2 × 10$^{11}$Pa. (**5.07KN**)

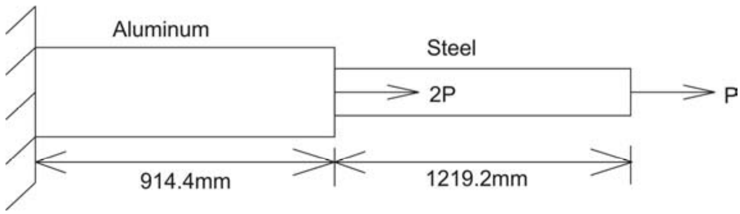
**1.7** The aluminum bar with cross sectional area of 160mm$^2$ carries the axial loads at the positions as shown in the Fig. Given that E = 70GPa. Compute the total change in length of the bar.

(**– 1.607mm**)



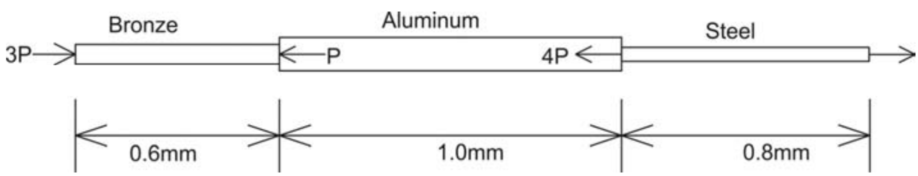

**Problem 1.7**

**1.8** The compound bar carries the axial forces P and 2P. Find the maximum allowable value of P if the working stresses are 275.6MPa for steel and 137.8MPa for aluminum and the total elongation of the bar is not to exceed 5.08mm. The areas of Aluminum and steel rods are 387mm$^2$ and 258mm$^2$. $E_{Al}$ = 70GPa, $E_{St}$ = 210GPa. (**17.8kN)**




**Problem 1.8**

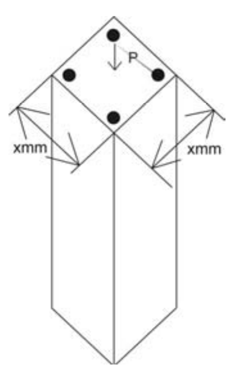
**1.9** Find the maximum values of P that will not exceed an overall deformation of is 2mm and the following stresses: Steel 140MPa, Bronze 120MPa and Aluminum 80MPa. Take $E_{St}$ = 200GPa, $E_{Br}$ = 83GPa, $E_{Al}$ =70GPa. $A_{st}$=300mm$^2$, $A_{Br}$=450mm$^2$ and $A_{Al}$ = 600mm$^2$. (**18.0KN**)




**Problem 1.9**

**1.10** A circular rod 0.2m long tapers from 20mm diameter at one end to 10mm diameter at the other end. On applying an axial pull of 6kN, it was found to extend by 0.068mm. find the Young's modulus of the material of the rod (**112.3GPa**)

**1.11** A concrete pedestal of square cross section (x = 152.4mm) is reinforced with 19.05mm diameter steel bars as shown in the figure. Calculate the maximum possible load P based upon allowable stresses in the steel and concrete of 124.02MPa and 137.MPa respectively. Take $E_C = 2.4 \times 10^{10}$Pa, $E_S = 2 \times 10^{11}$Pa. (**440KN**)




**Problem 1.11 and 1.12**

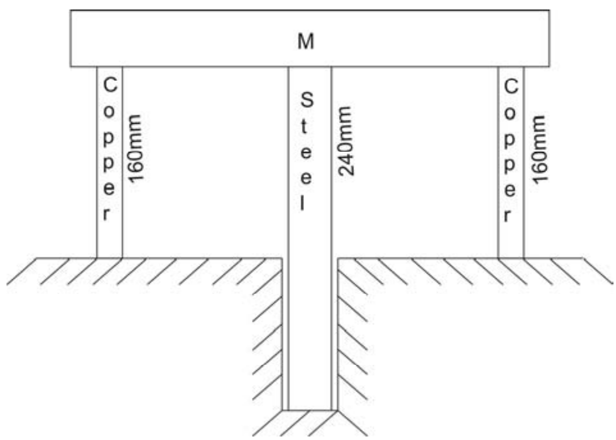
**1.12** The concrete post (x = 300mm) is reinforced axially with four symmetrically placed steel bars as shown in the figure, each of cross sectional area 900mm$^2$. Calculate the stresses in each material when the axial load P = 100kN is applied. The modulus of elasticity is 200GPa for steel and 14GPa for concrete.

