CHAPTER

1 Linear Wave Shaping

1.1 INTRODUCTION

Let us consider a transmission network consisting of linear elements. Sinusoidal signal is applied to a network, the output signal is sinusoidal in the steady state conditions. The influence of the network circuit on the signal may be completely specified by the ratio of output to input amplitude and phase angle between output and input waveform. No other periodic waveform preserves its shape. Generally when transmitted through a linear network the output signal may have a little resemblance to the input signal.

"The process whereby the shapes of non sinusoidal signals are shaped by passing the signal through the linear network is called linear wave shaping".

1.2 HIGH PASS RC CIRCUIT

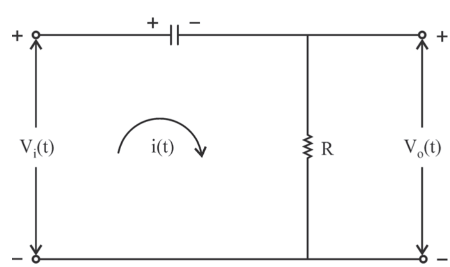


FIGURE 1.1 High pass RC circuit

The high pass RC circuit is shown in Fig.1.1. The input is denoted by $V_i(t)$, and the output as $V_o(t)$, 'a' is the charge of the capacitor.

At zero frequency the capacitor has infinite reactance and hence open circuited. Therefore, the capacitor blocks the dc signal not allowing it to reach output. Hence the capacitor is called blocking capacitor. The coupling circuit provides dc isolator between input and output.

Since the reactance of the capacitor decreases with increasing frequency the end output increases.

Thus the circuit abstracts the low-frequency and it allows the high frequency to reach the output. Hence this circuit is called high pass RC circuit.

1.3 SINUSOIDAL INPUT

The sinusoidal input V_i (t) is mathematically defined as Vi (t) = V_m sin wt

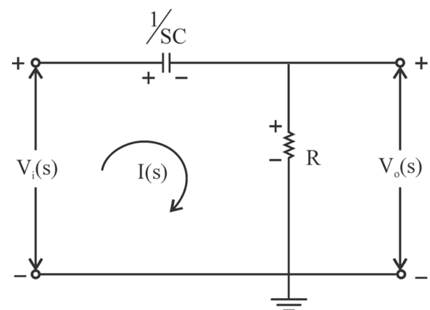


FIGURE 1.2 Laplace Network of high passe RC circuit

In the analysis of Network to sinusoidal input is obtained using Laplace transform as shown in Figure 1.2 applying KVL around the circuit.

$$-1/sc\ I(s) - I(s)\,R + Vi(s) = 0$$

$$Is = \frac{Vi(s)}{\left[R + \frac{1}{sc}\right]}$$

$$V_o(s) = I(s)R = \frac{Vi(s)}{\left[R + \frac{1}{sc}\right]} \times R$$

$$V_o(s) = \frac{Vi(s)R}{\frac{scR+1}{sc}} \Rightarrow \frac{Vi(s)R}{1} \times \frac{sc}{(scR+1)} = \frac{scRVi(s)}{scR+1}$$

$$A = \frac{V_o(s)}{V_i(s)} = \frac{1}{1 + \frac{1}{scR}} \Rightarrow \text{Transfer function}$$

Numerator and De-numerator divided by SCR applying sinusoidal input varying its frequency 0 to α, S = jw

$$A = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1}{1 + \frac{1}{j\omega RC}j} \qquad \frac{1}{j} = -j, \quad j^2 = -1$$

$$\omega = 2\prod f$$

$$A = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1}{1 + j\frac{1}{2\prod fRC}} \text{ Frequency domain transfer function}$$

$$|A| = \left|\frac{V_o(j\omega)}{V_i(j\omega)}\right| = \frac{1}{\sqrt{1 + \left(\frac{1}{2\prod fRC}\right)^2}} \qquad \theta = -\tan^{-1}\frac{1}{2\prod fRC}$$

At lower cut-off frequency f_1,

$$|A| = \frac{1}{\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + \left(\frac{1}{2\prod f_1 RC}\right)^2}}$$

$$\frac{1}{2} = \frac{1}{1 + \left(\frac{1}{2\prod f_1 RC}\right)^2}$$

$$2 = 1 + \left(\frac{1}{2\prod f_1 RC}\right)^2$$

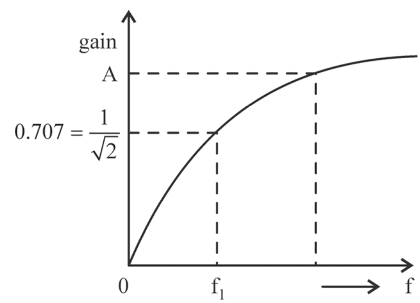


Fig 1.3 0 to f_1 – cut off Jone gain frequency plot

Equating the Denominators 2nfRC = 1

$$f_1 = \frac{1}{2\prod fRC} = \text{lower cut of frequency of high pass RC circuit}$$

$$A = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1}{\sqrt{1 + \left(\frac{1}{2\prod fRC}\right)^2}} = \frac{1}{\sqrt{1 + \left(\frac{f_1}{f}\right)^2}} \qquad \theta = \tan^{-1}\left(\frac{f_1}{f}\right)$$

1.4 STEP INPUT VOLTAGE

Let us consider that the step input voltage of Magnitude a voltage is applied as an input to the high pass RC circuit. When the input step is applied to the circuit, the current starts flowing instantaneously, then the capacitor changes exponentially and the current decays exponentially. Due to which the output voltage also decays exponentially. When capacitor charges equal to the input voltage level of voltage, current stops and the output voltage attains zero values in steady state conditions.

Let us mathematically analyse the output voltage as

$$V_o(t) = B_1 + B_2 e^{-t/\tau}$$

B_1 B_2 , constants

τ is the time constant of the circuit

$$\tau = RC$$

The output voltage consists of two parts

1. B_1 is the steady state value of the output voltage

   $t \to \infty$,

   $V_o(\infty) \to B_1$

2. The transient part represented by expression decaying term B_2 $e^{-t/T}$

   The circuit is said to achieve steady state

   When the transient part completely dies out i.e., $t \to \infty$

$$\text{Limt } t \overset{V_o(t)}{\rightarrow} \alpha(t) = t \overset{\lim}{\rightarrow} \alpha(B_1 + B_2 e^{-t/\tau})$$

$$= B_1 \text{ as Lim } t \overset{\lim}{\rightarrow} e^{-t/\tau} = 0$$

Let the steady state value of output voltage v_f

$B_1 = V_f$

To determine the B_2 (constant)

t = 0 consider initial output voltage

t = 0 be V_i

$$V_o(t)\big|_{t=0} \Rightarrow B_1 + B_2 = V_i$$

$V_i = V_f + B_2$

$B_2 = V_i - V_f$

Substituting the value B_1 and B_2

$$V_o(t) = V_f + (V_i - V_f)e^{-t/\tau}$$

Thus $t \to \infty$ the capacitor blocks d.c, hence the final steady state output voltage is zero

$V_f = 0$

The voltage across the capacitor cannot change instantaneously

$t = 0^+$ i.e., just after t = 0

The voltage across capacitor is zero. It can't change. Hence the output voltage at t = 0^+ is same as the input voltage equal to A volt. When the capacitor is initially unchanged then the output is same as of input $t = 0^+$

V_i = A voltage

$$V_o(t) = V_f + (V_i - V_f)\, e^{-t/\tau}$$
$$= 0 + (A - 0)e^{-t/\tau}$$
$$= A\, e^{-t/\tau}$$

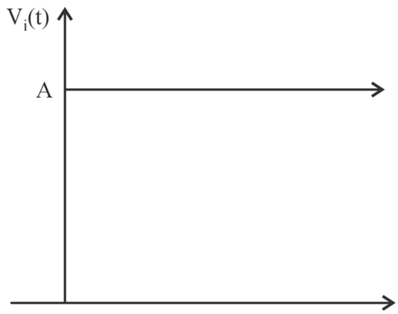


FIGURE 1.4 Step input

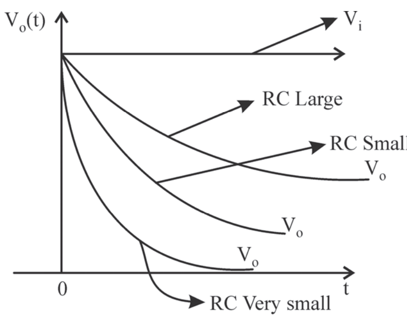


FIGURE 1.5 Step input for different time constants

1.5 PULSE INPUT

An ideal pulse has the waveform shown in Figure (1.6). The pulse amplitude is V and pulse duration is t_p.

It has been mentioned earlier that the pulse is the sum of the two step voltages.

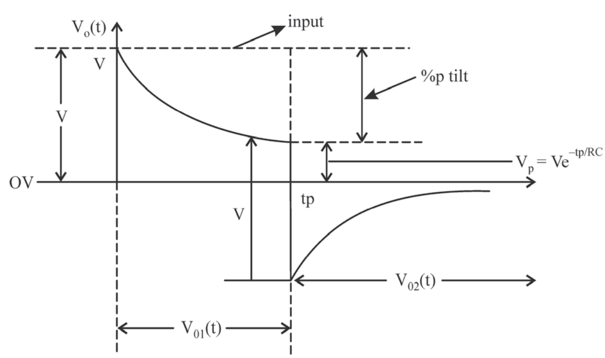


FIGURE 1.6 Pulse input waveform

So the response of the circuit $0 < t < t_p$ for the pulse input is same for a step input given by $V_{o1}(t) = Ve^{-t/RC}$.

At $t = t_p$ $\quad V_{o1}(t) = Ve^{-tp/RC} = V_p$

Now, consider the second part of the input for $t > t_p$. At $t = t_p$. As the input falls by V volts suddenly and the capacitor voltages can't change instantaneously, the output has to drop by a V volts to $V_p - V$

$t = t_p$ i.e, t_p^+

Hence the output drop by V from V_p at $t = t_p^t$ the capacitor voltage changes the output voltage decays exponentially to 0

For the second part of the pulse

$t = t_p^t \quad V_{o2}(t_p^t) = V_p - V$

$V_{o2}(t_p^t) = Ve^{-tp/RC} - V$

$V_{o2}(t_p^t) = V\,(e^{-tp/RC} - V)$

This is the initial output voltage for the second part of pulse

$V_i = V(e^{-tp/RC} - 1)$

The output voltage final value is zero

$V_f = 0$

$V_{o2}(t) = V_f + (V_i - V_f)\, e^{-t/RC}$

$V_{o2}(t) = V(e^{-tp/RC} - 1)\,(e^{-(t-tp)/RC}$

The output waveform RC >>t_p, RC comparable to t_p, and RC << t_p shown in figure 1.7, 1.8, 1.9

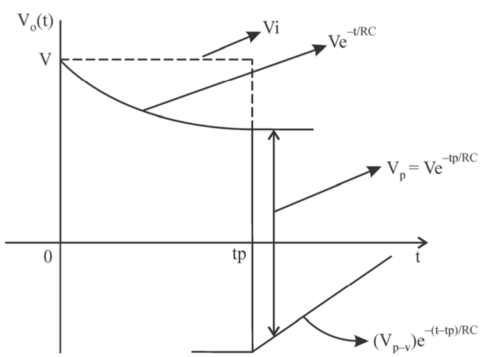


FIGURE 1.7 RC >> t_p

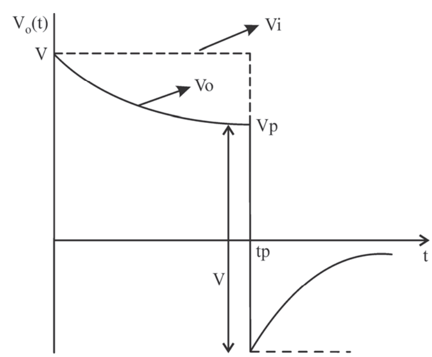


FIGURE 1.8 RC comparable to t_p

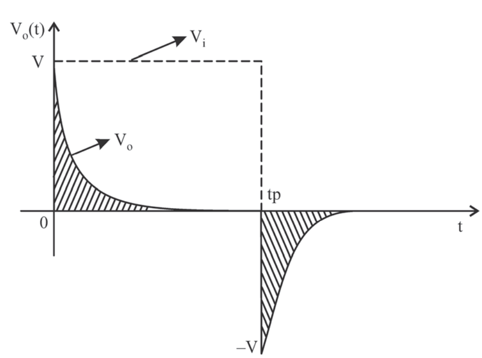


FIGURE 1.9 RC <<t_p

The response with large time constant RC ie, $RC/T_p >>1$ is as shown in figure (1.7)

It can be observed that large time constant, the tilt is very small and undershoot also is very small, both the linear destruction are small. However the negative portion decreases very slowly

The response with small time constant $RC/t_p <<1$ is shown in Fig. (1.9). The output consists of a positive spike of amplitude V at the beginning of the pulse and a negative spike of the same size at the end of the pulse. This process of converting pulse into spikes using a circuit of small time constant is called peaking.

1.6 SQUARE-WAVE INPUT

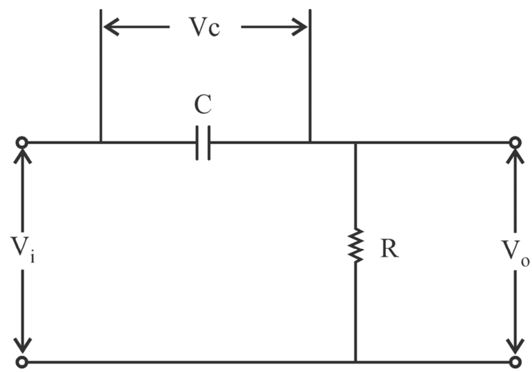


FIGURE 1.10 RC Circuit

Consider the various voltages present in high pass RC circuit as shown in the fig 1.10

q = charge on the capacitor

Apply Kirchhoff law $\qquad c = \frac{q}{V} \quad V = \frac{q}{c}$

$$V_i = V_c + V_o$$

$$V_i = \frac{q}{c} + vo$$

Differentiating the equation

$$\frac{dV_i}{dt} = \frac{1}{c}\frac{dq}{dt} + \frac{dv_o}{dt} \qquad i = \frac{dq}{dt}$$

$$\frac{dV_i}{dt} = \frac{1}{c}(i) + \frac{dV_o}{dt}$$

$$Vo = iR \quad i = \frac{V_o}{R}$$

Substituting in equation

$$\frac{dV_i}{dt} = \frac{V_o}{RC} + \frac{dV_o}{dt}$$

Both sides multiplied by the dt

$$dV_i = \frac{V_o}{RC} dt\ dV_o$$

Integrating the time period from 0 to T

$$\int_0^T dV_i = \frac{1}{RC}\int_0^T V_o dt + \int_0^T dV_o$$

$$[V_i]_0^T = \frac{1}{RC}\int_0^T V_o dt + [V_o]_0^T$$

$$V_i(T) - V_i(0) = \frac{1}{RC}\int_0^T V_o dt + V_o(T) - V_o(0)$$

Under steady-state conditions, the output waveform is repetitive with a time period T

$$V_i(T) = V_i(0) \text{ and } V_o(T) = V_o(0)$$

Hence $\int_0^T v_o(t)dt = 0$. This integral represents this area under the output waveform over one cycle i.e, the average value of output response, substituting the equations.

$$\frac{1}{RC}\int_0^T V_o dt = 0$$

The average level of the steady state output signal is always zero

[1] The average level of the output signal is always zero irrespective of the average level of the input. The output must extend in both positive and negative direction with respect to the zero voltage axis and area of the part of the waveform above the zero axis must equal the area below the zero axis.

[2] When input changes continuously by amount V, the output also changes by the same amount in the same direction.

[3] During any finite time interval where the input maintains a constant level, the output decays exponentially towards zero voltage.

They are in the limiting case, when the ratios RC/T_1 and RC/T_2 are both very large with respect to unity, the output waveform is exactly same as the input.

Now, consider the extreme case when RC/T_1 and RC/T_2 are very small as compared to unity.

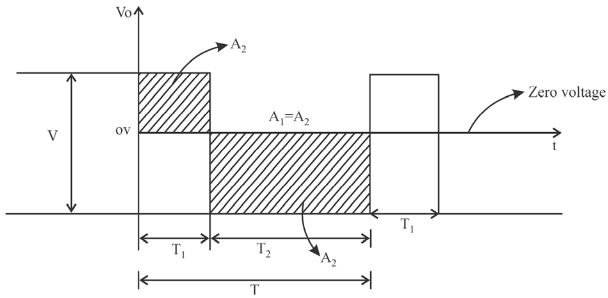


FIGURE 1.11

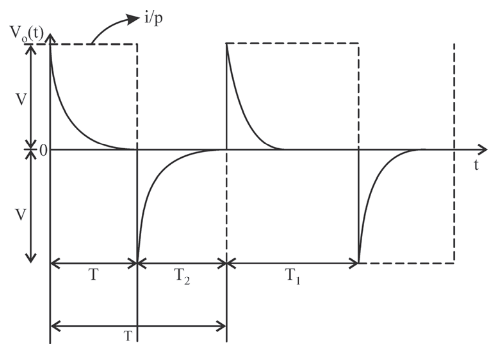


FIGURE 1.12 The high pass RC circuit with small time constant producer spikes circuit

Under steady state condition the capacitor charger and discharges to the same voltage level in each cycle.

For $0 < t < T_1$ the output is given by $V_{o1} = V_1 e^{-t/RC}$

$$\text{At } t = T_1 \qquad Vo_1 = V_1^1 = V_1 e^{-T_1/RC}$$

For $T_1 < t < T_1 + T_2$ the output is $V_{o2} = V_2 e^{-(t-T_1\text{-})/RC}$

$$\text{At } t = T_1 + T_2\text{ , } Vo_2 = V_2^1 = V_2 e^{-T_2/RC}$$

$$V_1^1 - V_2 = V \text{ and } V_1 - V_2^1 = V$$

Expression for the percentage tilt:

The Tilt is defined as the decay in the amplitude of the output voltage wave when the input maintains its level constant.

Mathematically the percentage tilt p is defined as

$$p = \frac{V_1 - V_1^1}{\text{input amplitude}} \times 100$$

When the time constant RC of the constant is very large compared to the period of the input waveform RC>>T

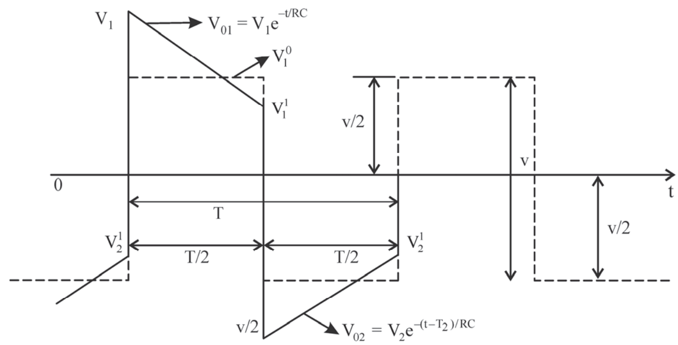


FIGURE 1.13 Tip tilt of a symmetrical square wave when RC >>T

For a symmetrical square wave with zero average value

$$V_1 = -V_2,\ \text{i.e, } V_1 = |V_2|,\ V_1^1 = -V_2^1\ \text{ i.e, } V_1^1 = |V_2|$$

and $T_1 = T_2 = T/2$

RC >>T shown in figure 1.13

$$V_1^1 = V_1 e^{-T/2RC} \text{ and } V_2^1 = V_2 e^{-T/2RC}$$

$$V_1 - V_2^1 = V$$

$$V_1 + V_1^1 = V \qquad [\text{Note} - V_2^1 = V_1^1]$$

$$V_1 + V_1 e^{-T/2RC} = V$$

$$V_1 = \frac{V}{1+e^{-T/2RC}} \text{ or (a) } V = V_1(1+e^{-T/2RC})$$

$$\%\text{ tilt } p = \frac{V_1 - V_1^1}{V/2} \times 100\%$$

Input amplitude = v/2

$$= \frac{V_1 - V_1 e^{-T/2RC}}{V(1+e^{-T/2RC})} \times 200\%$$

$$= \frac{1-e^{-T/2RC}}{1+e^{-T/2RC}} \times 200\%$$

When the time constant is very large T/2RC <<1

$$p = \frac{1-\left[1+(-T/2RC)+(-T/2RC)^2\frac{1}{2!}\right]}{1+1+(-T/2RC)+(-T/2RC)^2\frac{1}{2!}} \times 200\%$$

$$\% \ p = \frac{1-(1-T/2RC)}{1+(1-T/2RC)} \times 200\% \qquad e^{-T/2RC} = 1 - \frac{T}{2RC}$$

$$p = \frac{\frac{T}{2RC}}{2} \times 200\%$$

$$= \frac{T}{2RC} \times 100\% = \frac{\pi f_1}{f} \times 100\%$$

$f_1 = \dfrac{1}{2\pi RC}$ is the lower cut off frequency of the high pulse RC circuit

1.7 RAMP INPUT

A waveform which is zero for $t < 0$ and which increases linearly with the time for $t > 0$ is called ramp (or) sweep voltage. Ramp input can be mathematically written as

$$Vi(t) = \begin{cases} 0 \text{for } t < 0 \\ \alpha t \text{ to } t > 0 \end{cases}$$

Where α is the slope of the ramp

$$V_i = \frac{q}{c} + V_0 \Rightarrow V_i(t) = \frac{q}{c} + V_0(t)$$

$$Vi(t) = \alpha t = \text{input ramp}$$

$$\alpha t = \frac{q}{c} + V_0(t)$$

Differentiating the equation both side w.r.t t

$$\alpha = \frac{dq}{cdt} + \frac{dV_o}{dt}$$

$$\left[\text{Note } \frac{dq}{dt} = i, \frac{dq}{dt} = \frac{V_o}{R}, V_o = iR,\ i = \frac{V_o}{R}\right]$$