**($\sigma_{con}$ = 7.255MPa, $\sigma_{st}$ = 103.6MPa)**

**1.13** The rigid block of mass M is supported by the three symmetrically placed rods. Determine the largest allowable value of M if the area of the steel rods is 1200mm$^2$ and the copper rod is 900mm$^2$. The allowable stresses in copper and steel rods are 70MPa and 140MPa. $E_s$ = 200GPa, $E_c$ = 120GPa.

(Hint: $2P_c+P_s = Mg$ and $\delta_s = \delta_c$)

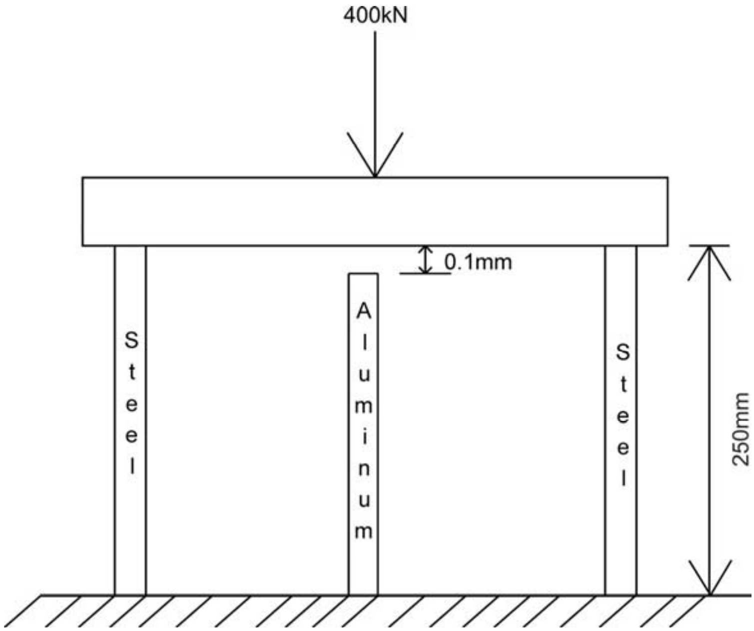
**(22.34kN)**




**Problem 1.13**

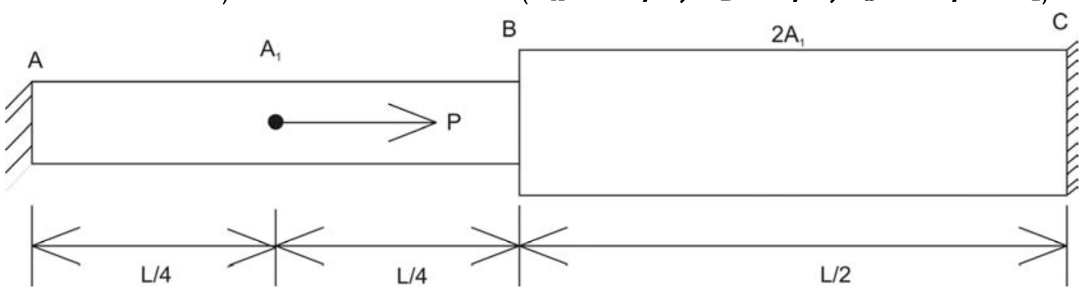
**1.14** Before 400kN load is applied, the platform rests on two steel bars each of 1200mm$^2$ cross sectional area as shown in the Figure. The cross sectional area of the Aluminum bar is 2400mm$^2$. Determine the stresses in the aluminum bar after the load is applied. $E_s$ = 200GPa and $E_A$ = 70GPa.

(Hint: $2P_s+P_a = 400kN$ and $\delta_s = \delta_a + 0.1$) **(22.5MPa)**



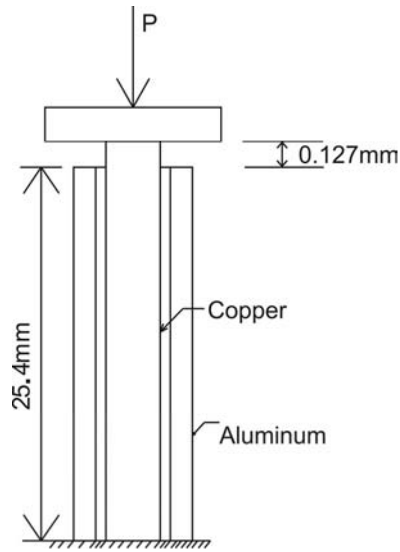

Problem 1.14

**1.15** The axially loaded bar AB shown in figure held between rigid supports. The bar has cross sectional area $A_1$ from A to C and $2A_1$ from C to B. Find the reactions at supports A, B and finds the deflection at point D if the load P acts. Take E as Young's modulus) **($R_A = 2P/3$, $R_B = P/3$, $\delta_d = PL/6EA_1$)**