Substituting in the equation

$$\alpha = \frac{V_o}{RC} + \frac{dV_o}{dt}$$

Initially capacitor is zero $V_o(0) = 0$

take Laplace form

$$\frac{dV_o(t)}{dt} + \frac{V_o(t)}{RC} = \alpha$$

$$V_o(s)\ S + \frac{1}{RC} V_o(s) = \frac{\alpha}{S}$$

$$\left[\ S + \frac{1}{RC}\right] V_o(s) = \frac{\alpha}{S}$$

$$V_o(s) = \frac{\alpha}{S(\ S + \frac{1}{RC})}$$

$$V_o(s) = \alpha RC\left[\ 1/S - \frac{1}{S + 1/RC}\right]$$

$$V_o(t) = \alpha RC\left[\ 1 - e^{-t/RC}\right]$$

$$V_o(t) = 0 \qquad t = 0$$

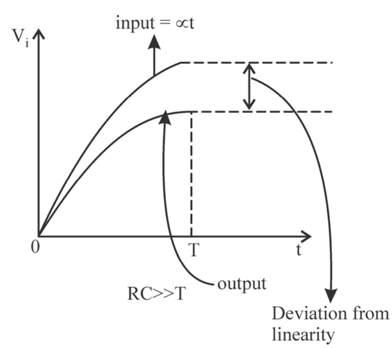


FIGURE 1.14 Deviation from linearity

$$V_o(t) = \alpha RC\left[\ 1 - \left\{1 + (-t/RC) + \frac{(-t)^2}{(RC)^2 2!} + \frac{(-t)^3}{(RC)^3 3!} + \right\}\right]$$

$$= \alpha RC\left[\ t/2RC - \frac{t^2}{2(RC)^2}\right] = \frac{\alpha t - \alpha t^2}{2RC} = \alpha t\left[\ t - \frac{t}{2RC}\right]$$

The falling away of output from input is called deviation from linearity

This departure of output from linearity is called the trangenmussion error denoted as e_t.

$$e_t = \left.\frac{Vi - Vo}{Vi}\right|_{t=T} = \frac{\alpha t - \alpha t\left(1 - \frac{t}{2RC}\right)}{\alpha t} = \frac{T}{2RC} = \pi f_1 T$$

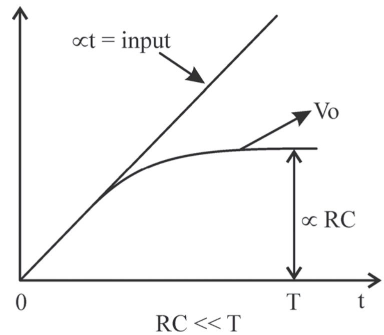


FIGURE 1.15 RC << T

1.8 EXPONENTIAL INPUT (HPF)

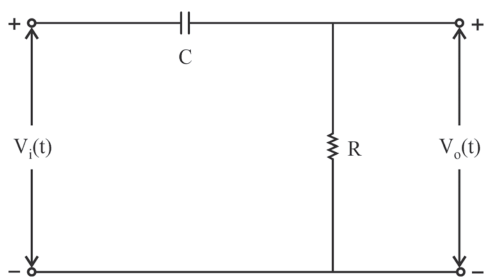


FIGURE 1.16 HPF

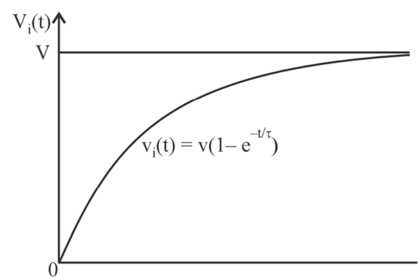


FIGURE 1.17 Exponential waveform

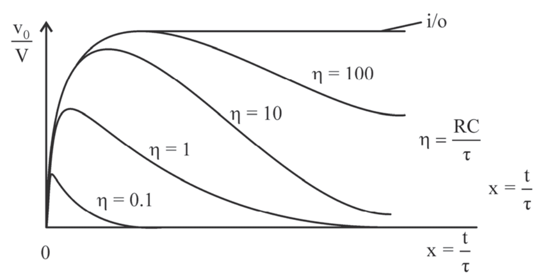


FIGURE 1.18 Output waveform

Let us consider the RC-high pass circuit and the exponential input denoted in the figure (1.16, 1.17, 1.18)

The exponential input can be expressed as

$$Vi(t) = V(1 - e^{-t/\tau})$$

Repeating similar steps of the previous sections

$$\frac{dV_i(t)}{dt} = \frac{V_o(t)}{RC} + \frac{dV_o(t)}{dt}$$

Substituting the equation for exponential input w consider 'w'

$$\frac{V}{\tau} e^{-t/T} = \frac{V_o(t)}{RC} + \frac{dV_o(t)}{dt}$$

The intial output is zero as the intial voltage on the capacitor is zero.

$V_o(0) = 0$ which makes the Laplace transform approach suitable to solve the above differential equations the equation is rewritten as

$$\frac{dv_o(t)}{dt} + \frac{v_o(t)}{RC} = \frac{V}{\tau} e^{-t/\tau}$$

Taking Laplace transform both sides

$$\left[S + \frac{1}{RC}\right] v_o(s) = \frac{V}{\tau} \frac{1}{(s + \frac{1}{\tau})}$$

$$v_o(s) = \frac{V}{\tau} \frac{1}{(s + \frac{1}{\tau})\ (S + \frac{1}{RC})}$$

$$V_o(s) = \frac{V}{\tau} \frac{1}{(\frac{1}{\tau}) - (\frac{1}{RC})} \left[\frac{1}{S + RC} - \frac{1}{S + \frac{1}{\tau}}\right]$$

$$v_o(t) = \frac{VRC\tau}{\tau(RC - \tau)} \left(e^{-t/RC} - e^{-t/\tau}\right)$$

$$v_o(t) = \frac{VRC}{RC - \tau} \left(e^{-t/RC} - e^{-t/\tau}\right)$$

$$v_o(t) = \frac{\frac{VRC}{\tau}}{\frac{RC}{\tau} - 1} \left(e^{-t/RC} - e^{-t/\tau}\right)$$

x and n defined as

$$x = \frac{t}{\tau} \qquad\qquad n = \frac{RC}{\tau}$$

Do note that RC is the circuit time constant and τ is the input time constant

X may be Normalised time and n interpreted as the Normalised time constant

$$\frac{x}{n}=\frac{t}{RC}$$

The modified expression can be written as

$$v_o(t)=\frac{V_n}{n-1}\left(e^{-x/n}-e^{-x}\right), \qquad n=1$$

$$v_o(t)=\frac{V_n}{n-1}\left(e^{-x/n}-e^{-x}\right), \qquad n\neq 1$$

it make use of L hospital rule

$$v_o(t)=\frac{\overset{\lim}{n\to 1}\frac{d}{dn}\left[V_n(e^{-x/n}-e^{-x})\right]}{\overset{\lim}{n\to 1}\frac{d}{dn}(n-1)}$$

$$v_o(t)=\overset{\lim}{n\to 1}\frac{\left[V(e^{-x/n}-e^{-x})+V_n(-e^{-x/n})\left[\frac{-x}{n^2}\right]\right]}{1}$$

$$v_o(t)=\overset{\lim}{n\to 1}\left[V(e^{-x/n}-e^{-x})+V_n(-e^{-x/n})\left[\frac{-x}{n^2}\right]\right]$$

$$v_o(t)=Vxe^{-x}$$

It conclude our derivation by starting that the response of the RC high pass circuit for as exponential waveform is given by

$$v_o(t)=\frac{Vn}{n-1}(e^{-x/n}-e^{-x}) \qquad \text{for } n\neq 1$$

$$v_o(t)=Vxe^{-x} \qquad \text{for } n=1$$

[1] When n is large the response has larger peak amplitude as well as a wider pulse width.

[2] Similarly when the n response is smaller and has as smaller peak amplitude provided the width of the pulse is narrow n has an effect on both peak value and the width of the output pulse.

1.9 SINUSOIDAL INPUT

The analysis of the High pass RC circuit to sinusoidal input is obtained using Laplace transform approach applying KVL to the circuit.

$$-I(s)\frac{1}{sc}-I(s)R+Vi(s)=0$$

$$Vi(s) = \frac{1(s)}{SC} + I(s)R$$

$$Vi(s) = I(s)\left[\frac{1}{SC} + R\right]$$

$$I(s) = \frac{Vi(s)}{SC + R}$$

$$Vo(s) = I(s)R = \frac{Vi(s)}{SC + R} \times R$$

$$\frac{Vo(s)}{Vi(s)} = \frac{R}{R + \frac{1}{SC}} = \frac{1}{1 + \frac{1}{S + RC}} = \text{ Tranfer function}$$

Frequency varies from 0 to ∞ s replaced by $j\omega$

$$\frac{Vo(j\omega)}{Vi(j\omega)} = \frac{1}{1 + \frac{1}{j\omega RC}}$$

$$\frac{Vo(j\omega)}{Vi(j\omega)} = \frac{1}{1 - \frac{1}{j2\pi fRC}}$$

$$\frac{1}{1 + \frac{j}{2\pi fRC}} \Rightarrow$$

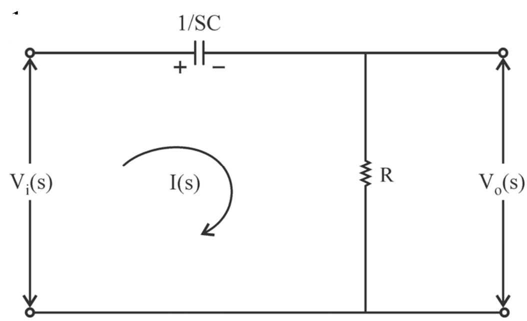


FIGURE 1.19 High pulse RC circuit

Frequency domain transfer function

$$A = \left|\frac{Vo(j\omega)}{Vi(j\omega)}\right| = \frac{1}{\sqrt{1 + \left(\frac{1}{2\pi fRC}\right)^2}}$$

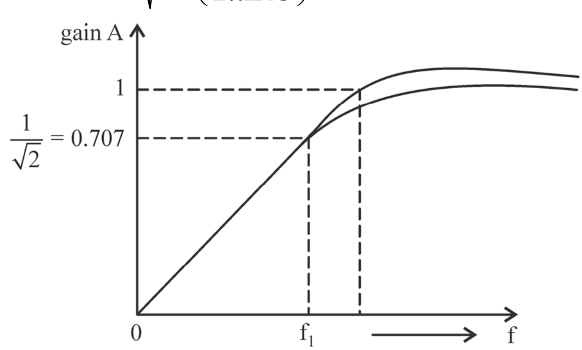


FIGURE 1.20 output waveform

Frequency increases the gain A approaches to unity. Initially output increases as the frequency increases and becomes equal to input at high frequency. As $f \to \infty, A \to 1$. To allow high-frequencies to pass.

A gain is $1/\sqrt{2}$ is called lower cout of frequency f_1 of the circuit.

$0 - f_1$ is cut off/zone

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+\left(\frac{1}{2\pi f_1 RC}\right)^2}}$$

$$\frac{1}{2} = \frac{1}{1+\left(\frac{1}{2\pi f_1 RC}\right)^2}$$

$$2 = 1+\left(\frac{1}{2\pi f_1 RC}\right)^2$$

$$2\pi f_1 RC = 1 \qquad f_1 = \frac{1}{2\pi RC} = \text{lower cut off frequency}$$

1.10 HIGH PASS RC CIRCUIT AS A DIFFERENTIATOR

For high pass RC circuit of time constant is very small in comparison with the time required for the input signal to make an appreciable change, the circuit is called differentiator.

Under this case, the drop across R is negligible compared to drop across C. Hence the total input vi(t) appears across C.

The current i is given

$$i(t) = C\frac{dvi(t)}{dt}$$

Hence the output which drops across R is

$$V_o = iR$$

$$V_o(t) = RC\frac{dvi(t)}{dt}$$

The output is proportional to the derivative of the input. A criteria for good differentiation in terms of steady sate sinusoidal analysis is that if a sinusoidal is applied to the high pass RC circuit, the output will be a sine wave shifted by a leading angle θ such that $\tan\theta = \frac{\alpha c}{R}\frac{1}{wRC}$ the output will be proportional to sin (w_t + θ). In order to have true differentiation we must obtain cos w_t. In other words θ must be equal to 90^0. This result can be obtained only if R = 0 or C = 0. However if ω RC = 0.01, then 1/ωCR = 100 and θ = 89.4^0 and for some applications this may be close enough to 90^0.

If the peak value of input is V_m, the output is

$$V_o = \frac{V_m R}{\sqrt{R^2 + \frac{1}{w^2 C^2}}} \sin(\omega t + \theta)$$

and if ωRC <<1, then the output is approximately $V_m \omega RC \cos\omega t$. This results agrees with the expected value RC $\frac{dvi(t)}{dt}$. If ωRC = 0.01 then the output amplitude is 0.01 times the input amplitude.

These facts prove that with a small time constant the high pass RC circuit behaves as a differentiator.

The time constant RC of the circuit should be much smaller than the time period of the input signal RC<<T.

Application: RC>>T is employed in R-C completely of amplifier where distortion and differentiation of waveform is to be avoided, multi libratory, flip flap

1.11 LOW-PASS RC CIRCUIT

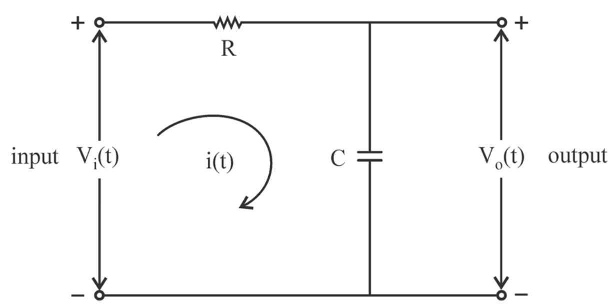


FIGURE 1.21 low pass RC circuit

Fig.1.21 shows a low pass RC circuit. The circuit passes the low frequencies readily, but attenuates high-frequencies because the reactance of the capacitor C decreases with increasing frequency. At very high frequencies the capacitor acts as virtual short-circuited and the output fall to zero. Thus, the high frequencies get attenuated. At zero frequency the reactance of the capacitor is infinity (capacitor is open circuit).

Sinusoidal input:

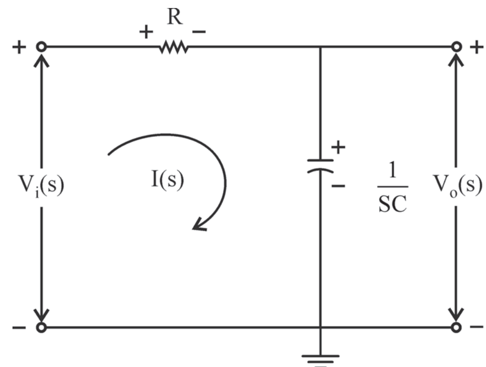


FIGURE 1.22 low RC circuit Laplace

If the input voltage is sinusoidal Vi(t) expressed as,

$$Vi(t) = V_m \sin \omega t$$

It can make use of the Laplace transform and analyse the circuit in s-domain. Since there is no change on the capacitor.

Applying KVL to the circuit as shown in figure

We can write

$$Vi(s) - I(s)R = \frac{1(s)}{SC} = 0$$

$$Vi(s) = I(s)R + \frac{1(s)}{SC}$$

$$Vi(s) = I(s)\left[R + \frac{1}{SC}\right]$$

$$I(s) = \frac{Vi(s)}{\left[R + \frac{1}{SC}\right]}$$

$$Vo(s) = I(s) \times \frac{1}{SC}$$

I(s) is substituting the Vo(s)

$$\frac{Vo(s)}{Vi(s)} == \frac{1}{\left[R + \frac{1}{SC}\right]} \times \frac{1}{(SC)} = \frac{1}{(SCR + \frac{\cancel{SC}}{\cancel{SC}})}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1 + SRC} \text{ Transfer function}$$

For analysing frequency response replace S by jω

$$\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1}{1+j\omega RC} = \frac{1}{1+j2\pi fRC} \text{ Frequency domain of transfer function}$$

$$A = \left|\frac{V_o(j\omega)}{V_i(j\omega)}\right| = \frac{1}{\sqrt{1+(2\pi fRC)^2}} = \text{ gain of the circuit}$$

At the upper is off frequency f_2, $|A| = \frac{1}{\sqrt{2}}$

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+(2\pi fRC)^2}}$$

$$\frac{1}{2} = \frac{1}{1+(2\pi f_2 RC)^2} \quad \text{Equating denominator}$$

$$2 = 1+(2\pi f_2 RC)^2$$

$$f_2 = \frac{1}{2\pi RC} = \text{upper cut off frequency}$$

cut off zone and from f_2 on wards

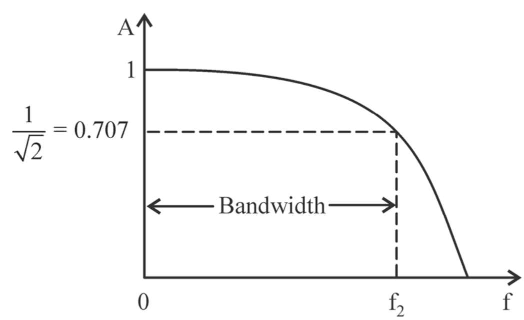


FIGURE 1.23 output wave form

The magnitude of the steady state gain A and the angle θ by which output leads the input is given by

$$A = \frac{1}{1+j\left(\frac{f}{f_2}\right)} \quad \text{and} \quad |A| = \frac{1}{1+\left(\frac{f}{f_2}\right)^2}$$

$$\theta = -\tan^{-1}\left(\frac{f}{f_2}\right)^2 \qquad f_2 = \frac{1}{2\pi RC}$$

It can explain output signal i $V_o(t) = AV_m\sin(\omega t + \theta)$, hence the phase angle θ is Negative

1.12 STEP VOLTAGE INPUT

Consider the step input voltage of magnitude A is applied to the low pass RC circuit having a time constant RC. A step voltage V (t) can be mathematically written as

$$V(t) = \begin{cases} 0 & \text{for } t > 0 \\ V & \text{for } t \geq 0 \end{cases}$$

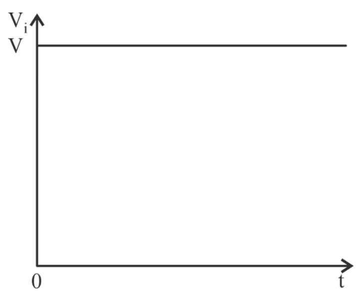


FIGURE 1.24 Step input

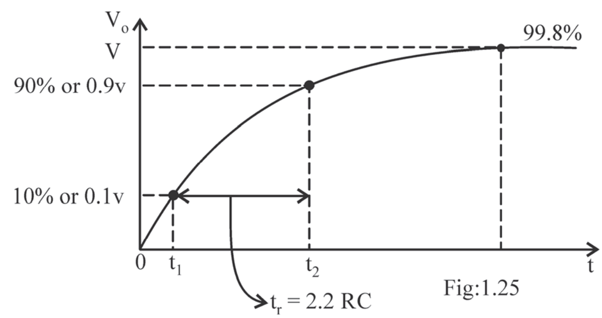


FIGURE 1.25 Step response Low pass RC current

If the capacitor is initially uncharged when a step input voltage is applied. The voltage across the capacitor can't charge instantaneously the output will be zero at t = 0. When the capacitor charges the output, voltage rises exponentially towards the steady state value V with.

Let V^1 is the initial voltage across the capacitor

Writing KVL around loop

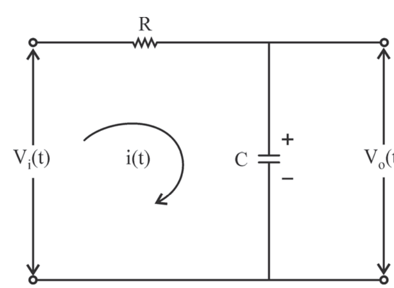


FIGURE 1.26 Low pass RC circuit

$$Vi\ (t) = i(t)\, R + \frac{1}{c}\int i(t)dt$$

Differentiating the equation

$$\frac{dVi(t)}{dt} = \frac{Rdi(t)}{dt} + \frac{1}{c}i(t)$$

$$Vi(t) = V,\ \frac{dVi(t)}{dt} = 0$$

Note

$$\frac{di}{dt} = (\text{final value-intional value})$$

$L(t) = 1/s$

$L(1/t) = s$

Take Laplace transform both side

$$0 = \frac{Rdi(t)}{dt} + \frac{1}{C}i(t)$$

$$R[SI(s) - I(0^+)] + \frac{1}{c}I(s)$$

$L(I(t)) = I(s)$

$$I_{0^+} = Is\left[s + \frac{1}{RC}\right]$$

$$L\left[\frac{di(t)}{dt}\right] = \underset{\text{Final}}{\underset{\downarrow}{SI}}\ (s) - \underset{\text{Initial}}{\underset{\downarrow}{(I_0^+)}}s$$

The initial current I_0^+ is given by

$$I_0^+ = \frac{V - V^1}{R} \qquad I(s) = \frac{I(0^+)}{S + \frac{1}{RC}} = \frac{V - V^1}{R(S + \frac{1}{RC})}$$

$V_0(s) = Vi(s) - I(s)R$

$$= V/S - \frac{(V - V^1)}{\cancel{R}(S + \frac{1}{RC})}\cancel{R} = \frac{V}{S} - \frac{V - V^1}{S + \frac{1}{RC}}$$

Taking Inverse Laplace transform both sides

$$V_0(t) = V - (V - V^1)\ e^{-t/RC}$$

V is the final voltage (v final) when the capacitor is charged

V^1 is the internal voltage across the capacitor

$V_0(t) = V\ \text{final} - (V\ \text{final} - V\ \text{final})e^{-t/RC}$

The capacitor fanatically uncharged than

$$V_0(t) = V(1 - e^{-t/RC})$$

Expression for rise time:

The rise time t_r is defined as the time it take the voltage to rise from 0.1 to 0.9 of its final value. It gives an indication of how fast the circuit can respond to a discontinuity in voltage.