Problem 1.15

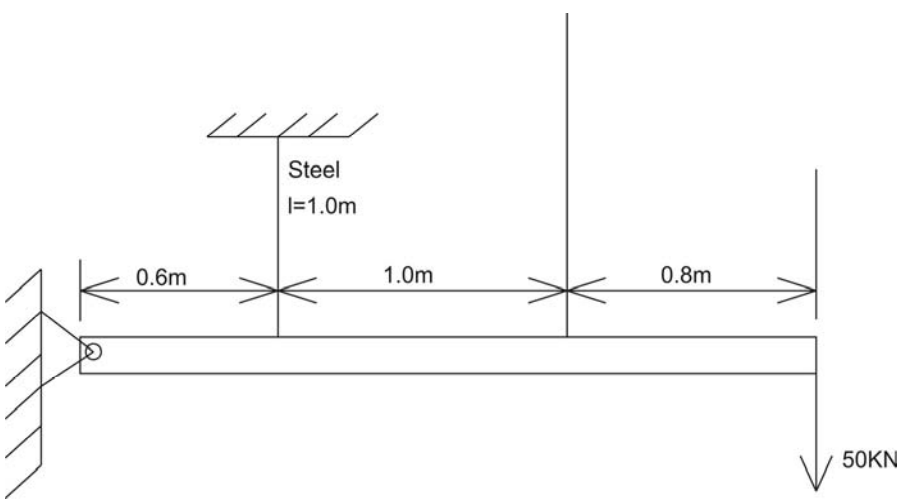
**1.16** A copper rod is placed in an aluminum sleeve as shown in the figure. The rod is 0.127mm longer than the sleeve. Find the maximum safe load P that can applied to the structure if the allowable stress for copper is 137.8MPa and for aluminum it is 70MPa. Area of Copper rod and Aluminum sleeve is 1290mm$^2$ and 1935mm$^2$ respectively. $E_c$ = 117GPa, $E_a$ = 70GPa. **(415.5MN)**




**Problem 1.16**

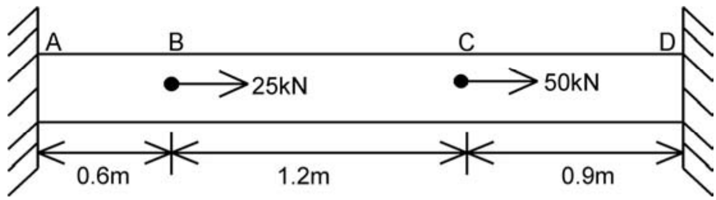
**1.17** Figure shows a rigid bar that is supported by a pin at A and two rods, one made up of steel and other bronze. Compute the stresses in each rod caused by the 50kN load. Area of Steel and Bronze is 600mm$^2$ and 300mm$^2$ respectively. $E_s$ = 200GPa, $E_b$ = 83GPa

**($\sigma_s$ = 191.8MPa, $\sigma_b$ = 106.1MPa)**




**Problem 1.17**

**1.18** Determine the stress in the middle portion BC of a homogenous bar with a cross sectional area of 500mm$^2$ is fixed at both the ends and loaded as shown in the figure. **(22.2MPa)**




**Problem 1.18**

**1.19** A compound tube consists of a steel tube 170mm external diameter and 10mm thickness and an outer brass tube 190mm external diameter and 10mm thickness. The length of the each tube is 0.15m. This compound tube carries a central load of 1000kN. Find the stresses carried by each tube.