Assuming the capacitor is initially uncharged.

The time required for the output to achieve 10% of its final value can be obtained

$$V_o(t) = V(1 - e^{-t/RC})$$

at $t = t_1$ $V_o(t) = 10\%$ (or) $V = 0.1\ V$

$$0.1\ V = V(1 - e^{-t/RC})$$

$$0.1 = 1 - e^{-t/RC}$$

$$e^{-t1/RC} = \ln 0.9$$

$$\frac{-t_1}{RC} = \ln(0.9) \qquad t_1 = 0.1\ RC$$

Similarly the time required for the o/p to achieve 90% of its final value output

$t = t_2$ $V_o(t) \simeq 90\%$ (or) $V = 0.9V$

$$0.9\ \cancel{V} = \cancel{V}(1 - e^{-t_2/RC})$$

$$0.9 = 1 - e^{-t_2/RC}$$

$$e^{-t_2/RC} = 0.1$$

$$\frac{-t_2}{RC} = \ln(0.1)$$

$$t_2 = 2.3\ RC$$

$$t_r = t_2 - t_1$$

rise time $t_r = 2.3\ RC - 0.1\ RC = 2.2\ RC$

Relation between upper 3 dB frequency and rise time

$$f_2 = \frac{1}{2\pi RC} \text{ (or) } RC = \frac{1}{2\pi f_2}$$

$$\text{Rise time} = 2.2\ RC = \frac{2.2}{2\pi f_2} = \frac{0.35}{f_2} = \frac{035}{BW}$$

Rise time is inversely proportional to the upper 3 dB frequency and directly proportional to the time constant RC.

τ = time constant = RC in RC circuits

1.13 PULSE INPUT VOLTAGE

Consider the pulse input voltage having pulse width t_p, applied as input to the RC circuit the pulse sum the two step voltages the response to a pulse for times less than the pulse width t_p is the same as that for a step input because pulse signal is same as the step input for $t < t_p$. However at the end of the pulse as the input become zero. The output also drops exponentially to zero as capacitor voltage falls exponentially to zero as the input becomes zero.

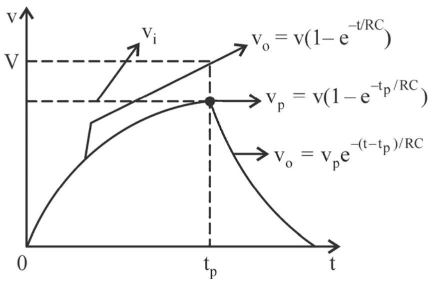


FIGURE 1.27 RC >> t_p

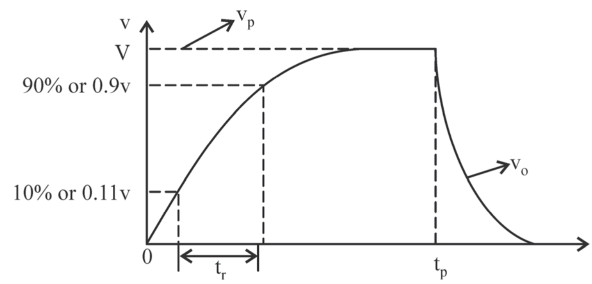


FIGURE 1.28 RC < t_p

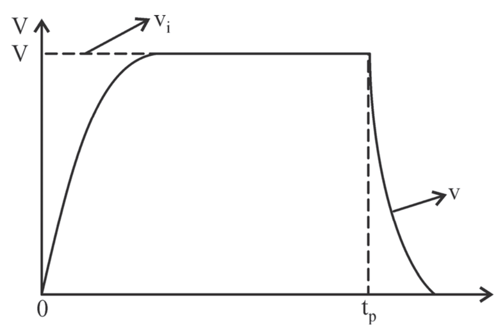


FIGURE 1.29 RC <<t_p

Output for pulse input is given by

$$V_{out} = V(1 - e^{-t/RC})\ t < t_p$$

$$V_{out} = V(1 - e^{-t/RC}) = V_p \text{ (say)}$$

$t = t_p$, input voltage becomes zero but the voltage across a capacitor can't charge instantaneously. Output remain the same as it is $t = t_p$. After that capacitor starts getting discharged through resistance R and voltage across it drops exponentially to zero.

The output voltage $t > t_p$

$$V_{out} = V_p\, e^{-(t-tp)/RC}$$

The above equation, discharging equation of capacitor delayed by time t_p. The output voltage must be decreasing towards to zero.

The output voltage will always extend beyond pulse width t_p. This is because charge stored on capacitor during pulse cannot leak off instantaneously.

To minimize the distortion, the resistance must be small compared with the pulse width t_p.

$$f_2 = \frac{1}{t_p} \quad t_r = 0.35 f_2$$

The upper 3-dB frequency f_2 is chosen equal to the reciprocal of the pulse width t_p.

1.14 SQUARE WAVE INPUT

Consider a periodic waveform whose instantaneous value is constant at 'V' with respect to ground for a V"T_1 and changes abruptly for time T_2 at regular interval $T = T_1 + T_2$. A reasonable reproduction of the input is obtained if the resistance tr is small compared with the pulse width.

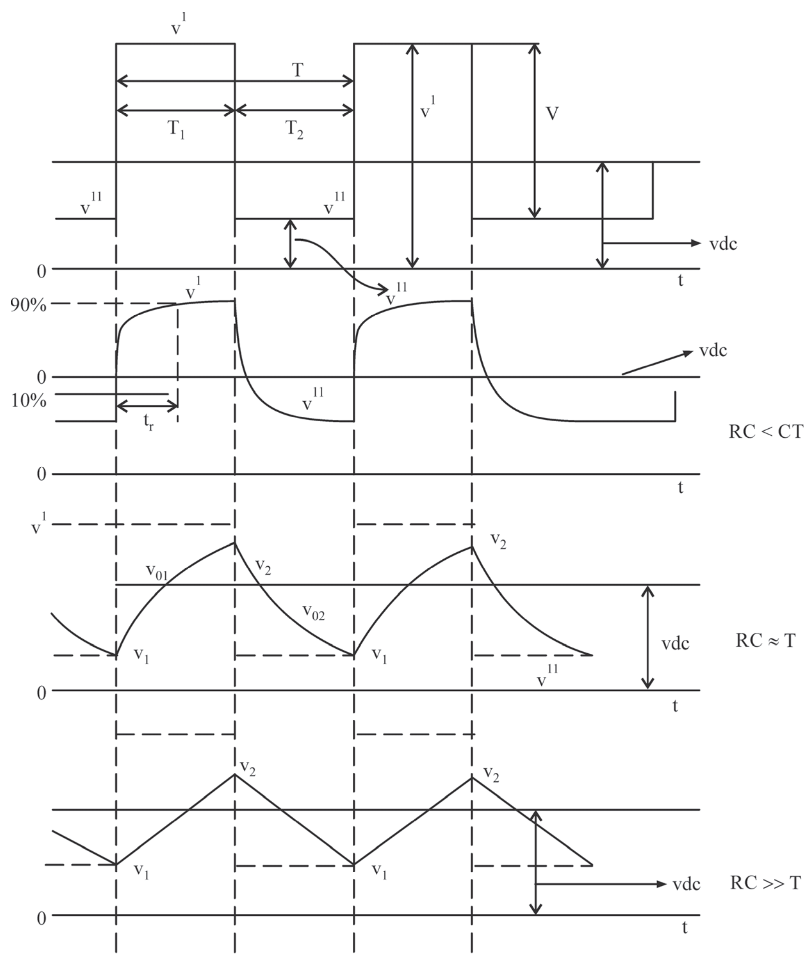


FIGURE 1.30 Different time constant of square output waveform

The steady state response is drawn in fig (b)

If the time constant RC is comparable with the period of the input square wave, the output will have the appearance shown in fig (c) if the time constant is very large compared with the input wave period, the output is exponentially linear as illustrated in fig (d).

In fig (c) rising portion of the equation

$$V_{o1}(t)V' - (V' - V_2)e^{-t/RC}$$

Where V_2 is the voltage across the capacitor at t = 0 and V' is the level of the capacitor charge.

So, the output voltage for $0 < t < T_1$

$$V_{o1}(t)V' + (V_1 - V')e^{-t-/RC}$$

Similarly for $T_1 \le t \le T_2$, if intial voltage across capacitor is V_2 and input voltage is constant at it V'' and output voltage = V_{o2}

$$V_{o2} - V'' + (V_2 - V'')e^{-(t-T_1)RC}$$

$$t = T_1,\ V_{o1} = V_2$$

$$t = T_1 + T_2,\ \ V_{o1} = V_2 = V^1 + (V_1 - V')e^{-T_1/RC}$$

then $\quad t = T_1 \quad V_{o2} = V_1$

$$V_2 = V' + (1 - 0^{-T_1/RC}) + V_1 e^{-T/RC}$$

$$V_1 = V'' + (V_2 - V'')e^{-T_2/RC}$$

For symmetrical wall

$$V_1 = -V_2, \quad V' = -V''$$

$$T_1 = T_2 = T/2$$

$$V_1 = -V' + (-V_1 + V')e^{-T/RC_2}$$

$$V_1 = V' - V_1 e^{-T/RC_2} + V' e^{-T/RC_2}$$

$$V_1 = (1 + e^{-T/2RC}) = V'(e^{-T/RC_2} - 1)$$

$$V_1 = \frac{V'(e^{-T/RC_2} - 1)}{e^{-T/RC_2} + 1}$$

Input square wave of peak to peak voltage V

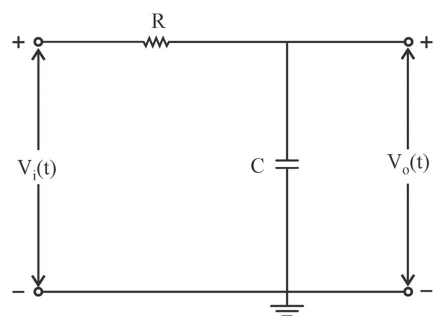
$$V' = V/2$$

$$\frac{V}{2} = \frac{(e^{-T/RC_2} - 1)}{(e^{-T/2RC} + 1)}$$

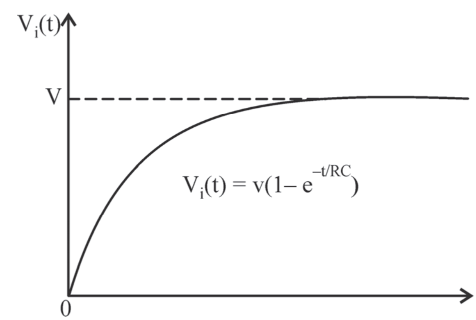
$$V_2 = \frac{-V(e^{-T/2RC} - 1)}{2(e^{-T/2RC} + 1)}$$

$$= \frac{V}{2}\left[\frac{1 - e^{-T/2RC}}{1 + e^{-T/2RC}}\right]$$

1.15 EXPONENTIAL INPUT : (LPF)



(A)



(B)

FIGURE 1.31 The exponential wave form (a,b)

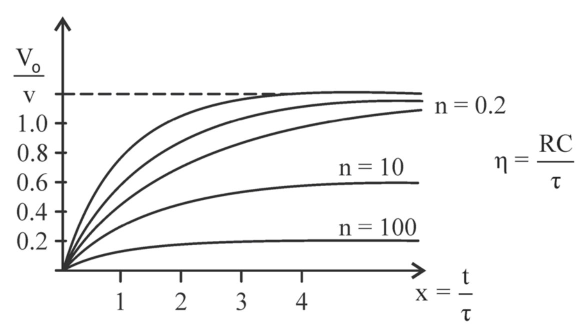


FIGURE 1.32 The output of the RC low pass circuit

apply KVL to the circuit we can write

$$Vi(t) = i(t)R + \frac{1}{c}\int_0^t i(t)dt$$

$$V_i(t) = RC + \frac{dV_o}{dt} + V_o(t)$$

$$V(1-e^{-t/\tau}) = RC \times \frac{dv_o(t)}{dt} + V_o(t)$$

Apply Laplace transform both sides

$$\frac{V}{s} - \frac{V}{s+1/\tau} = RCSV_o(s) + V_o(s)$$

$$V_o(s) = \frac{1}{RC\tau s(s+1/\tau)(s+1/RC)}$$

$$V_o(s) = \frac{V}{S} + \frac{V}{(\frac{RC}{\tau}-1)(S+\frac{1}{\tau})} - \frac{V}{(1-\frac{\tau}{RC})(S+\frac{1}{RC})}$$

$$X = \frac{t}{\tau} \text{ and } n = \frac{RC}{\tau}$$

Substituting expression, we obtain the output waveform in both the cases.

$$V_o(t) = \frac{V}{S} + \frac{V}{(n-1)(S+\frac{1}{\tau})} - \frac{V}{(1-\frac{1}{n})(S+\frac{1}{RC})}$$

Taking in inverse Laplace transform on both side

$$V_o(t) = V\left[1 + \frac{1}{(n-1)}e^{-t/\tau} - \frac{n}{(n-1)}e^{-t/RC}\right]$$

$$V_o(t) = V\left[1 + \frac{1}{(n-1)}e^{-x} - \frac{n}{(n-1)}e^{-x/n}\right] \qquad \text{for } n \neq 1$$

This equation is not valid when n = 1

We can find the expression for output n = 1by using L'Hospital Rule

$$V_o(t) = \frac{\lim\limits_{n\to 1} \frac{d}{dn}\left[V(n-1+e^{-x} - ne^{-x/n}\right]}{\lim\limits_{n\to 1} \frac{d}{dn}(n-1)}$$

$$V_o(t) = \lim_{n\to 1} \frac{\left[V(1-e^{-x/n}) - V_n\left[e^{-x/n}\left(\frac{-x}{n^2}\right)\right]\right]}{1}$$

$$V_o(t) = \lim_{n\to 1}\left[V(1-e^{-x/n}) - V_n\left[e^{-x/n}\left(\frac{-x}{n^2}\right)\right]\right]$$

$$V_o(t) = V(1+(1+x)e^{-x}) \qquad \text{for } n = 1$$

τ = is the input time constant

RC is the circuit time constant

x may be tread normalised time x may be interpreted as the normalised time constant

$$\frac{x}{n} = \frac{t}{RC}$$

$$V_o(t) = \left[1 + \frac{1}{n-1} e^{-x} - \frac{n}{n-1} e^{-x/n}\right] \qquad \text{for } n \neq 1$$

$$V_o(t) = 1\left(1 + (1+x)e^{-x}\right) \qquad \text{for } n = 1$$

If the time constant of this response is τ then the rise time of this exponential waveform can be written as $t_r = 2.2\tau$.

1.16 LOW PASS RC CIRCUIT AS AN INTEGRATOR

For low pass RC circuit, if the time constant is very large when compared to the time required by the input signal to make an appreciable change compared, the circuit acts as an integrator. The voltage drops across C will be very small in comparison to the drop across R and it may consider that the total input appears across R, then the current is Vi(t)/R and the output signal across C is

$$V_o(t) = \frac{1}{C}\int i(t)dt = \frac{1}{C}\int \frac{vi(t)}{R}dt = \frac{1}{RC}\int vi(t)dt$$

Hence the output is proportional to the integral of the input

$$Vi(t) = \alpha t,\ \text{the result is } \alpha t^2 / 2RC$$

$$Vo(t) = \frac{\alpha t^2}{2RC}$$

As time increases, the drop across will not remain negligible compared with that across R and the output will not remain the integral of the input. The output will change from quadrate to a linear function of time.

Low pass RC circuit time constant is very large in compression with the time required for the input signal the circuit acts as a integrator.

Integrator is almost invariably preferred over differentiation in analogue computer application. These resource are given below.

(i) An integrator is less sensitive to noise voltage than a differentiator because of its limited band with.

(ii) It is more convenient to introduce initial conditions in an integrator.

(iii) The differentiator overloads the amplifier if the input changes rapidly. This is not the case for an integrator.

(iv) The gain of an integrator decrease as the frequency. Hence easy to stabilise. The gain of the differentiator increase as the frequency, hence suffers from the problem of stability.

1.17 ATTENUATORS

It consider now the simple resistance attenuator which is used to reduce the amplitude of single waveform the single resistance combination of fig (1.33) would multiply the input signal by the ratio a = $\frac{R_2}{R_1 + R_2}$ independent of frequency.

The potential decoder consisting of two resistances R_1 and R_2, used as an attenuator.

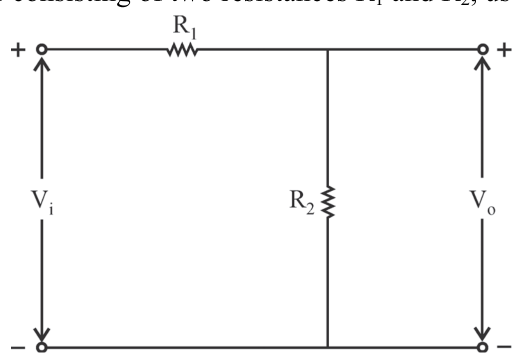


FIGURE 1.33 Simple attenuator

If the output of the attenuator the input capacitance C_2 of the amplifying will be the stray capacitor and attenuator, the resistor R_2 of the attenuator as shown in figure.1.31

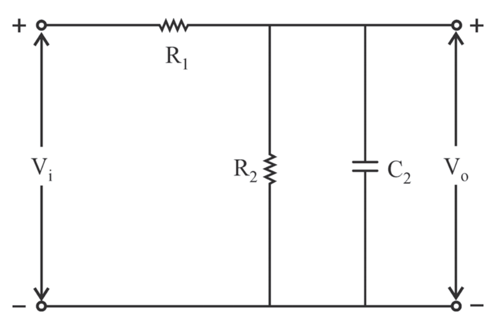


FIGURE 1.34 Actual attenuator

Attenuator equivalent circuit as shown in Fig 1.3

Thevenin voltage is A v_i which attenuated voltage and R is equal to the parallel combination of R_1 and R_2. Generally R_1 and R_2 are very large so that the nominal input impedance of the attenuator may be large enough to prevent loading down the input signal, the time constant RC_2 of the circuit is large which is totally unacceptable. Due to large time constant, resistance is also large which causes destruction. The high frequency components get attenuated. Hence attenuator no longer remains independent of the frequency.

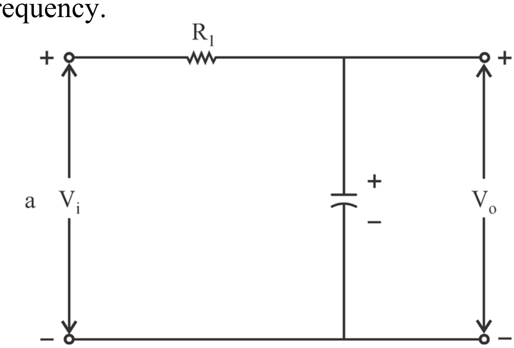


FIGURE 1.35 Attenuator equal at circuit

The attenuator may compensate so that, its attenuation is once again independent of frequency, by shunting R_1 by a capacitor C_1 as shown in figure 1.36.

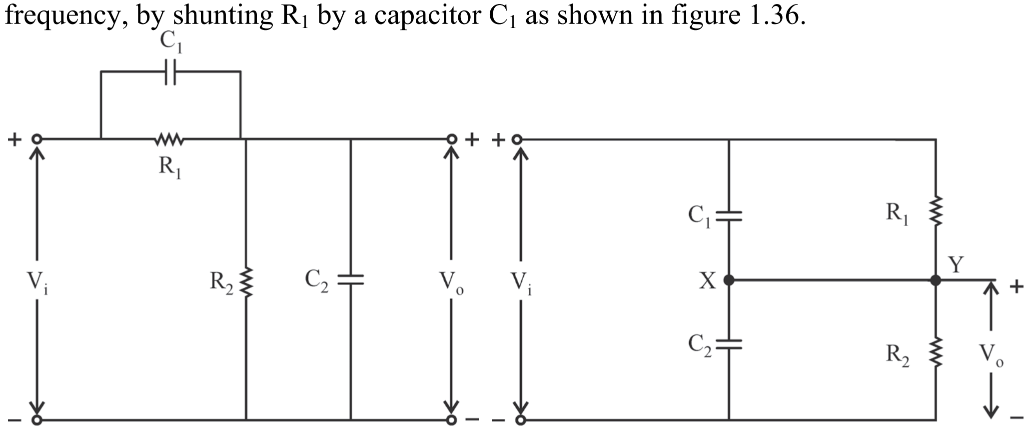


Figure 1.36 (a) (b) Compensated attenuator

The circuit can be redrawn such that the two resistors and two capacitors act as four across of a bridge figure (1.36 (b))

$$R_1C_1 = R_2C_2$$

Under the balanced condition no current can flow through the branched joining the terminal x and y. Hence, for calculating output, the branch x-y can be omitted under balanced bridge condition. This output is equal to v_i independent of frequency.

1.18 STEP INPUT RESPONSE

Let us find out the output waveform, when the step voltage is applied to the compensated attenuator. The step input has amplitude V, applied at t = 0 so the input change from 0 to V instantaneously at t = 0.

Now the voltage across C_1 and C_2 must change abruptly. But the voltage across capacitor cannot charge instantaneously if the current remain finite. Infinite current exists at t = 0. For an infinitesimal time so the finite charge $q = \int_{0-}^{0+} i(t)dt$ is delivered to each capacitor. So just after t = 0 ie at 0^+.

$$A = \frac{q}{c_1} + \frac{q}{c_2} \qquad C = \frac{Q}{V}$$

$$A = q\left[\frac{C_1 + C_2}{C_1C_2}\right] \qquad V = \frac{Q}{C}$$

$$q = A / \left[\frac{C_1 + C_2}{C_1C_2}\right]$$

Output voltage at $t = 0^+$ is voltage across the capacitor C_2 at $t = 0^+$

$$V_0(0^+) = \frac{q}{C_2}$$

$$V_0(0^+) = \frac{AC_1\cancel{C_2}}{(C_1 + C_2)\cancel{C_2}}$$

$$V_0(0^+) = \frac{AC_1}{(C_1 + C_2)}$$

In the steady state as $t \to \alpha$ both the capacitors act as open circuited. Hence, the final value of the output voltage a totally by the resistor

$$V_0(\alpha) = \frac{AR_2}{R_1 + R_2}$$

For the perfect compensation

$$V_0(0^+) = V_0(\alpha)$$

$$\frac{C_2}{C_1 + C_2}\cancel{A} = \frac{R_2}{R_1 + R_2}\cancel{A}$$

$$(R_1+R_2)\, C_1 = R_2\, (C_1 + C_2)$$

$$R_1C_1 = R_2C_2$$

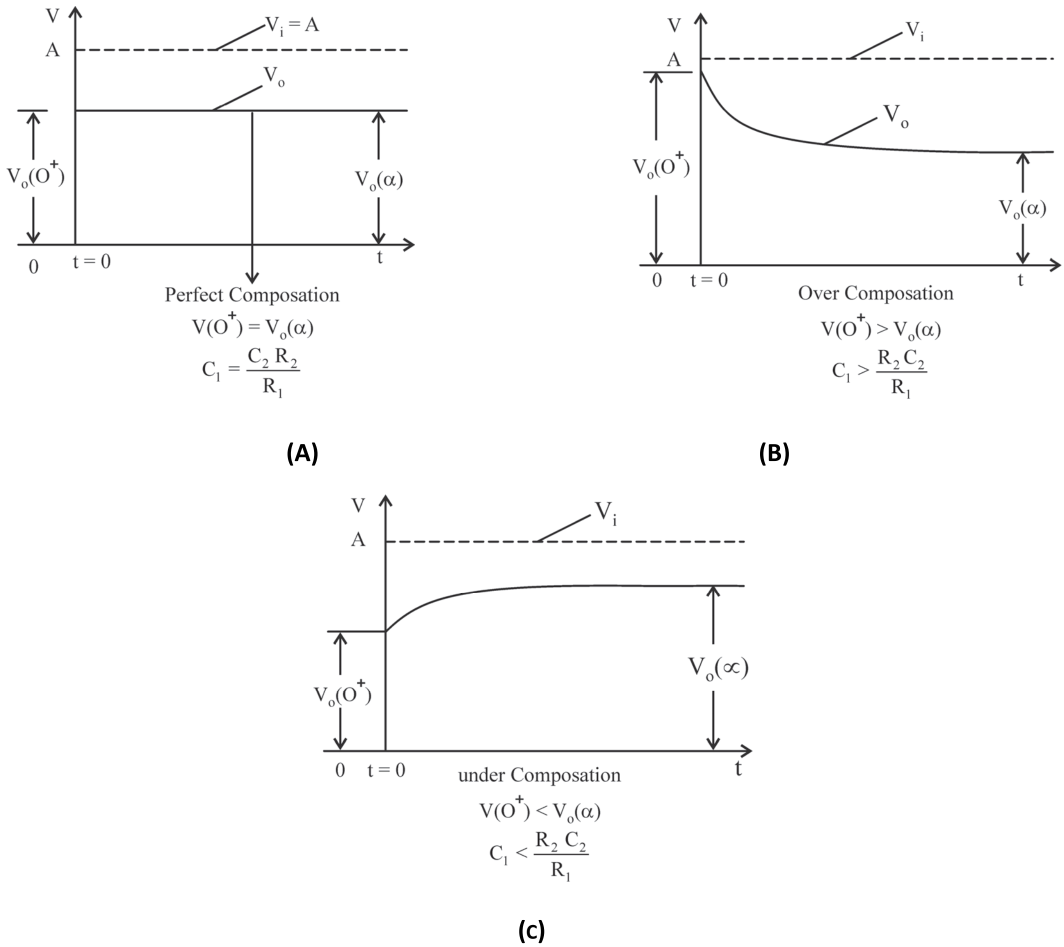


FIGURE 1.37 Different compensator of output waveform a, b, c

1.19 HIGH PASS RC CIRCUIT

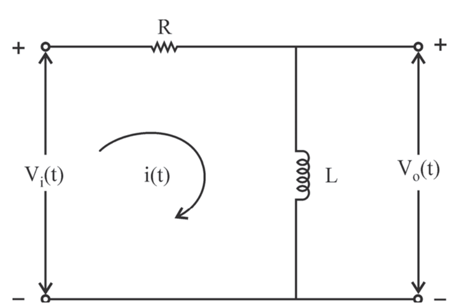


FIGURE 1.38 High pass RL circuit

By applying KVL we can write that

$$Vi(t) = VR(t) + VC(t)$$

$$Vi(t) = i(t)\,R + \frac{Ldi(t)}{dt}$$

Applying Laplace transform on both sides, we can write that

$$Vi(s) = RI(s) + SLI(s)$$

$$I(s) = \frac{Vi(s)}{R + LS}$$

$$V_o(t) + L\frac{di(t)}{dt} \qquad \text{we can write}$$

$$V_o(s) + LsI(s)$$

$$V_o(s) + Ls\frac{vi(s)}{R + LS}$$

$$\frac{V_o(s)}{Vi(s)} + \frac{LS}{R + LS}$$

$$G(s) = \frac{V_o(s)}{Vi(s)} + \frac{LS / R}{1 + LS / R}$$

G(s) as the transfer function of the circuit. Frequency function can be obtained G(f) by replacing $s = j\omega$ $s = j2\pi f$

$$G(f) = \frac{1}{1 - j\dfrac{R}{2\pi fL}} = \frac{1}{1 - j(f_1 / f)}$$

$$G(f) = \frac{1}{1 - (f_1 / f)}$$

Hence f_1 represents the lower cut off frequency

$$f_1 = \ = \frac{1}{2\pi L / R} \qquad \text{G(f) in terms of } f_1$$

we can write $A = \dfrac{1}{\sqrt{1 + (f_1 / f)^2}}$ $\qquad G(f) = |G(f)|\underline{|G(f)} = A\underline{|\phi}$

$$\phi = \tan^{-1}(f_1 / f)$$

Hence A is the magnitude; ϕ is the phase angle of the frequency function.

1.20 STEP INPUT VOLTAGE OF HIGH PASS RL CIRCUIT

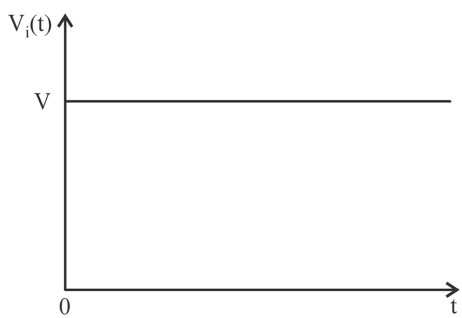


FIGURE 1.39 Step wave form

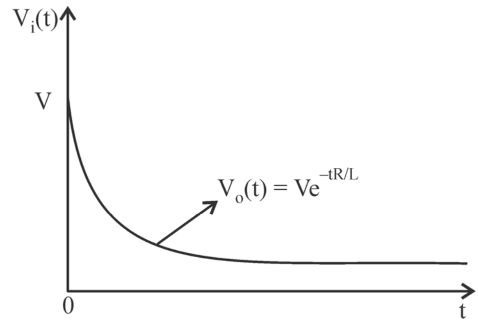


Fig 1.40 Out wave form RL HP circuit

Consider the RC high pass R_2 circuit as shown in fig 39, 40 applied to the step input to the high pass R_2 circuit the step function of amplitude RL can be mathematically writ written as Vi (t) = Vo(t) assume that the initial condition is over zero we know that the Laplace transform of the function

$$Vi(s) = \frac{V}{s}$$

The transfer function of the RC high pass circuit has been obtained

$$G(s) = \frac{Vo(s)}{Vi(A)} = \frac{Ls}{R + Ls} = \frac{S}{S + \frac{R}{L}}$$

$$Vo(s) = Vi(s)\ G(s) = \left(\frac{V}{S}\right)\left(\frac{S}{S + \frac{R}{L}}\right) = \frac{V}{S + \frac{R}{L}}$$

$$Vo(s) = \frac{V}{S + \frac{R}{L}}$$

In time domain equation can be written as

$$Vo(s) = Ve^{-Rt/L}$$

1.21 LOW PASS RL CIRCUIT

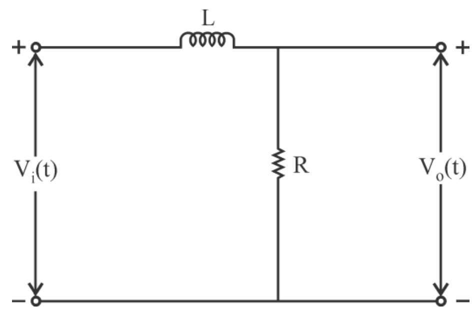


FIGURE 1.41 Low pass RL circuit

By applying KVL we can write that

$$Vi(t) = V_R(t) + V_L(t)$$

$$Vi(t) = i(t)R + \frac{Ldi(t)}{dt}$$

Applying Laplace transform on both side

$$Vi(s) = RI(s) + LSI(s) = Is(R+LS)$$

$$I(s) = \frac{Vi(s)}{R + Ls}$$

$$Vo(t) = i(t)\ R$$

We can write

$$Vo(s) = I(s)R$$

$$Vo(s) = R\frac{Vi(s)}{(R + Ls)}$$

$$G(s) = \frac{Vo(s)}{Vi(s)} = \frac{R}{R + Ls}$$

$$G(s) = \frac{1}{1 + \frac{LS}{R}} \Rightarrow \text{Transfer function of the circuit}$$

S is replaced by $jw = j2\prod f$

$$G(t) = \frac{1}{1 + j2\pi f(\frac{L}{R})}$$

$$G(t) == \frac{1}{1 + j(f / f_2)}$$

Hence f_2 is representing the upper cut off frequency

$$f_2 = \frac{1}{2\pi(L/R)}$$

The frequency function G(t) in terms of f_2

$$G(f) = |G(f)| \underline{|G(f)} = A\underline{|\phi}$$

$$A = \frac{1}{\sqrt{1+(f/f_2)^2}}$$

$$\phi = \tan^{-1}(f/f_2)$$

Hence A is the magnitude

ϕ is the phase angle of the frequency function.

1.22 STEP INPUT VOLTAGE OF LOW PASS RC CIRCUIT

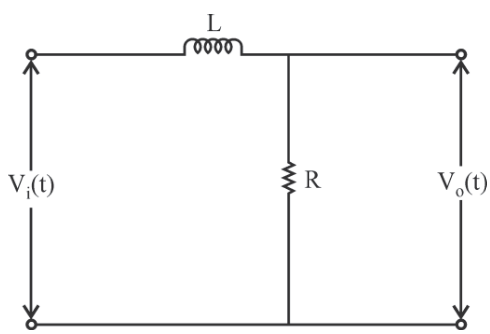


FIGURE 1.42 Low pass RC circuit

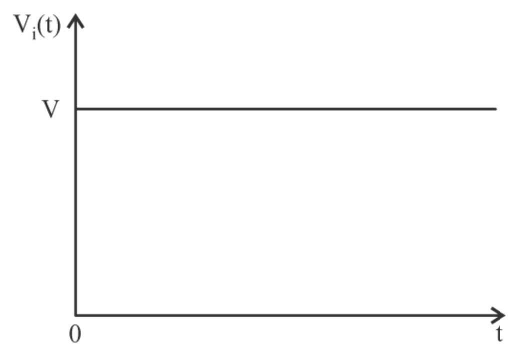


FIGURE 1.43 The step waveform of input

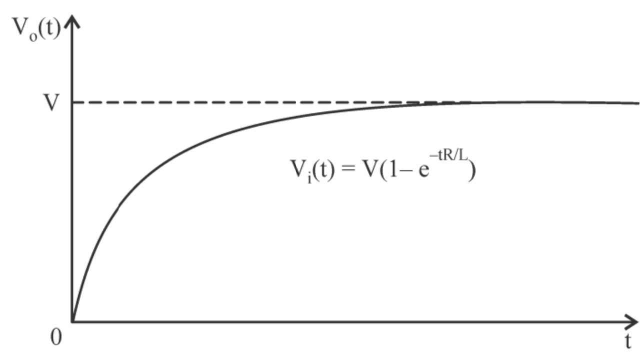


FIGURE 1.44 Output wave form of step

Consider the low pass RC circuit indicated in fig 42, 43, 44, step input voltage is applied to the RL low pass circuit. The step function of amplitude V can be mathematically written as Vi(t) = V_o(t).

Assume that the initial condition is zero. So that we can obtain the tranfer function of the circuit. We know the Laplace transform of the this function

$$\mathrm{Vi(s)} = \frac{\mathrm{V}}{\mathrm{s}}$$

$$\mathrm{G(s)} = \frac{\mathrm{Vo(s)}}{\mathrm{Vi(s)}} = \frac{\mathrm{R}}{\mathrm{R+LS}} = \frac{\frac{\mathrm{R}}{\mathrm{L}}}{\mathrm{S}+\frac{\mathrm{R}}{\mathrm{L}}}$$

$$\mathrm{Vo(s)} = \mathrm{Vi(s)\ G(s)} = \left(\frac{\mathrm{V}}{\mathrm{s}}\right)\times\left(\frac{\mathrm{R/L}}{\mathrm{S+R/L}}\right)$$

$$\left(\frac{\mathrm{R/L}}{\mathrm{S(S+R/L)}}\right) = \mathrm{V}\left(\frac{1}{\mathrm{S}}+\frac{1}{\mathrm{S}+\frac{\mathrm{R}}{\mathrm{L}}}\right)$$

$$\mathrm{Vo(s)} = \mathrm{V}\left(\frac{1}{\mathrm{S}}+\frac{1}{\mathrm{S}+\frac{\mathrm{R}}{\mathrm{L}}}\right)$$

In time domain this equation can be written as

$$\mathrm{Vo(t)} = \mathrm{V}(1-e^{-\mathrm{Rt/L}})$$

1.23 RINGING CIRCUIT

The RLC circuit which produces nearly undamped oscillations is called ringing circuit. The RLC circuit undamped ratio ξ reduces, the oscillation responses increases.

When ξ tends to zero the circuit oscillations for long time and performes many cycles, the oscillations reduces ϕ = is ringing circuit value, N is the number of cycles

$$\mathrm{Q} = \pi\mathrm{N}$$

$$\mathrm{N} = \frac{\mathrm{Q}}{\pi}$$

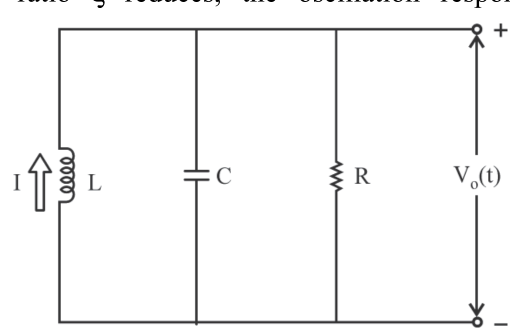


FIGURE 1.45 Ringing circuit

The ringing circuit as shown in figure 1.45

For initially capacitance is uncharged and inductor carries an initial current I.

When damping is made very small the output becomes undamped and takes the form of sine wave which on oscillated in magnetic energy gets stored in an inductor during one part of the cycle. It is converted into electrostatic energy stored in capacitor during next part of cycle.

Then amplitude of the oscillation is

$$\frac{1}{2}cv^2{}_{max} = \frac{1}{2}LI^2$$

$$V_{max} = I\sqrt{\frac{L}{C}}$$

Application: The ringing circuit is used to generate the sequence of pulse.

1.24 RLC SERIES CIRCUIT

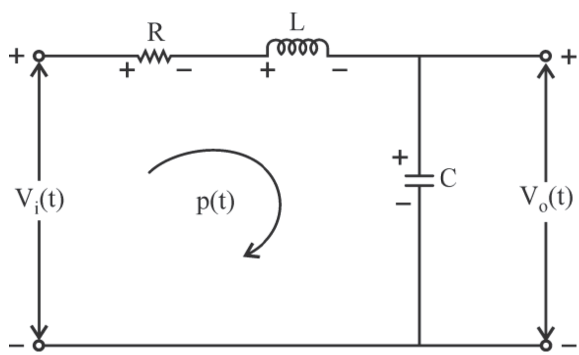


FIGURE 1.46 RLC series circuit

RLC series circuit is shown in fig 1.46

The output taken across capacitor 'C'

Applying KVL Loop

$$Vi(t) - i\,(t)\;R - \frac{Ldi(t)}{dt} - \frac{1}{c}\int i(t)dt = 0$$

Take Laplace transform of the above equation. Initially capacitor is unchanged, inductor current is zero

$$Vi(s) = I(s)R + LSI(s) + \frac{1}{c}\frac{I(s)}{S}$$

$$Vi(s) = I(s)(R + LS) + \frac{1}{cS}$$

$$I(s) = \frac{Vi(s)}{R + LS + \frac{1}{cS}}$$

From the circuit the output equation is

$$V_o(t) = \frac{1}{C}\int i(t)dt$$

$$V_o(s) = \frac{I(s)}{SC}$$

I(s) substitutes the above of output equation

$$V_o(s) = \frac{1}{SC}\left[\frac{VI(s)}{R + LS + \frac{1}{SC}}\right]$$

$$\frac{V_o(s)}{Vi(s)} = \frac{1}{S^2LC + SRC + 1}$$

Numerator and denominator divided by the $\frac{1}{LC}$

$$\frac{V_o(s)}{Vi(s)} = \frac{\frac{1}{LC}}{S^2 + \frac{RS}{L} + \frac{1}{LC}}$$

The ratio of Vo(s) to Vi(s) is called transfer function in the circuit.

The equation obtained by equating denominator polynomial of a transform function is zero.

$$S^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$S_1, S_2 = \frac{-\frac{R}{L} \pm \sqrt{\left[\frac{R}{L}^2\right] - \frac{4}{LC}}}{2} \qquad \text{Note: } ax^2 + bx + c \quad \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= -\frac{R}{2L} \pm \sqrt{\left[\frac{R^2}{2L}\right]^2 - \frac{1}{LC}}$$

Critical resistance RCr:

This resistance of value which reduce square root term to zero. Giving real, equal and negative roots.

$$\frac{RC}{2L} = \sqrt{\frac{1}{LC}}$$

$$RC_r = 2\sqrt{\frac{L}{C}}$$

(i) Damping ratio (ξ): it is denoted by Greek letter zeta (ξ)

The ratio of a dual resistance in the circuit to the critical resistant.

$$\xi = \frac{R}{RC_r} = \frac{R}{2}\sqrt{C/L}$$

(ii) Characteristic independence: the term $\sqrt{L/C}$ is called characteristic independence

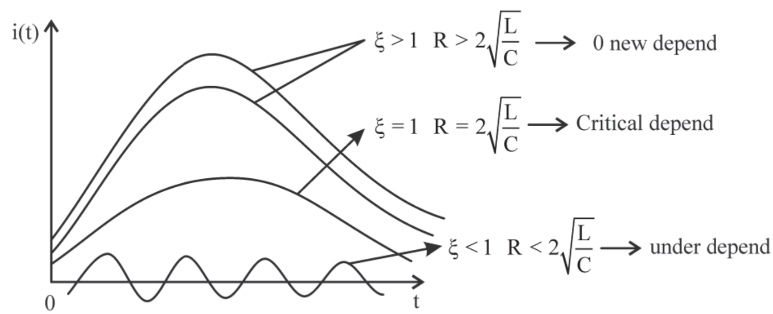


FIGURE 1.47 RLC Series circuit of current response

(iii) Natural frequency w(n)

$$\omega(n) = \frac{1}{\sqrt{LC}}$$

1.25 RLC PARALLEL CIRCUIT

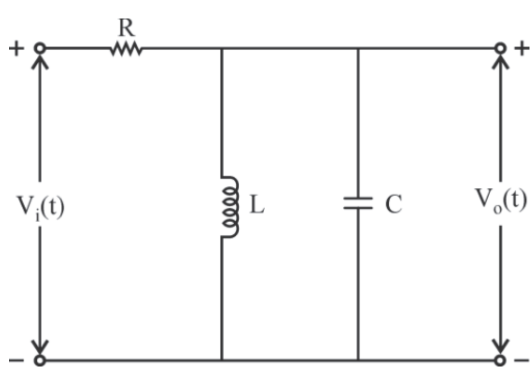


FIGURE 1.48 RLC parallel circuit

SOLVED EXAMPLES

EXAMPLE 1.1

An oscilloscope test probe is indicated in the fig. 1.49. Assume that cable capacitance is 100 pF. The input impedance of the scope is 2 MΩ in parallel with 10 pF. What is,

a) Attenuation of the probe and
b) C for best response

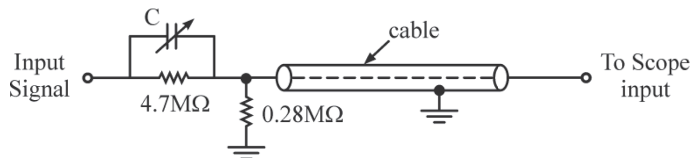


Figure 1.49

SOLUTION

Considering the cable capacitance and the scope input impedances equivalent circuit can be obtained as shown in the fig. 1.50.