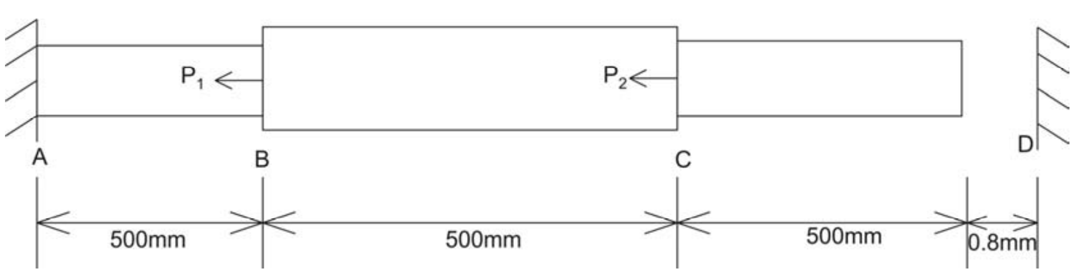
**($\sigma_s$ = 127.34MPa, $\sigma_b$ = 63.67MPa)**

**1.20** A 28mm diameter steel bar 400mm length is placed centrally within a brass tube having an inside diameter of 30mm and outside diameter of 40mm. The bar is shorter in length than the tube by 0.12mm. While the bar and tubes are held vertically on a rigid platform, a compressive force of 60kN is applied at the top of the tube through a rigid plate. Determine the stresses induced in both the bar and tube

**($\sigma_s$ = 48.85MPa, $\sigma_b$ = 54.42MPa)**

**1.21** A steel rod is subjected to an axial loads $P_1$ = 150kN and $P_2$ = 90kN as shown in the figure. Calculate the axial force in each segment if the wall at D yields (moves to the left) 0.8mm, E = 200GPa. Areas for the segments AB, BC, CD are 900mm$^2$, 2000mm$^2$ and 1200mm$^2$ respectively.

**($P_{AB}$ = - 148.9kN, $P_{BC}$ = 1.11kN, $P_{CD}$ = 91.1kN)**




**Problem 1.21**

**1.22** A high tensile rod E = 200GPa, μ = 0.3 is compressed by an axial force P(along the length). When there is no load the diameter of the rod is 50mm. in order to maintain certain clearances, the diameter of the rod must not exceed 50.02mm. What is the largest possible value of load P?

(Hint: Lateral strain = 0.02/50 = $4 \times 10^{-4}$) (**P = 524kN**)

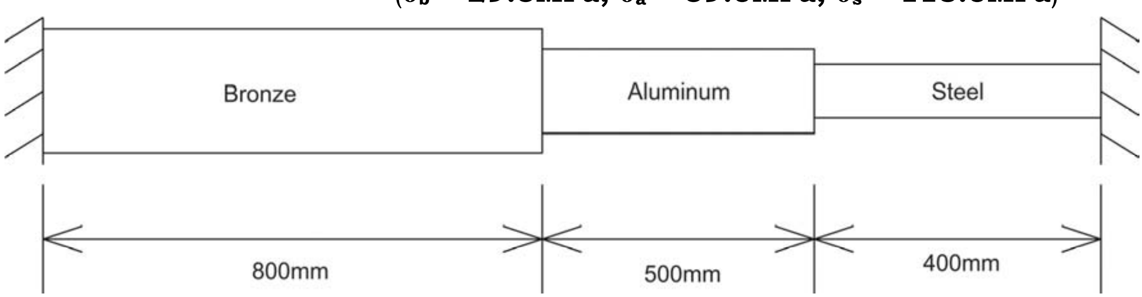
**1.23** A solid circular cast iron (E = $0.87 \times 10^{11}$Pa, μ = 0.3) of diameter 57.15mm and length 381mm is compressed by an axial force (along the length) of 200.25kN. Find the increase in diameter and decrease in the volume of the bar (**0.0155mm, 360.51mm³**)

**1.24** A steel bar 300mm long, 50mm wide and 12mm thick is subjected to an axial pull of 84kN. Find the change in length, width, thickness and volume of the bar. E = 200GPa and μ=0.32

(**δl=0.21mm, δw=-0.0112mm, δt=-0.0027mm, δv=45.36mm³**)

**1.25** A compound bar composed of three segments as shown in the figure. Compute the stress in each material if the temperature suddenly drops by 30°C. Areas of the bronze, Aluminum and steel segments are 2400mm², 1200mm² and 600mm² respectively. Take $E_b$ = 83GPa, $E_a$ = 70GPa, $E_s$ = 200GPa. $\alpha_b = 19 \times 10^{-6}/^{o}C$, $\alpha_a = 10.5 \times 10^{-6}/^{o}C$, $\alpha_s = 6.5 \times 10^{-6}/^{o}C$.