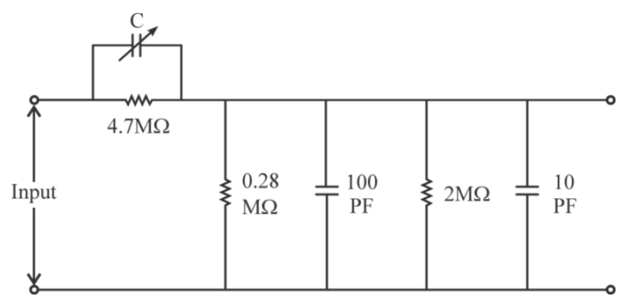


Figure 1.50

Combining the resistances in parallel and capacitors in parallel.

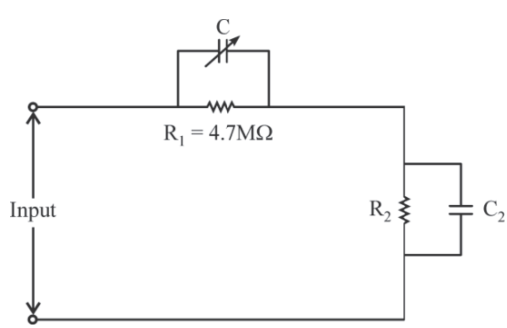


Figure 1.51

$$R_2 = \frac{0.28 \times 2}{0.28 + 2} = 0.2456\ M\Omega$$

$$C_2 = 100 + 10 = 110\ pF$$

$$R_1 = 4.7\ M\Omega$$

a) The attenuation of the probe is,

$$a = \frac{R_2}{R_1 + R_2}$$

$$= \frac{0.2456 \times 10^6}{0.2456 \times 10^6 + 4.7 \times 10^6} = 0.04966$$

b) C for best response is,

$$C = \frac{R_2}{R_1} \times C_2$$

$$= \frac{0.2456 \times 10^6}{4.7 \times 10^6} \times 110 \times 18^{-12}$$

$$= 5.748\ pF$$

EXAMPLE 1.2

A step input of 10 V when applied to the low pass RC circuit produces the output with a rise time of 200 µsec. Calculate the upper 3-dB frequency of the circuit. If the circuit uses a capacitor of 0.47 µF, determine the value of the resistance.

SOLUTION

The rise time of the output is given by the equation,

$$t_r = \frac{2.2}{2\pi f_2} \quad \text{where } f_2 \text{ is upper 3-dB frequency}$$

$$f_2 = \frac{2.2}{2\pi t_r} = \frac{2.2}{2\pi 200 \times 10^{-3}}$$

$$= 1.75\ KHz$$

Now $$f_2 = \frac{1}{2\pi RC}$$

$$1.75\ KHz = \frac{1}{2\pi R \times 0.47 \times 10^{-6}}$$

$$R = 193.5\ \Omega$$

EXAMPLE 1.3

A 10 KHz square wave is applied to high pass RC circuit which produces the output with a tilt of 3.8%. Calculate the lower 3-aB frequency of the circuit. If the circuit uses a capacitor of 0.47 μF, determine the value of the resistance.

SOLUTION

The % tilt in the output is given by the equation, % P $= \frac{\pi f_1}{f} \times 100$ where f_1 is lower 3-dB frequency

$$0.038 = \frac{\pi f_1}{10 \times 10^3}$$

$$f_1 = \frac{10 \times 10^3 \times 0.038}{\pi} = 120.95\text{Hz}$$

Now $$f_1 = \frac{1}{2\pi RC}$$

$$120.95 = \frac{1}{2\pi \times R \times 0.47 \times 10^{-6}}$$

$$R = 2.8\ \text{K}\Omega.$$

EXAMPLE 1.4

A 1 KHz symmetrical square wave of ± 10 V is applied to RC circuit having 1 msec time constant. Calculate and plot the output to the scale for RC configurations as,

(i) High pass circuit

(ii) Low pass circuit

SOLUTION

(i) High pass RC circuit

The general response of high pass RC circuit to square wave input is described by the equations,

$$V_1 = A_1 e^{-T_1/RC}$$

$$A_2 = V_1^{\,1} - A$$

$$V_2^{\,1} = A_2 e^{-T_2/RC}$$

$$A_1 = V_2^{\,1} + A$$

For symmetrical square wave,

$A_1 = - A_2$

$V_1{}^1 = - V_2{}^1$

$T_1 = T_2 = T/2$

Substituting this into above equations and solving for A_1 and $V_1{}^1$ we get,

$$A_1 = \frac{A}{1 + e^{-T/2RC}}$$

$$V_1^1 = \frac{A}{1 + e^{T/2RC}}$$

For a given square wave,

$$T = \frac{1}{f} = \frac{1}{1 \times 10^3} = 1\text{ msec}$$

RC = 1 msec

and $A = 10 - (-10)$

$= 20 =$ peak to peak of input

$$\therefore \quad A_1 = \frac{20}{1 + e^{-0.5}} = 12.45\text{ V}$$

$$\text{and} \quad V_1^1 = \frac{20}{1 + e^{+0.5}} = 7.55\text{ V}$$

$$\therefore \quad V_2^1 = -V_1^1 = -7.55\text{ V}$$

and $A_2 = - A_1 = - 12.45$ V

Hence the output can be shown as in the fig. 1.52

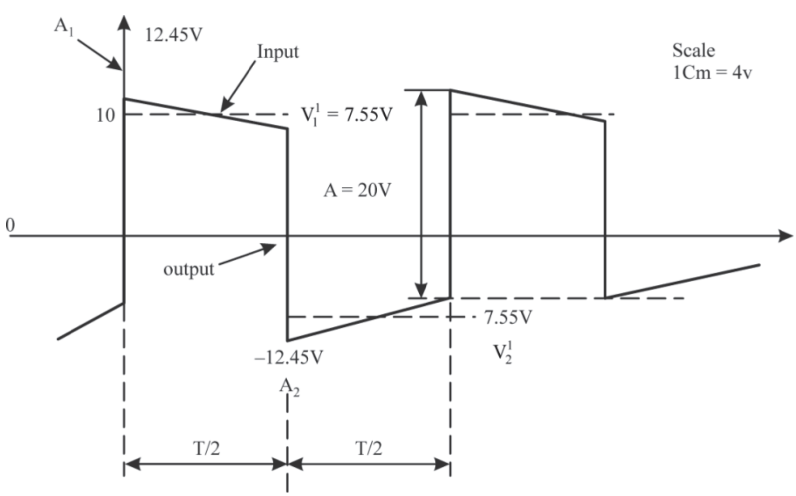


FIGURE 1.52

(ii) Low pass RC circuit

For symmetrical square wave,

$$V_2 = \frac{A}{2}\frac{e^{2x-1}}{e^{2x+1}}$$

where $x = \dfrac{T}{4RC}$

and $V_1 = -V_2$

$$\therefore \quad x = \frac{1\times10^{-3}}{4\times1\times10^{-3}} = 0.25$$

$$\therefore \quad V_2 = \frac{20}{2}\frac{e^{0.5-1}}{e^{0.5+1}} = 7.45\text{ V}$$

and $V_1 = -7.45$ V

the response is shown in the fig. 1.53

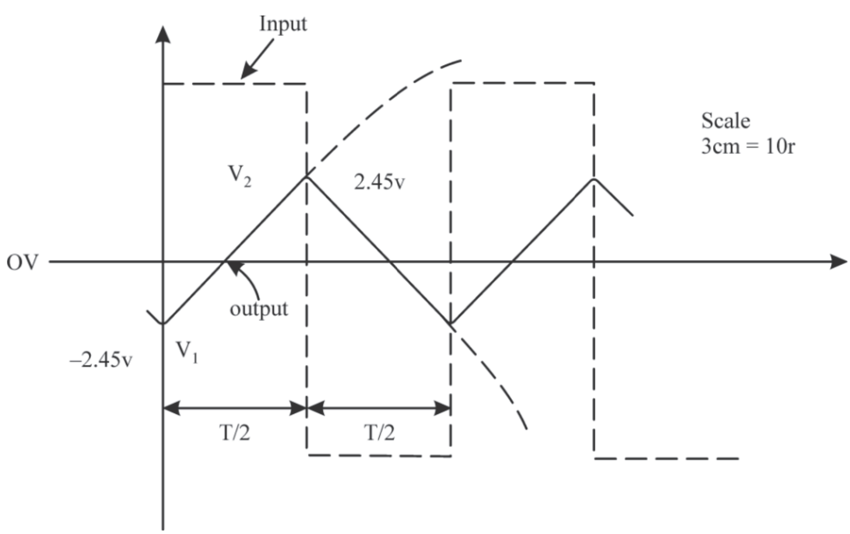


FIGURE 1.53

EXAMPLE 1.5

For the attenuator circuit shown in the fig. 1.54, calculate and plot the output for the cases :

a) C_1 50 pF and C_1 = 150pF
   The input V_i is a step of 10V.

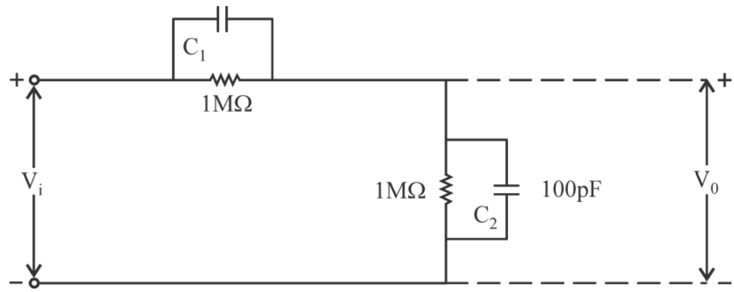


FIGURE 1.54

SOLUTION

From the circuit above

$$R_1 = R_2 = 1 \text{ M}\Omega$$

$$C_2 = 100 \text{ pF}$$

For bridge balance,

$$C_1 = \frac{R_2}{R_1} C_2 = 100 \text{ pF} = Cp$$

$$C_1 = 150 \text{ pF}$$

Case I

The capacitor $C_1 < Cp$, hence it is under compensated.

The input step is of magnitude A = 10

$$\therefore \quad V_0(0+) = \frac{AC_1}{C_1 + C_2} = \frac{10 \times 50}{(50 + 100)} = 3.33 \text{ V}$$

While $$V_o(\infty) = \frac{R_1}{R_1 + R_2} A = \frac{1}{1+1} \times 10 = 5 \text{ V}$$

The rise from $V_0(0+)$ to $V_o(\infty)$ is exponential in nature.

$$R = \frac{R_1 R_2}{R_1 + R_2} = 0.5 \text{ M}\Omega$$

$$C = C_1 + C_2 = 150 \text{ pF}$$

$$\therefore \quad t = RC = 75 \text{ µsec}$$

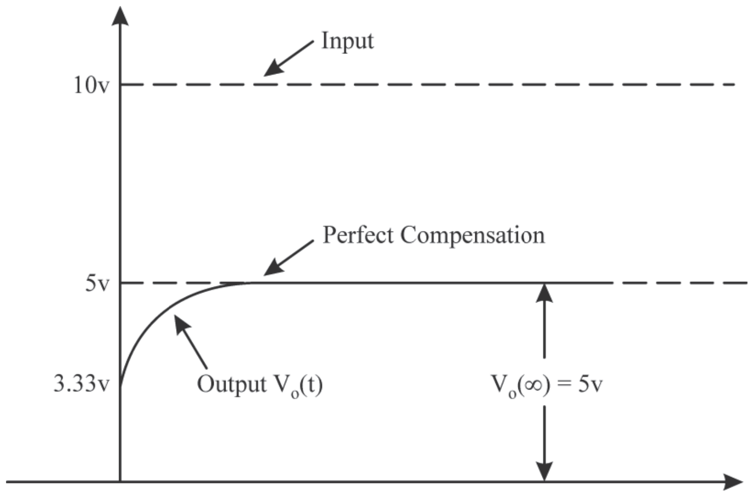


FIGURE 1.55

Case II

C_1 150 pF

The capacitor $C_1 > C_p$, hence it is over compensated

$$\therefore \qquad V_0(0+) = \frac{AC_1}{C_1 + C_2} = \frac{10 \times 150}{(100 + 150)} = 6 \text{ V}$$

and $\qquad V_0(\infty) = 5 \text{ V}$

$$R = \frac{R_1 R_2}{R_1 + R_2} = 0.5\text{M}\Omega$$

$$C = C_1 + C_2 = 250 \text{ pF}$$

Output decays exponentially from $V_0(0+)$ to $V_0(\infty)$ as shown in the fig 1.56

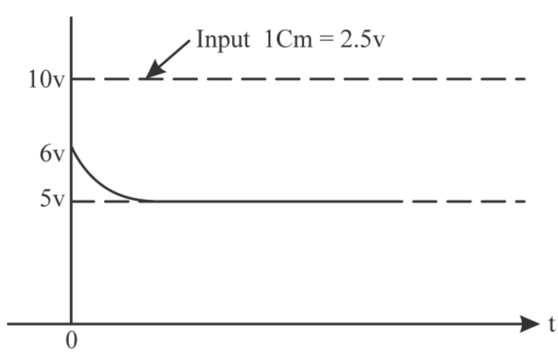


FIGURE 1.56

EXAMPLE 1.6

The input to the attenuator shown in the fig 1.57 is a step of 20 V. Calculate and plot the output for i) perfect compensation and ii) over compensation case.

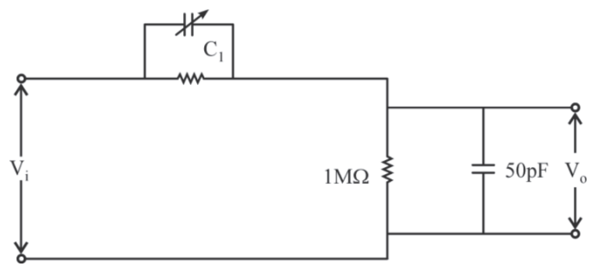


FIGURE 1.57

SOLUTION

From the figure above

$$R_1 = R_2 = 1M\,\Omega$$
$$C_2 = 50 \text{ pF}$$

(i) Perfect compensation

$$C_1 = \frac{R_2}{R_1} C_2 = \frac{1}{1} \times 50 = 50 \text{ pF}$$

The output will be perfect step response

$$a = \frac{R_2}{R_1 + R_2} = \frac{1}{1+1} = 0.5$$

$$\therefore \quad V_0(\infty) = aA = 0.5 \times 20 = 10V$$

The response is as shown in the fig 1.58

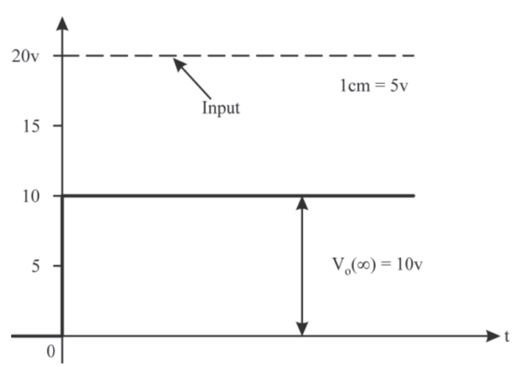


FIGURE 1.58

(ii) Now $Cp = \frac{R_2}{R_1} C_2 = 50 \text{ pF}$

For over compensation $C_1 > Cp$

Let $C_1 = 100\text{pF}$

$$\therefore \quad V_0(0+) = \frac{AC_2}{C_1 + C_2} \quad \text{where } A = 20$$

$$= \frac{20 \times 100}{100 + 50} = 13.33 \text{ V}$$

And $$V_0(\infty) = \frac{AR_2}{R_1 + R_2} = \frac{20 \times 1}{(1+1)} = 10 \text{ V}$$

The response is as shown in the fig 1.59.

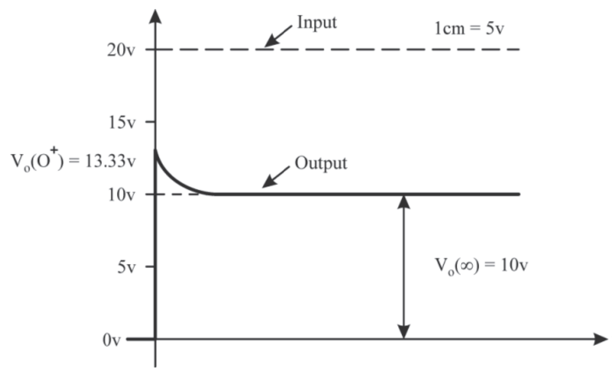


FIGURE 1.59

EXAMPLE 1.7

For the attenuator shown in the fig 1.60draw the output wave forms for C_1 = 50 pF, C_1 = 75pF and C_1 = 25 pF. The input is a 20 V step

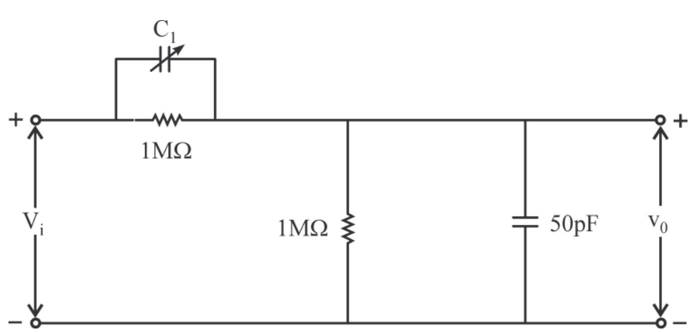


FIGURE 1.60

SOLUTION

From the above figure

$$R_1 = R_2 = 1M\,\Omega$$
$$C_2 = 50 \text{ pF}$$

Consider the various value of C_1. But before that calculate C_p

$$Cp = \frac{R_2}{R_1} C_1 = \frac{1}{1} \times 50 = 50 \text{ pF}$$

Case I $\quad C_1 = 50 \text{ pF} = Cp$

This is perfectly compensated attenuator.

$$\therefore \quad V_0(0+) = V_0(\infty) = aVi$$

$$a = \frac{R_2}{R_1 + R_2} = \frac{1}{2}$$

and $\quad Vi = A = 20$ V

$$\therefore \quad V_0(\infty) = 20 \times \frac{1}{2} = 10r$$

The response is as shown in the fig 1.82

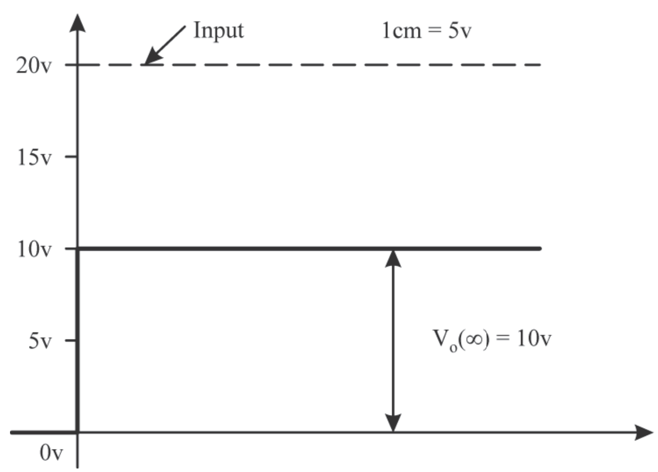


Figure 1.61

Case II $\quad C_1 = 75 \text{ pF} > Cp$

This is over compensated attenuator

$$\therefore \quad V_0(0+) = \frac{AC_1}{C_1 + C_2} = \frac{20 \times 75}{75 + 50} = 12 \text{ V}$$

$$V_0(\infty) = 20 \times \frac{1}{2} = 10 \text{ V}$$

The response is exponential from $V_0(0+)$ to $V_0(\infty)$ as shown in the fig. 1.62.

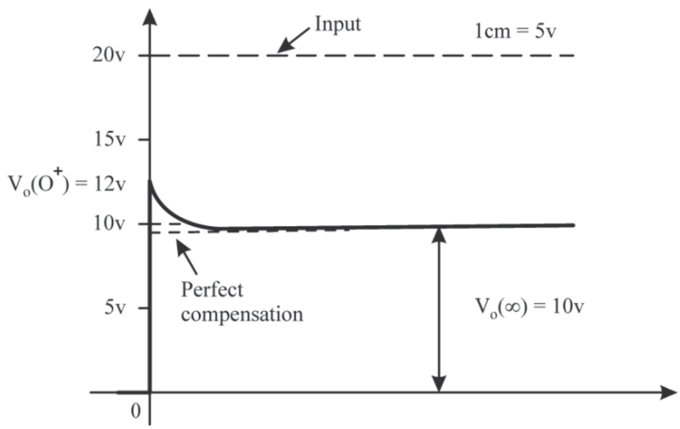


FIGURE 1.62

Case III $\quad C_1 = 25$ pF < Cp

This under compensated attenuator

$$V_0(0+) = \frac{AC_1}{C_1 + C_2} = \frac{20 \times 25}{25 + 50} = 6.67 \text{ V}$$

$$V_0(\infty) = 10 \text{ V}$$

The response is exponentially rising from V_0 (0+) to $V_0(\infty)$ as shown in the fig. 1.62

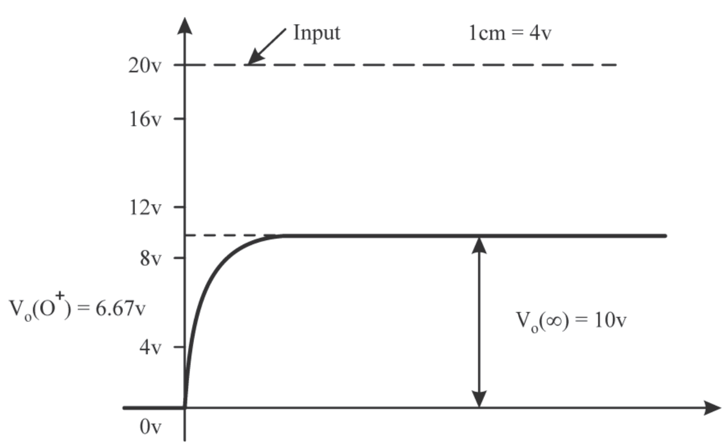


FIGURE 1.63

EXAMPLE 1.8

A 10 Hz square ware is fed to an amplifier. Calculate and plot the output waveform under the following conditions.

The lower 3 dB frequency is i) 0.3 Hz ii) 3 Hz iii) 30 Hz.

SOLUTION

The lower 3 dB frequency indicates that the amplifier acts as a high pass circuit.

F = 10 Hz and f_1 = lower 3dB frequency

i.e., T = 1/f = 0.1 sec.

Let the amplitude of square wave input is A.

$$A_1 = \frac{A}{1+e^{-T/2RC}} \text{ and } V_1^1 = A_1 e^{-T/2RC}$$

(i) $f_1 = 0.3$ Hz

$$\therefore \quad f_1 = \frac{1}{2\pi RC} = \text{i.e } RC = 0.5305$$

$$\therefore \quad A_1 = \frac{A}{1+e^{-0.1/2\times 0.5305}} = 0.5235 \text{ A}$$

$$\therefore \quad V_1{}^1 = 0.5235\text{A } e^{-0.1/2\times 0.5305} = 0.4764 \text{ A}$$

(ii) $f_1 = 3$ Hz

$$\therefore \quad f_1 = \frac{1}{2\pi RC} \text{ i.e., } RC = 0.05305$$

$$\therefore \quad A_1 = \frac{A}{1+e^{-0.1/2\times 0.5305}} = 0.7196 \text{ V}$$

$$\therefore \quad V_1{}^1 = 0.7196 \text{ A } e^{-0.1/2\times 0.5305} = 0.2803 \text{ V}$$

(iii) $f_1 = 30$ Hz

$$\therefore \quad f_1 = \frac{1}{2\pi RC} \text{ i.e., } RC = 5.305\times 10^{-3}$$

$$\therefore \quad A_1 = \frac{A}{1+e^{-0.1/2\times 0.5305\times 10^{-3}}} = 0.999 \text{ V}$$

$$\therefore \quad V_1{}^1 = 0.999 \text{ A } e^{-0.1/2\times 5.305\times 10-3} = 8.059\times 10^{-5} \text{ A}$$

The wave forms are shown in the fig.1.64

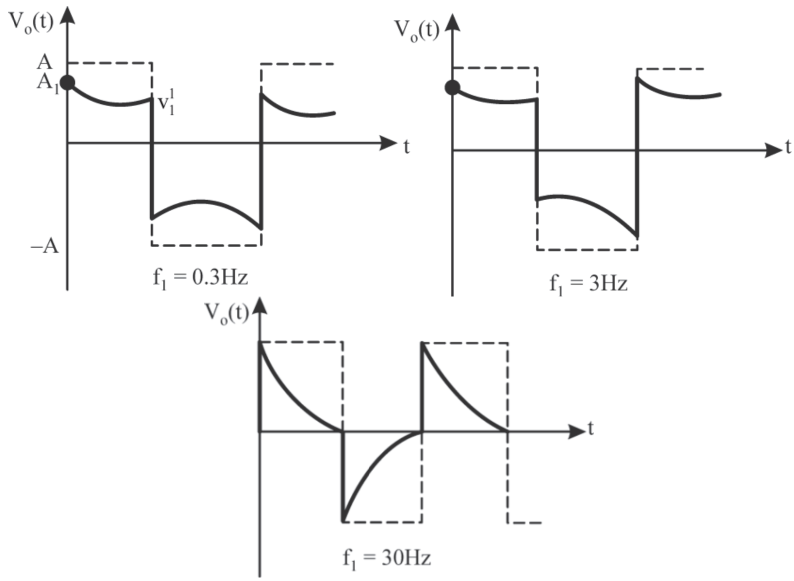


FIGURE 1.64

EXAMPLE 1.9

For a low pass RC circuit, with a pulse input, prove that the area under the pulse is same as the area under the output waveform across the capacitor.

SOLUTION

For a low pass circuit

$$V_{o1}(t) = A(1–e^{-t/RC}).....0 < t < t_p$$

$$V_{o2}(t) = V_p\, e^{-(t-tp)/RC} .....t > t_p$$

Where $V_p = A\,(1– e^{-tp/RC)}$

The waveform is shown in the figure 1.65

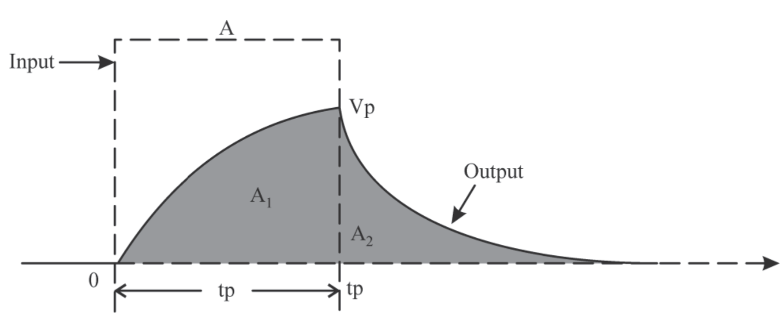


FIGURE 1.65

Area means to find integration

$$\therefore \quad A_1 = \int_0^{t_p} V_{o1}(t)dt = \int_0^{t_p} A(1 - e^{-t/RC})dt = A\left[t + \frac{e^{-t/RC}}{(1/RC)}\right]_0^{t_p}$$

$$= At_p + ARC\, e^{-tp/RC} - ARC = At_p - ARC\,(1 - e^{-tp/RC})$$

$$= At_p - V_p\, RC$$

And
$$A_2 = \int_{t_p}^{\infty} V_{o2}(t)dt = \int_{t_p}^{\infty} V_p e^{-(t-t_p)/RC} dt = V_p e^{t_p/RC}\left[\frac{-e^{-t/RC}}{(1/RC)}\right]_{t_p}^{\infty}$$

$$= V_p e^{tp/RC}\,[0 + RC\, e^{-tp/RC}] = V_p\, RC$$

Thus
$$A_1 + A_2 = At^{-1}{}_p - V_pRC + V_pRC = At_p$$

= Area under the input pulse

= Area under the output curve.

EXAMPLE 1.10

An ideal 1 μs pulse is fed to an amplifier. Calculate and plot the output waveform under the following conditions,

The upper 3 dB frequency is: (i) 10 MHz (ii) 0.1 MHz

SOLUTION

The upper 3dB frequency indicated that an amplifier is a low pass circuit. For pulse input with low pass RC circuit,

$$V_{o1}(t) = A\,(1 - e^{-t/RC})$$

$$V_{o2}\,(t) = V_p\, e^{-(t - tp)/RC}$$

where $V_p = A\,(1 - e^{-tp/RC})$

Given : $t_p = 1\mu s$ and $f_2 = \dfrac{1}{2\pi RC}$

(a) $f_2 = 10$ MHz

$$\therefore \quad RC = \frac{1}{2\pi \times 10 \times 10^6} = 1.5915 \times 10^{-8}$$

$$\therefore \quad V_p = A\,[1 - e^{-1\times 6-10} / 1.5915 \times 10^{-8}] = A$$

The capacitor charges very quickly to A and then discharges. The waveform is shown in the fig. 1.66 (a)

(b) $f_2 = 0.1$ MHz

$$\therefore \quad RC = \frac{1}{2\pi \times 0.1 \times 10^6} = 1.591 \times 10^{-6}$$

$$\therefore \qquad V_p = A\,[1 - e^{-1\times10^{-6}} / 1.591\times10^{-6}] = 0.4665\ A$$

From V_{p1} capacitor discharges according to an equation,

$$V_o\,(t) = 0.4665\ A\ [\,e^{-(t-1\times10^{-6})} / 1.59 \times 10^{-6}]$$

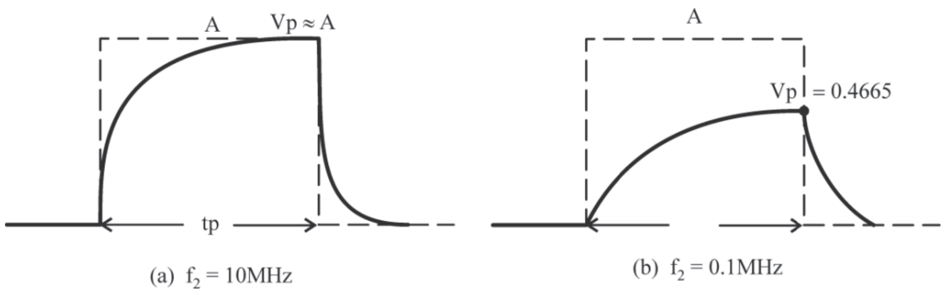


FIGURE 1.66

EXAMPLE 1.11

The limited ramp shown in the fig. 1.67 is applied to a RC differentiator. Draw the wave forms for the case,

(i) T = 0.2 RC ii) T = RC and iii) T = 5 RC.

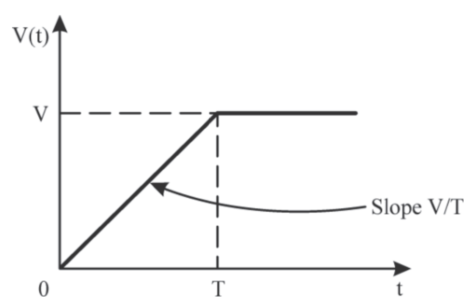


FIGURE 1.67

SOLUTION

The differentiator is high pass RC circuit. The output equation for the ramp of slope α is given by,

$$V_o(t) = \alpha\,RC\,(1 - e^{-t/RC})$$

(i) T = 0.2 RC so output at t = T is,

$$V_o(t) = \alpha\,RC\,(1 - e^{-0.2RC/RC})\ \alpha\,RC \times 0.1812$$

But $\qquad \alpha = \dfrac{V}{T}$ and $RC = \dfrac{T}{0.2} = 5T$

$$\therefore \qquad V_o(t)\Big|_{t=T} = \frac{V}{T}\times 5T\times 0.1812 = 0.9063\ \text{V}$$

After t = T_1 the output falls exponentially.

(ii) T = RC so output at t = T is,

$$V_o(t)\Big|_{t=T} = \frac{V}{T}\times T\times[1-e^{-1}] = 0.6321\ \text{V}$$

(iii) T = 5 RC so output at t = T is,

$$V_o(t)\Big|_{t=T} = \frac{V}{T}\times\frac{T}{5}\times[1-e^{-5RC/RC}] = 0.1986\ \text{V}$$

The waveforms are shown in the fig 1.68

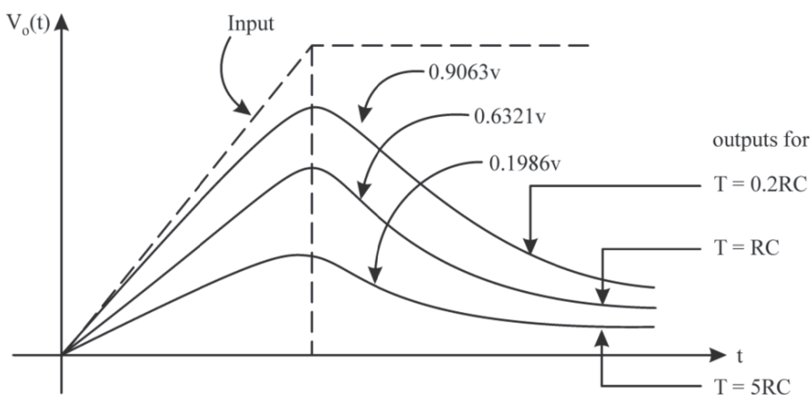


FIGURE 1.68

EXAMPLE 1.12

The periodic waveform shown is applied to an RC integrating circuit whose time constant is 10 μs. Sketch the output. Calculate the maximum and minimum values of the output voltage with respect to ground under steady state conditions.

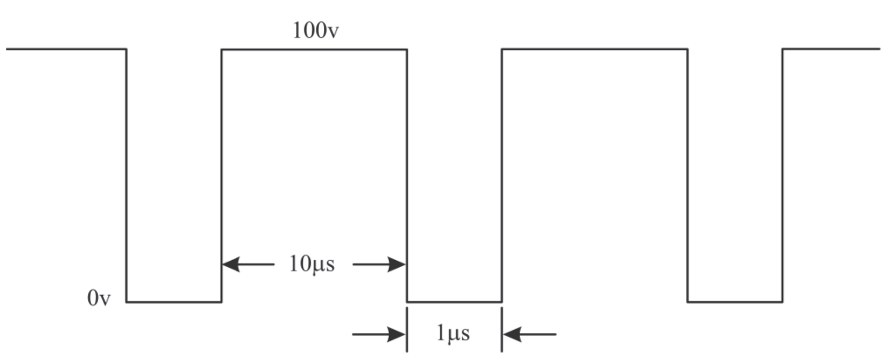


Figure 1.69

SOLUTION

Let $T_1 = 10\ \mu s$ and $T_2 = 1\ \mu s$

When input is 100 V, capacitor charges to 100 V from initial voltage V_2

$$\therefore \quad V_i = V_2,\ V_f = 100\ V$$

$$\therefore \quad V_{o1} = V_f = -(V_f - V_1)\ e^{-t/RC} = 100 - (100 - V_2)\ e^{-t/10\times10^{-6}}$$

At $t = 10\ \mu s$, $V_{o1} = 100 - (100 - V_2)\ e^{-10\times10^{-6}/10\times10^{-6}}$

But at $t = 10\ \mu s$, Vol = V_1 up to capacitor charges.

$$\therefore \quad V_1 = 100 - 100e^{-1} + V_2e^{-1}$$

$$\therefore \quad V_1 = -0.3678V_2 = 63.212$$

During $1\mu s$, the capacitor discharges from V_1 to V_2

$$\therefore \quad Vo_2 = 0 - (0 - V_1)e^{-t/RC}$$

At $t = 1\mu s$; $Vo_2 = V_2$

$$\therefore \quad V_2 - V_1e^{-(1\times10^{-6})/10\times10^{-6}} = 0.9048V_1$$

$V_1 = 94.742V$,

$V_2 = 85.722\ V$.

The wave form is shown in the fig. 1.70

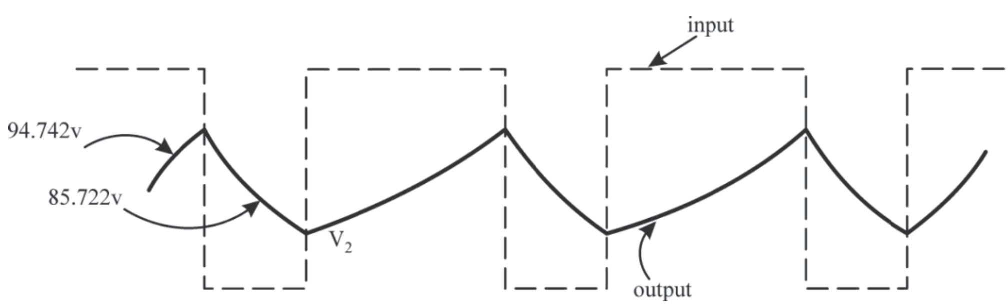


FIGURE 1.70

EXAMPLE 1.13

For a parallel RLC circuit, an input V_i is applied. Derive the Q factor of the circuit.

SOLUTION

The parallel RLC circuit and its Laplace network is shown in fig.1.71

$$Z'(s) = SL11\frac{1}{SC} = \frac{SL \times \frac{1}{SC}}{SL + \frac{1}{SC}}$$

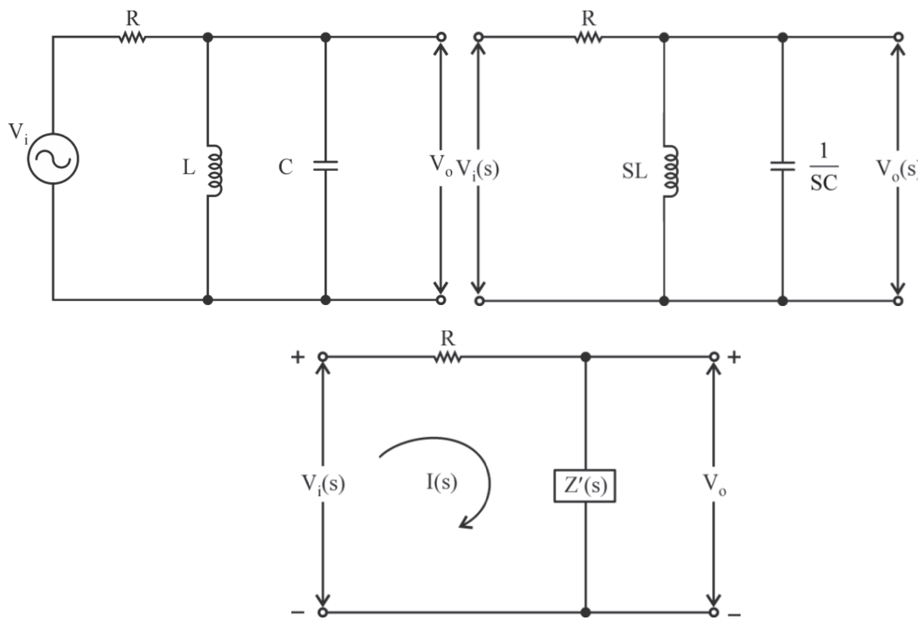


FIGURE 1.71 (a, b, c)

$$I(s) = \frac{V_i(s)}{R + Z'(s)} \text{ and } V_o(s) = I(s)Z'(s)$$

$$\therefore \quad Vo(s) = \frac{Vi(s)}{R + Z'(s)} \times Z'(s) = \frac{Vi(s)}{\left[R + \frac{SL}{S^2LC + 1}\right]} \times \left[\frac{SL}{S^2LC + 1}\right]$$

$$\therefore \quad \frac{Vo(s)}{Vi(s)} = \frac{SL}{S^2RLC + SL + R} = \frac{S\left(\frac{1}{RC}\right)}{S^2 + \frac{1}{RC}S + \frac{1}{LC}}$$

The characteristic equation is,

$$S^2 + \frac{1}{RC}S + \frac{1}{LC} = 0$$

$$\therefore \quad S_1, S_2 = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$$

$$R_{cr} = \frac{1}{2}\sqrt{\frac{L}{C}}$$

$$\xi = \text{damping constant} = \frac{R}{R_{cr}} = \frac{R}{\frac{1}{2}\sqrt{\frac{L}{C}}} = 2R\sqrt{\frac{C}{L}}$$

The Q factor of the parallel circuit is,

$$Q = W_n RC \text{ where } W_n = \frac{1}{\sqrt{\frac{L}{C}}} = \text{natural frequency}$$

$$\therefore \quad Q = \frac{RC}{\sqrt{\frac{L}{C}}} = R\sqrt{\frac{C}{L}}$$

$$\therefore \quad Q = 2\xi \quad \text{..... from damping constant.}$$

EXAMPLE 1.14

An ideal 1μs pulse is fed to an amplifier. Calculate and plot the output waveform under the following conditions. The upper 3-dB frequency is (a) 8 MHz (b) 2 MHz (c) 0.2 MHz

SOLUTION

The upper 3-dB frequency indicates that the amplifier acts as a low-pass circuit so the pulse shown in fig 1.72 (a) is applied to the RC low pass circuit shown in fig 1.72 (b)

(a) When the upper 3-dB frequency $f_2 = 8$ MHz

Time constant of the circuit $RC = \frac{1}{2\pi f_2} = \frac{1}{2\pi \times 8 \times 10^6} = 0.0198\ \mu s$

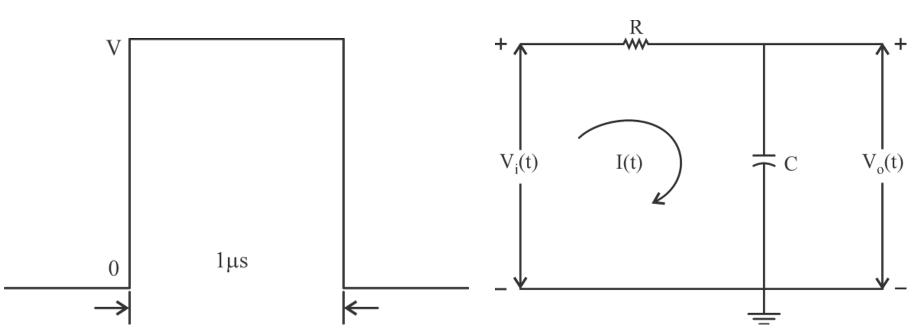


FIGURE 1.72 (a) input wave form (b) circuit diagram

Since $t_p = 1\ \mu s$ and RC = 0.0198 μs

Since the time constant is very small in comparison with the pulse width, the capacitor C charges rapidly with a rise time.

$$t_r = 2.2 \times 0.0198 = 0.043\ \mu s$$

The output V_o is given by Vo – V $(1 - e^{-t/RC})$ where V is the amplitude of the pulse

At $\quad t = t_p$

$$Vo = V\,(1 - e^{-tp/RC}) = V\,(1 - e^{-1/0.0198}) = V$$

The output waveform is shown in figure 1.73 (a)

(b) When $f_2 = 2$ MHz

$$RC = \frac{1}{2\pi f_2} = \frac{1}{2\pi \times 2} = 0.0795\mu s$$

$$t_p = 1\mu s \text{ and } RC = 0.0795\ \mu s$$

$$\therefore RC < t_p$$

Since the time constant is small the capacitor charges fast

Rise time $\quad t_r = 2.2\ RC = 2.2 \times 0.0795 = 0.17\ \mu s$

The output is given by $Vo = (1 - e^{-t/RC})$

$$\text{At} = t = t_p,\ Vo = V\,(1 - e^{-tp/RC}) = V\,(1 - e^{-1/0.0795})$$
$$= 0.999\ V$$

The output wave form in shown in figure 1.73 (b)

(c) When $\quad f_2 = 0.2$ MHz

$$RC = \frac{1}{2\pi f_2} = \frac{1}{2\pi \times 0.2} = 0.0795\ \mu s$$

So $t_p = 1\mu s$ and RC = 0.0795 μs. RC is comparable to t_p since the time constant is comparable to the pulse width, in the interval $0 < t < t_p$ the capacitor charges exponentially according to the equation

$$V_o = V\,(1 - e^{-t/RC})$$

At $t = t_p$ the output voltage $V_o = V_p$

$$V_p = V(1 - e^{t_p/RC})$$
$$V_p = (1 - e^{-1/0.079}) = 0.999\ V$$

For $t > t_p$, V_o decreases according to the equation

$$V_o = V_p\, e^{-(t_p/RC)}$$
$$= 0.999\ Ve^{-(t-1)/0.079}$$

The output wave form is shown in figure 1.73 (c)

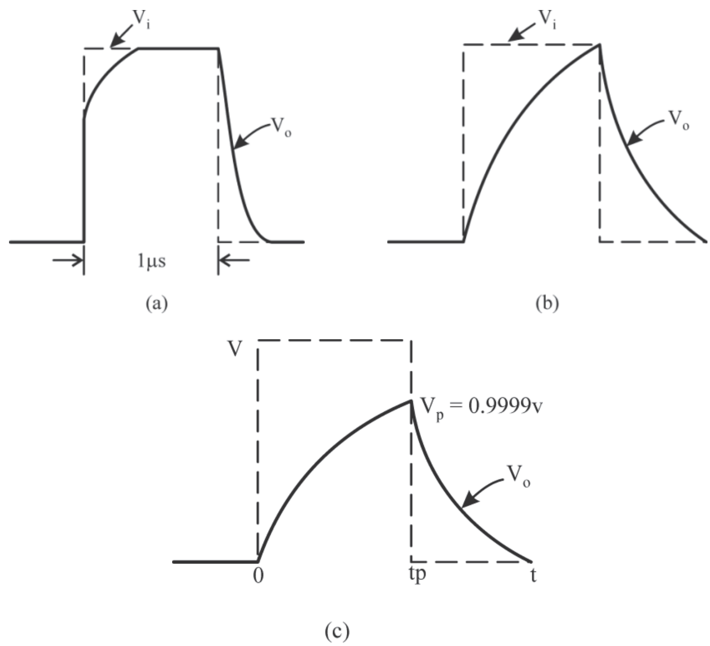


FIGURE 1.73 (a), (b) and **(c)** output wave forms.