(**$\sigma_b$ = 29.5MPa, $\sigma_a$ = 59.0MPa, $\sigma_s$ = 118.0MPa**)




**Problem 1.25**

**1.26** A steel rod 30mm diameter and 300mm long is subjected to tensile force P acting axially. The temperature of the rod is then raised through 80°C and the total extension measured as 0.35mm. Calculate value of P. E = 200GPa, $\alpha = 12 \times 10^{-6}/^{o}C$.

(**6.29kN**)

**1.27** A steel rod of 20mm diameter passes centrally through a copper of internal diameter 20mm and external diameter 40m. Both the ends are tightened by nuts. The nuts are tightened till the compressive load on the tube is 50°C. Find the stresses in the rod and the tube. $E_s$ = 210GPa, $E_c$ = 100GPa, $\alpha_s = 12 \times 10^{-6}/^oC$, $\alpha_c = 18 \times 10^{-6}/^oC$ [Hint See example:1.50]

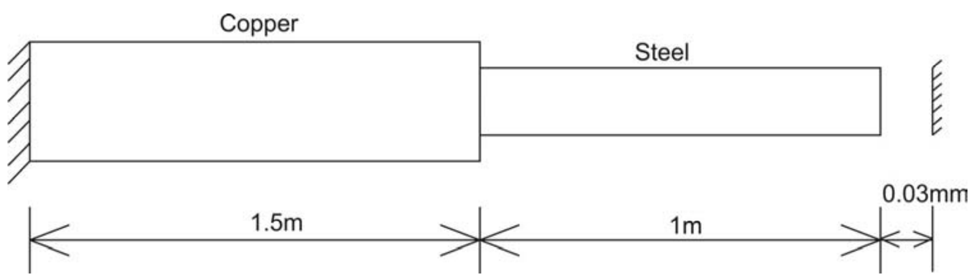
**($\sigma_s$=123.2MPa, $\sigma_c$=41.1MPa)**

**1.28** Three pillars, each 500mm² support a weight of 200kN. The central pillar is of steel and the outer ones are of copper. The pillars are so adjusted that the distance between each pillar is same at 20°C. The temperature is then raised to 120°C. Estimate the stress in each pillar at 120°C. $E_s$ = 210GPa, $E_c$ = 80GPa, $\alpha_s = 12 \times 10^{-6}/^oC$, $\alpha_c = 18.5 \times 10^{-6}/^oC$ [Hint: example 1.49]

**($\sigma_s$= - 75.5MPa, $\sigma_c$=162.22MPa)**

**1.29** Determine the stress in the bar shown in the Figure if the temperature raised by 30°C. Areas of Copper and Steel rods are 200cm² and 100cm² respectively. $E_s$ = 200GPa, $E_c$ = 100GPa, $\alpha_s = 12.5 \times 10^{-6}/^oC$, $\alpha_c = 16.5 \times 10^{-6}/^oC$.

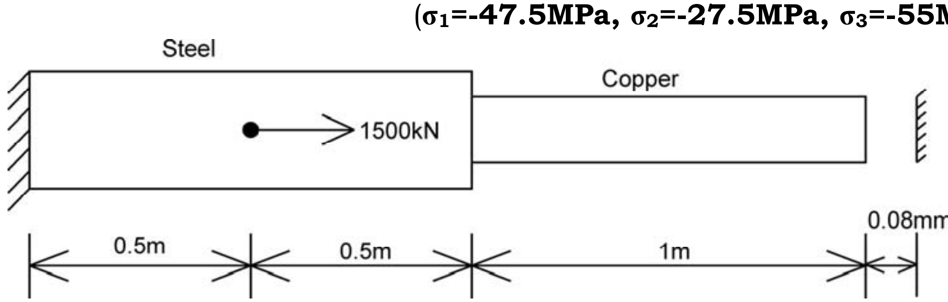
**($\sigma_s$=-87MPa, $\sigma_c$=-43.5MPa)**




**Problem 1.29**

**1.30** Determine the stress in the three parts of the bar shown in the Figure if the temperature raised by 20°C. Areas of Copper and Steel rods are 100cm² and 200cm² respectively. $E_s$ = 200GPa, $E_c$ = 100GPa, $\alpha_s = 12.5 \times 10^{-6}/^oC$, $\alpha_c = 16.5 \times 10^{-6}/^oC$.

**($\sigma_1$=-47.5MPa, $\sigma_2$=-27.5MPa, $\sigma_3$=-55MPa)**




**Problem 1.30**