EXAMPLE 1.15

A symmetrical square wave of amplitude $\pm 4V$ and frequency 3 KHz is impressed on an RC low pass circuit. If R = 4 KΩ, X = 0.1 μf. Calculate and plot the steady state output with respect to time.

SOLUTION

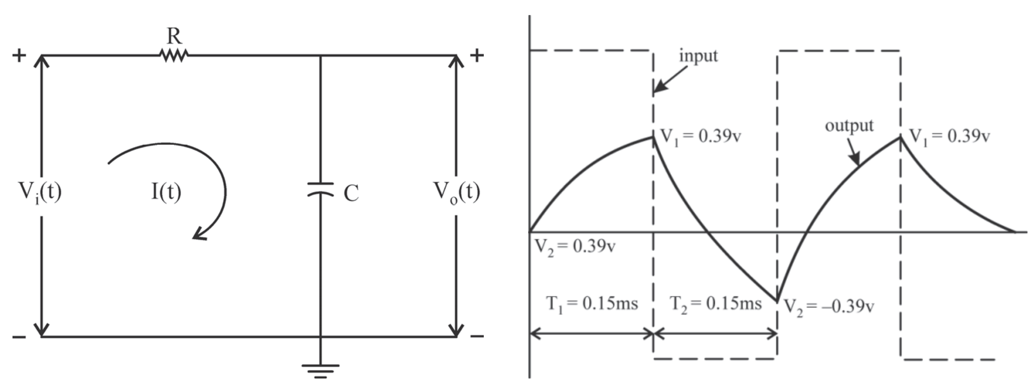


FIGURE 1.74

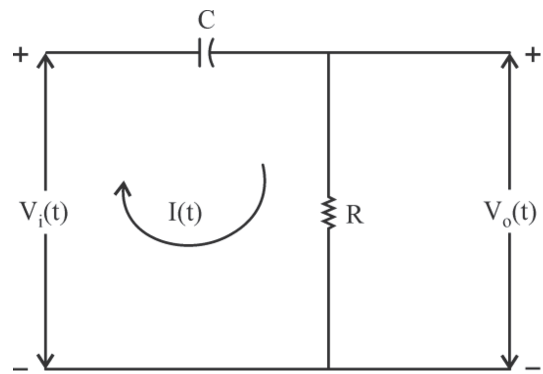


FIGURE 1.75

Given $\quad f = 3\text{ KHz},\ T = \dfrac{1}{3\text{ KHz}} = 0.3\text{ ms}$

$$\therefore \quad T_1 = T_2 = \frac{T}{2} = \frac{0.3\text{ ms}}{2} = 0.15\text{ ms}$$

The time constant of the circuit T = RC = $4 \times 10^3 \times 0.1 \times 10^{-6} = 0.4$ ms

Since RC is comparable to $\dfrac{T}{2}$ the capacitor charges and discharges exponentially since the input is a symmetrical square wave

$$V_1 = \frac{V}{2} = \left[\frac{1-e^{-T/2RC}}{1+e^{-T/2RC}}\right]$$

where V is the peak-to-peak value of the input

$$\frac{4}{2} + \left[\frac{1-e^{-0.4/1}}{1-e^{-0.4/1}}\right]$$

$$= 0.39\text{ V}$$

$$V_2 = -V_1$$

$$= -0.39\text{ V}$$

$\therefore$ peak – to – peak wave of output

$$= 0.39\text{ V} - (-0.39\text{ V})$$

$$= 0.78\text{ V}$$

EXAMPLE 1.16

A 5 Hz square wave is fed to an amplifier. Calculate and plot the output wave form under the following condition. The lower 3dB frequency is (a) 0.2 Hz (b) 2 Hz (c) 0 Hz.

SOLUTION

Since the lower cut – off frequency is specified, the amplifier acts as an RC high pass circuit shown in figure 1.75

Given input frequency f = 5Hz

$$T = \frac{1}{f} = \frac{1}{5} = 0.2 \text{ s}$$

(a) When $f_1 = 0.2$ Hz

Time constant $RC = RC = \frac{1}{2\pi f_1} = \frac{1}{2\pi \times 0.2} = 0.7957$ s

$$V_1 = \frac{V}{1 + e^{-T/2RC}}$$

where Vo is the peak value of the input voltage

Substituting the values of T and RC

$$V_1 = \frac{V}{1 + e^{-0.2/(2\times 0.7957)}}$$

$$= \frac{1}{1 + e^{-0.2/(2\times 0.7957)}} = 0.5313 \text{ V}$$

$$= V_1{}^1 = V_1\, e^{-T1/RC} = V_1 e^{-T/2RC}$$

$$= (0.5789 \text{ V})\, e^{-0.25/2\times 0.7957}$$

$$= 0.4947 \text{ V volts}$$

$$V_2 = -V_1 = -0.5789 \text{ V volts}$$

$$V_2 = -V_1 = -0.4947 \text{ V volts}$$

Also

(b) When $f_1 = 2$Hz

$$RC = \frac{1}{2\pi f_1} = \frac{1}{2\pi \times 2}$$

$$= 0.079 \text{ S}$$

$$T/2 = 0.1 \text{ S}$$

Since RC comparable to T, the output rises and decays exponentially as shown in

For $0 < t < T_1$, Volts $= V_1 e^{-t/RC}$

At $t = T_1$, $V_o = V_1{}^1 = V_1 e^{-T1/RC} = V_1 e^{-T/2RC} = V_1 e^{-0.1/0.7957} = 0.510\ V_1$

For $T_1 < t < T_1 + T_2$, $V_{o2} = V_2 e^{-(t-T_1)/RC}$

At $t = T,\ V_{01} = V^1{}_2 = V_2 e^{-T_2/RC} = V_2\, e^{-0.1/0.7957} = 0.510\ V_2$

Since peak to –peak input is V Volts

$$V_1{}^1 - V_2 = V \text{ i.e } 0.510\ V_1 - V_2 = V$$

$$V_1 - V_2{}^1 = V \text{ i.e } V_1 - 0.510\ V_2 = V$$

$$0.510\ V_1 = V_2 - V_1 - 0.510\ V_2$$

$$V_1 = -V_2$$

$$V_1 = \frac{V}{1.510} = 0.6622V \text{ volts}$$

$$V_2 = -V_1 = -0.6622\ V \text{ volts}$$

$$V^1{}_1 = 0.510\ V_1 = 0.510 \times 0.6622\ V = 0.337\ V$$

$$V_2{}^1 = -V_1{}^1 = -0.337\ V$$

(c) When $f_1 = 10$ MHz

$$RC = \frac{1}{2\pi f_1} = \frac{1}{2\pi \times 10} = 0.0159\ S$$

$$RC << T$$

EXAMPLE 1.17

A 2 KHz symmetrical square wave of ± 5V is applied to an RC circuit having 1 ms constant .Calculate and plot the output for the RC configuration as (a) high-pass circuit.

SOLUTION

Given for f = 2 KHz

$$T = \text{ms}$$

$$T_{oN} = 0.5 \text{ ms and } T_{off} = 0.5 \text{ ms}$$

$$RC = 1 \text{ ms, peak-to-peak amplitude } V_{pp} = 5 - (-5) = 10\ V$$

Since RC is comparable to T the capacitor charges and discharges exponentially

(a) High pass-circuit:- when the 2 KHz square wave shown by dotted lines in fig (1.75 (a)) is applied to the RC high pass circuit shown in fig 1.76(a) under steady state conditions, the output wave form will be as shown by the thick line in fig 1.76 (b)

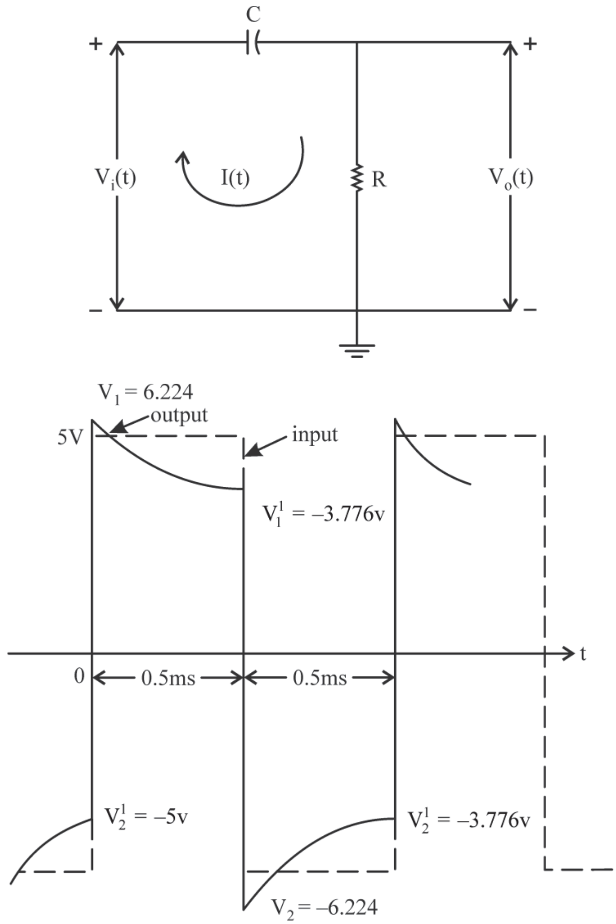


Figure 1.76 (high pass circuit) (a) circuit diagram and (b) output wave forms

The input signal is a symmetrical square wave,

$$V_1 = -V_2 \text{ and } V_1^{\,1} = -V_2^{\,1}$$

$$V^1{}_1 = V_1 e^{-T_2/RC} = V_1 e^{-0.5/1} = 0.6065\ V_1$$

$$V_2^{\,1} = V_2 e^{-T_2/RC} = V_2\ e^{-0.5/1} = 0.6065\ V_2$$

$$V_1^{\,1} + V_2 = 0$$

$$0.6065\ V_1 + V_1 = 10$$

$$V_1 = \frac{10}{1.6065} = 6.224\ \text{V}$$

$$V_2 = -V_1 = -6.224\ \text{V}$$

$$= 3.776\ \text{V}$$

$$V_2^1 = -V_1 = -3.776\ \text{V}$$

EXAMPLE 1.18

If a square of 4KHz is applied to an RC high-pass circuit and the resultant waveform measured on a CRO was tilled from 14V to 9V, find out the lower 3-dB frequency of the high pass circuit.

SOLUTION

The input and output wave form of the RC-high-pass circuit

$f = 4KHz$

$V_1 = 14V$

$V^1{}_1 = 9V$

$f_l = ?$ (lower 3dB frequency)

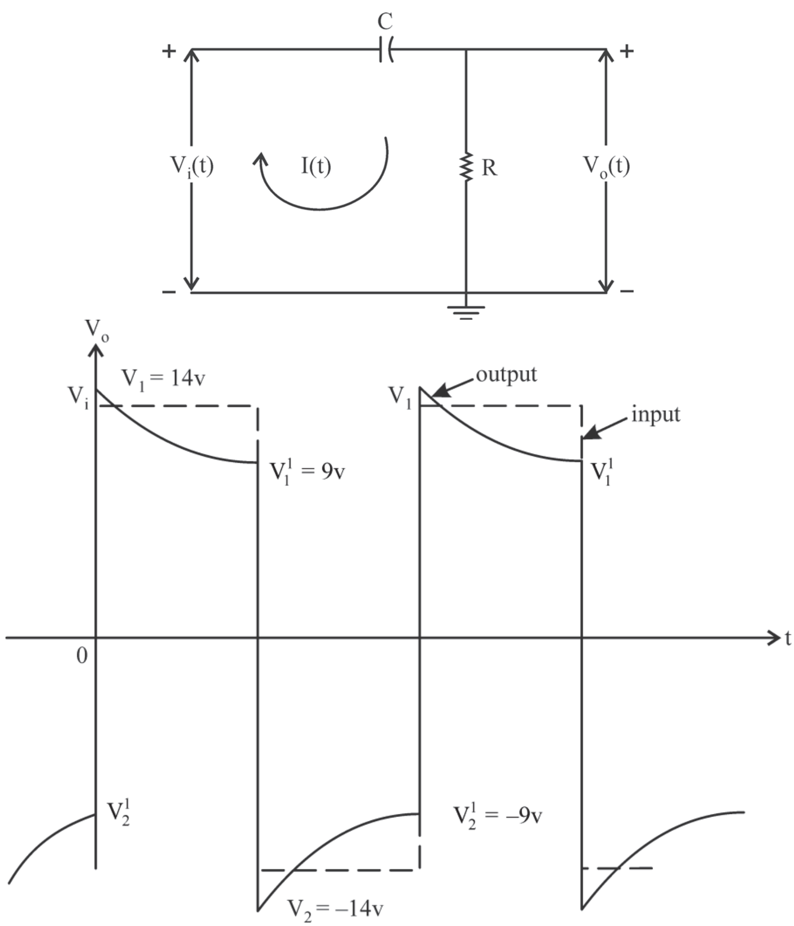


FIGURE 1.77 (a), (b)

peak-to-peak value of input $V = V_1 + V_2 = 14 + 14 = 28$ V

$$P = \frac{V_1 - V_1^1}{\frac{V}{2}} \times 100$$

$$P = \frac{14-9}{\frac{28}{2}}$$

$$= \frac{5}{14} = 0.357\ \%$$

Also % tilt

$$P = \frac{\pi f_1}{f} \times 100$$

$$0.357 = \frac{3.141 f_1}{4 \times 10^3} \times 100$$

$$f_1 = \frac{0.357 \times 4 \times 10^3}{3.141 \times 10^3} = 4.546 \text{ Hz}$$

EXAMPLE 1.19

A 1 KHz square wave output from an amplifier has rise time $t_r = 350$ ns and tilt = 5%. Determine the upper and the lower 3-dB frequencies.

SOLUTION

The amplifier has upper and lower 3-dB frequency so it acts as a combination of low-pass and high-pass filters, that is, as a band-pass filter. The upper cut-off frequency of a low-pass filter can be determined from the information about the rise time can we put t as 't'. The lower cut-off frequency of a high filter can be determined from the information about % tilt.

Rise time $$t_r = \frac{0.35}{f_2} = 2.2RC = 350 \text{ ns}$$

∴ The upper 3-dB frequency

$$f_2 = \frac{0.35}{t_r} = \frac{0.35}{350 \times 10^{-9}} = 1 \text{ MHz}$$

Percentage tilt $$= \frac{1}{2RC} \times 100$$

$$= \frac{\pi f_1}{f} \times 100 = 5$$

$\therefore$ The lower 3-dB frequency

$$f_1 = \frac{sf}{\pi \times 100} = 15.92 \text{ Hz}$$

EXAMPLE 1.20

A symmetrical square wave is applied to a high pass circuit having R = 10 KΩ and c = 0.04 μf

(a) If the frequency of the input signal is 2 KHz and the signal swings between ±4 V draw the output waveform and indicate the voltages.

(b) What happens if the frequency of the signal is reduced to 200 Hz? Show the output waveform.

SOLUTION

The square wave shown in figure 1.78 (b) is applied to the RC high pass circuit shown in figure 1.78 (a).

(a) The time constant of the circuit RC = 10 × 0.04 = 0.5ms f = 2 KHz

$$T = \frac{1}{2 \text{ KHz}} = 0.5 \text{ ms } T/2 = 0.25;$$

As the input is a symmetrical square wave

$$V_1 = \frac{V}{1 + e^{-T/2RC}} = \frac{20}{1 + e^{-0.25/1}} = \frac{10}{1 + e^{-0.25}} = 12.44\text{V}$$

$$V_2 = -V_1 = -12.44 \text{ V}$$

$$V_1^1 = \frac{V}{1 + e^{-T/2RC}} = \frac{20}{1 + e^{-0.25/2(0.5)}} = 8.75\text{V}$$

$$V^1{}_2 = -V_1{}^1 = -8.75 \text{ V}$$

(b) If f is reduced to 200Hz when

$$T = \frac{1}{f} = \frac{1}{200} = 20\text{ms}$$

$$V_1 = -\frac{V}{1 + e^{-T/2RC}} = \frac{20}{1 + e^{-0.25/2(0.5)}} = 11.24\text{V}$$

$$V_2 = -V_1 = -11.24 \text{ V}$$

$$V_1^1 = \frac{V}{1 + e^{-T/2RC}} = \frac{20}{1 + e^{-0.25/2(0.5)}} = 8.1756\text{V}$$

EXAMPLE 1.21

The limited ramp show in fig 1.56 is applied to an RC differentiator draw the output wave form for cases (a) T = 0.1 RC (b) T= RC and (c) T = 4RC.

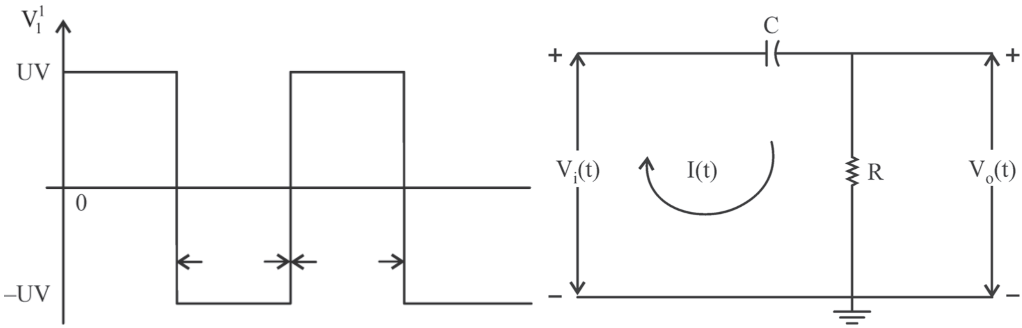


FIGURE 1.78 (a) circuit diagram and (b) input waveform

When the input signal frequency is reduced from 2 KHz to 200 Hz the circuit as differentiator as RC 10 S < T (0.25 ms).

SOLUTION

The limited ramp shows the dotted line in 1.56 (b) is applied to the RC high-pass circuit shown in figure 1.56 (a) for the ramp input slope α = V/T, where T, is the duration of ramp and V, the amplitude of ramp at t = T. The output for a ramp.

$$Vo(t) = \alpha \; RC\,(1 - e^{-t/RC})$$

(a) For T = 0.1 RC, the output at t = T is

$$= \frac{V}{T} \times 0.1T(1 - e^{-T/4T})$$

(b) For T = RC the output at t = T is

$$= \frac{V}{T} \times T(1 - e^{-T/T})$$

(c) For T = 4 RC, the output at t = T is

$$= \frac{V}{T} \times 4T(1 - e^{-T/(T/41)})$$

EXAMPLE 1.22

A 200Hz triangular wave with peak-to-peak amplitude 8 V is applied to a differentiating circuit with R = 2 MΩ and c = 200. PF Calculate and sketch the wave form of the output.

SOLUTION

The triangular wave shown by the dotted line in figure 1.80 is applied to the RC high-pass circuit shown in figure 1.80. The time constant of the RC circuit is

$$RC = 2 \times 10^{6} \times 200 \times 10^{-12} = 400\ \mu s$$

Frequency of the triangular wave f = 200 Hz

$$T = 1/f = 1/200 = 20\ ms$$

Since the time constant of the circuit is very small compared to the period of the input wave form, the circuit acts as a differentiator and the output waveform is in the form of a square wave having excursion from a RC (when slope is positive) to – dRC (when slope is negative) i.e.,

$$\alpha RC = \frac{8}{10} \times 200 \times 10^{-6} = 0.16V$$

$$Vo(t) = \alpha RC = +\ 0.16\ V \text{ when } \alpha \text{ is positive}$$

$$= -\ \alpha RC = -\ 0.16\ V \text{ when } \alpha \text{is negative}$$

The output is a square wave with levels + αRC and – αRC as show in figure 1.80

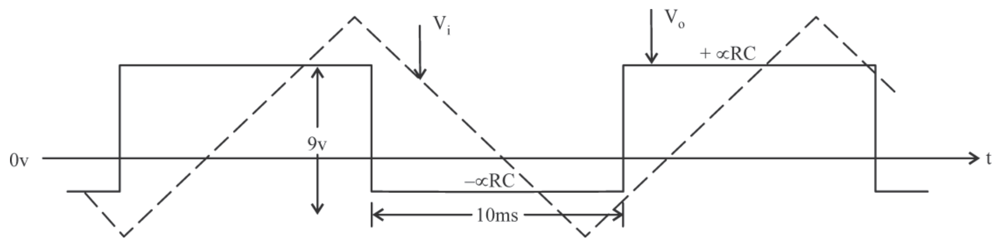


Figure 1.79

EXAMPLE 1.23

A pulse with a rise time t_r = 400 ns and a fall time t_f = 1 μs, pulse amplitude = 10 V and pulse width = 10 μs is applied to differentiating circuit with C = 100 pF and R = 400 Ω. Determine the amplitude of the differentiated output. Sketch the wave forms across R and C.

SOLUTION

The pulse shown in fig 1.81 (b) is applied to the RC high pass circuit shown in figure 1.81 (a). The input is exponential. The time constant of the ring waveform is

$$T = \frac{400\ ns}{2.2} = 181.81ns$$

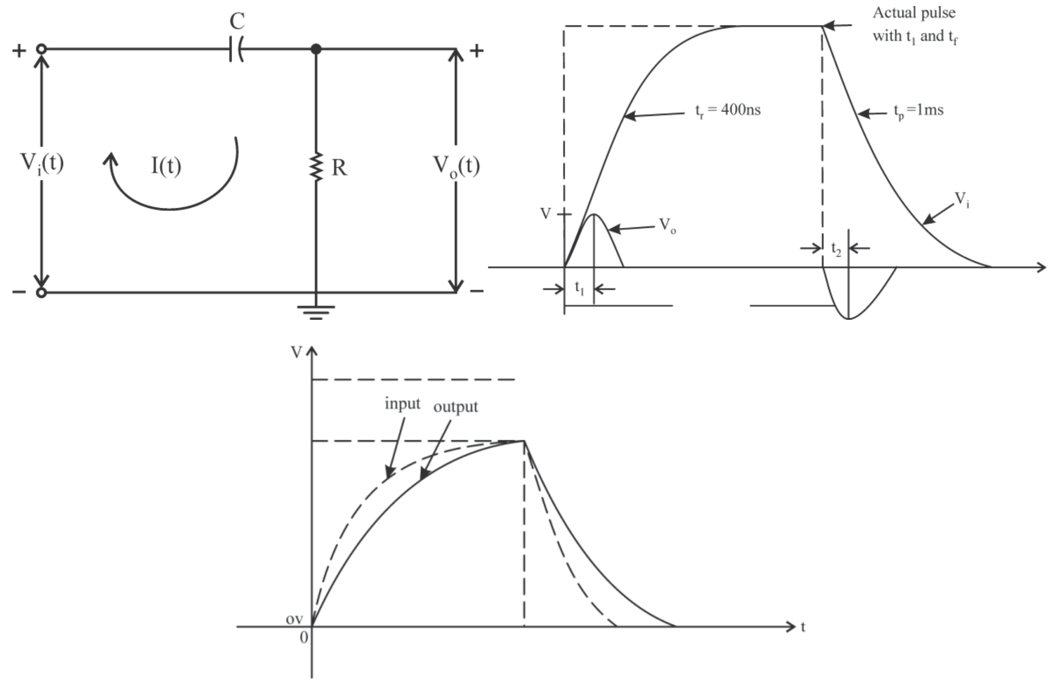


FIGURE 1.80 Output wave form (a) circuit diagram (b) voltage across R and voltage across C.

Time constant of the circuit RC = 400Ω × 200PF = 90ns

$$n = \frac{RC}{T} \cdot \frac{90}{181.81} = 0.4950,\ n < 1$$

The output is in the form of a pulse

Pulse amplitude (i.e. peak value during rise) is

$$V_n^{'(1-n)} = 10 \times 0.4950^{1(1-0.4950)} = 2.48\ \text{V}$$

Let the time at which the maximum output occurs be t_1. Then

$$x = \frac{t_1}{T} = \frac{n}{n-1} ln^n = \frac{0.4950}{0.4950-1} ln\ 0.4950 = 0.689$$

Or $\quad t_1 = 0.689\ T = 0.689 \times 181.81 = 125.26$ ns

The fail time $\quad t_f = 1\ \mu s = 2.2\ T_1$

Fall time constant $\quad T_1 = \frac{1\ \mu s}{2.2} = 0.4545\ \mu s$

$$n = \frac{RC}{T_1} = \frac{450\ \Omega \times 100\ \text{PF}}{0.4545\ \mu s} = 0.99$$

Peak value during fall is

$$V_o\ (\max) = -10 \times 0.99^{1(1-0.99)} = -9.9\ \text{V}$$

Let the time at which negative peak occurs be t_2

$$x = \frac{t_2}{T_1} = \frac{n}{n-1}\ln n = \frac{0.99}{0.99-1}\ln 0.99 = 0.99$$

$$t_2 = xT_1 = 0.99 \times 0.4545 = 0.4499\ \mu s$$

EXAMPLE 1.24

Prove that for the same input, the output from the two differentiating circuits shown in fig 1.80 will be the same if RC = L/R[1]. Assume Zero initial conditions.

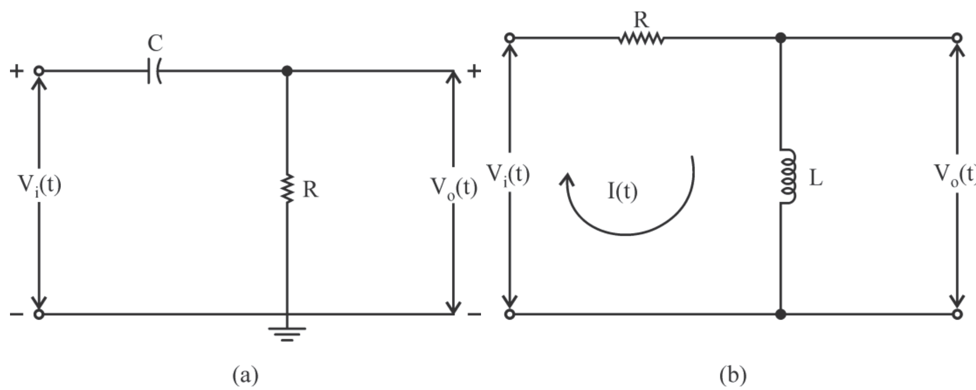


Figure 1.81

SOLUTION

(a) The RC high pass circuit shown in figure 1.81 (a)

$$V_i = \frac{1}{C}\int i dt + Ri$$

$$V_o = iR$$

$$i = \frac{V_o}{R}$$

$$V_i = V_o + \frac{1}{RC} = \int V_o dt$$

(b) For the RC high-pass circuit shown in figure 1.81 (b)

$$V_i = R_1^1 + L\frac{di}{dt}$$

$$V_o = L\ \frac{di}{dt}$$

$$\frac{di}{dt} = \frac{V_o}{L}$$

On integrating both sides

$$i = \frac{1}{L} = \int V_o dt$$

$\therefore$ from the above equations for V_i if RC = L/R', the output from the differentiator circuits are the same.

EXAMPLE 1.25

Compute and draw to scale the output waveform for (a) C_1 = 45 PF (b) = C_1 = 70 PF, and C_1 = 20 PF respectively for this circuit shown in figure 1.65. the input is a 10V step.

SOLUTION

In the circuit shown in figure 1.65 (a) for respect companion

$$C_1 = \frac{R_2C_2}{R_1} = \frac{1\ M\Omega}{1\ M\Omega} \times 45PF = 45\ PF$$

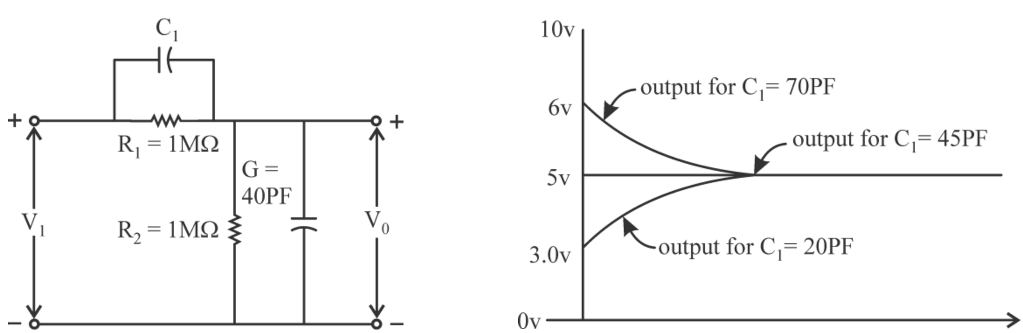


FIGURE 1.82 (a) circuit diagram and (b) output waveform

(a) Therefore, when C_1 = 45 PF the attenuator would be perfectly compensated. The rise time of the output waveform t_r = 0 the output would be an exact replica of the input but with reduced amplitude.

The attenuation constant $$\alpha = \frac{R_2}{R_1 + R_2} = \frac{1}{1+1} = 0.5$$

$$V_o(0^+) = V_o(\infty) = 0.5 \times 10 = 5\ V$$

(b) When C_1 = 70 PF, the attenuator is over compensated. Hence $V_o(0^+) > V_o(\infty)$

Initial response $$V_o(0^+) = V_i \frac{C_1}{C_1 + C_2} = 10 \times \frac{70}{70+45} = 6 \text{ V}$$

Final response $$V_o(\infty) = V_i \frac{R_1}{R_1 + R_2} = 10 \times \frac{1}{1+1} = 5 \text{ V}$$

Final time constant

$$RC = \left[\frac{R_1 R_2}{R_1 + R_2}\right](C_1 + C_2) = \left[\frac{1 \times 1}{1+1}\right] \times 10^6 \times (70+45) \times 10^{-12} = 57.5 \ \mu s$$

Fall time, $t_f = 2.2RC = 2.2 \times 57.5 = 126.5 \ \mu s$

(c) When $C_1 = 20$ PF the attenuator is under compensated.

Initial response $$V_o(0^+) = V_i \frac{C_1}{C_1 + C_2} = 10 \times \frac{20}{20+45} = 3.07 \text{ V}$$

Final response $$V_o(\infty) = V_i \frac{R_2}{R_1 + R_2} = 10 \times \frac{1}{1+1} = 5 \text{ V}$$

Rise time constant

$$RC = \left[\frac{R_1 R_2}{R_1 + R_2}\right](C_1 + C_2) = \left[\frac{1 \times 1}{1+1}\right] \times 10^6 \times (20+45) \times 10^{-12} = 32.5 \ \mu s$$

Rise time $t_r = 2.2 \text{ RC} = 2.2 \times 32.5 = 71.5 \ \mu s$

EXAMPLE 1.26

For the circuit shown in figure 1.91 (a) the input is a 10 V step. Calculate and plot to scale the output voltage.

SOLUTION

In the circuit shown in figure 1.91(a) for perfect compensation the required value of

$$C_1 = \frac{R_1 C_2}{R_1} = \frac{1 \times 50}{1} = 50 \text{ PF}$$

Though the input is a step it is not impressed on te attenuator network due to source resistance. The input to the attenuator is therefore, given by.

$$V_1^1 = V_i \frac{R_1 + R_2}{R_1 + R_2 + R_s} = 10 = \frac{1+1}{1+1+0.2} = 9.09\text{V} = 10 \text{ V}$$

This input step exhibits a rise time

$$t_r = 2.2[R_s(R_1 + R_2)]\left[\frac{C_1C_2}{C_1 + C_2}\right] = 2.2\left[\frac{0.2\times 2}{0.2+2}\right]\left[\frac{45\times 45}{45+45}\right] = 9\ \mu s$$

Since the attenuator is perfectly compensated, the output is a perfect replica of the input with reduced amplitude

$$V_o = \alpha V_i^1 = \frac{R_2}{R_1 + R_2} V_i^1 = \frac{1}{1+1} \times 9.09 = 4.545\ V$$

The rise time of the output = α × the rise time of the input =1/2 × 9 = 4.5 μs.

The input and output of the attenuator as shown in figure 1.71 (b)

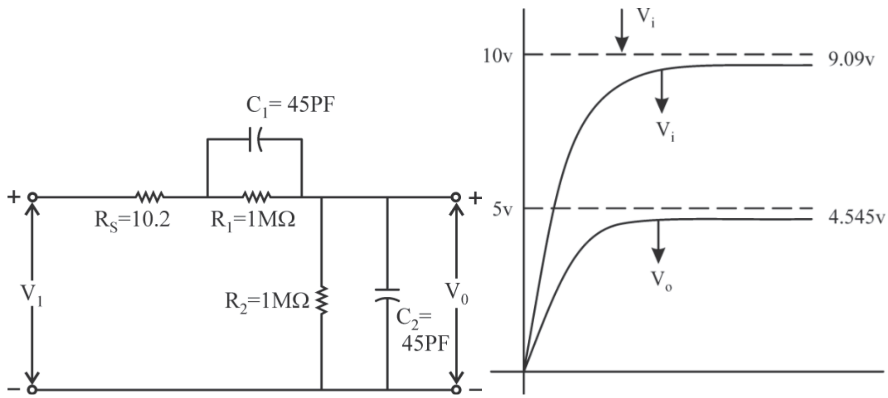


Figure 1.83

EXAMPLE 1.27

For the circuit and the input waveform shown in figure 1.92 (a) draw roughly the output waveform V_o. Make reasonable approximations and estimate the rise time of the waveform, the magnitude of the overshoot and the time constant of the decay to the final value.

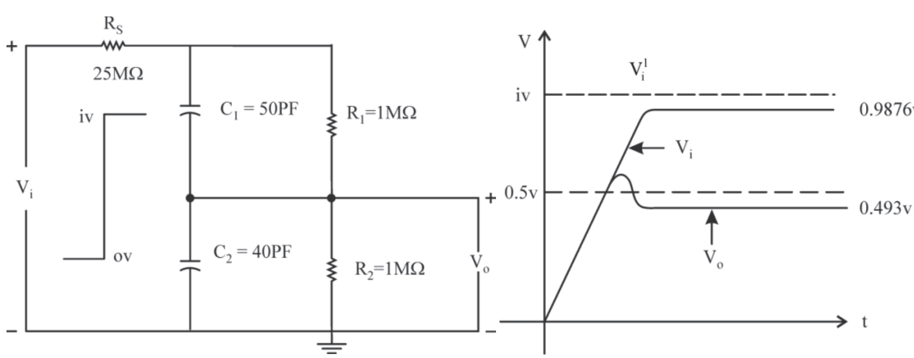


Figure 1.84

SOLUTION

In the circuit shown in fig 1.92 (a) for perfect compensation, the required value of

$$C_1 = \frac{R_2C_2}{R_1} = \frac{1\times 40}{1} = 40 \text{ PF}$$

The input to the attenuator therefore will be

$$V_i^1 = V_i \frac{R_1+R_2}{R_1+R_2+R_2} = 1\times = \frac{1+1}{1+1+0.025} = \frac{2}{2.025} = 0.9876 \text{ V}$$

The rise time of the input

$$t_r = 2.2[R_s 11(R_1+R_2)]\left[\frac{C_1C_2}{C_1+C_2}\right] = 2.2\left[\frac{0.025\times 2}{0.025+2}\right]\left[\frac{50\times 40}{50+40}\right] = 1.20 \text{ µs}$$

Initial response $\quad V_o(0^+) = V_i^1 \dfrac{C_1}{C_1+C_2} = \dfrac{0.9876\times 50}{50+40} = 0.548 \text{ V}$

Final response $\quad V_o(\infty) = V_i^1 \dfrac{R_1}{R_1+R_2} = \dfrac{0.9876\times 1}{1+1} = 0.493 \text{ V}$

Rise time of the output $= \dfrac{R_2}{R_1+R_2} \times$ rise time of input = 0.5 × 1.20µs = 0.6 µs

Fall time of output t_f = 2.2 RC = 2.2 $\left[\dfrac{R_1R_2}{R_1+R_2}\right](C_1+C_2) = 99$ µs

Overshoot = $V_o(0^+) - V_o(\infty)$ = 0.548 – 0.493 = 0.055 V

The input and output waveform of the attenuator are shown in figure 192 (b).

Multiple Choice Questions

1. How does a capacitor behave to sudden changes in voltage?

   (a) Offers reactance (b) Open circuit

   (c) Short circuit (d) Offers attenuation

2. When a sinusoidal wave = form is transmitted through a linear waveform circuit, the following feature does not change:

   (a) Time constant (b) Amplitude

   (c) Phase (d) Frequency

3. The expression for the lower cut off frequency of RC-high pass filter is

(a) $\frac{1}{2\pi RC}$ (b) $\sqrt{2}$ / RC

(c) RC (d) 0.707 RC

4. The function of a blocking a capacitor is

(a) It allows dc component (b) It does not allow the dc component

(c) It does not allow the dc and ac component (d) It allow both dc and ac component

5. The gain of a passive low pass filter at its lower cut off frequency is

(a) 3 dB up (b) $\sqrt{2}$

(c) Unity (d) 3 dB down

6. At high frequencies inductor behaves like a

(a) Short circuit (b) Open circuit

(c) Ordinary wave (d) Offers negligible reactance

7. A sinusoidal waveform is very useful in determining the following features in of a circuit

(a) Spectrum (b) Time constant

(c) Bandwidth (d) Linearity

8. The lower cult off frequency of any ideal high pass filter is

(a) f_1 (b) Zero

(c) Infinity (d) Cannot be determined

9. The average value of a output of a high-pass filter is

(a) Same as the input (b) Zero

(c) Depends on the waveform (d) Depends on the time constant

10. At high frequencies a capacitor behaves like a

(a) Short circuit (b) Open circle

(c) Behaves normally (d) Gives rise to a spike

11. The phase angle ϕ in an RC high-pass filter is

(a) Always lagging (b) Always leading

(c) Always out-of phase (d) In the same phase

12. Theoretically, a transient in a circuit reaches its final value at

(a) Zero (b) After one time constant

(c) After two time constants (d) infinity

13. When RC<<T, the output of a high-pass filter is
    (a) Constant (b) Proportional to the input
    (c) Derivative (d) Average value of the input
14. The expression for rise time t_r of a low pass filter is
    (a) $t_r = 1 + 0.35/f_2$ (b) $t_r = 1 - 0.35/f_2$
    (c) $t_r = 0.35/f_2$ (d) $t_r = 0.75/f_2$
15. In an RLC circuit, when K = 1, the circuit is
    (a) Over damped (b) Critically damped
    (c) Under damped (d) Undamped
16. In the case of the RLC circuit, popularly known as ringing circuit
    (a) $K \ll 1$ and $K \neq 0$ (b) $K \ll 1$
    (c) $K \gg 1$ and $K \neq 0$ (d) $K \gg 1$ & $K = 0$
17. In an ideal attenuator, the output voltage
    (a) Depends on frequency (b) Remains constant
    (c) Don't depend on frequency (d) Depends on the time constant
18. A square wave is transmitted through an RC low-pass filter when RC = T, then
    (a) The output and input have identical shape
    (b) The output is a series of spikes
    (c) The output is the integration of the input waveform
    (d) The output is a square wave with a tilt
19. The gain of a passive high-pass filter at its lower cut off frequency is
    (a) + 3 dB (b) 1
    (c) $1/\sqrt{2}$ (d) 1.14
20. In the case of an uncompensated attenuator with resistors, R_1 and R_2, the capacitance C_2 at the output terminals is neutralized when
    (a) $\dfrac{R_1}{C_1} = \dfrac{R_2}{C_2}$ (b) $R_1C_1 = R_2C_2$
    (c) $R_1C_2 = R_2C_1$ (d) $R_1C_1 \neq R_1C_2$
21. The delay time t_d is defined making use of an exponentially raising waveform
    (a) Time interval to rise from 10% to 90% of its final value
    (b) Time interval to rise from 10% to 50% of its final value
    (c) Time interval to rise from 0% to 90% of its final value
    (d) Time interval to rise from 0% to 50% of its final value

22. When RC>>T, the output of a high pass filter is
    (a) Constant (b) Proportional to the input
    (c) Derivative of the input (d) Average value of the input
23. In RLC circuit, when K < 1, the circuit is
    (a) Over damped (b) Critically damped
    (c) Under damped (d) Undamped
24. Condition for good differentiation of RC high-pass filter is
    (a) RC = 10T (b) RC >>T
    (c) RC = T (d) RC <<T
25. Condition for good integration of RC low-pass filter is
    (a) RC = 10T (b) RC >>T
    (c) RC = T (d) RC<<T
26. When RC <<1, the output of a low-pass filter is
    (a) Constant (b) Proportional to the input
    (c) Derivation of the input (d) Average value of the input
27. Condition for good differentiation for RL high pass filter is
    (a) L/R = 10T (b) 4L/R >>T
    (c) L/R = T (d) 4L/R <<T
28. In an RLC circuit, when K>1, the circuitis
    (a) Over damped (b) Critically damped
    (c) Under Damped (d) Undamped
29. Condition for good integration of RL low-pass filter is
    (a) L/R = 10T (b) L/R >>T
    (c) L/R = T (d) L/R <<T
30. The phase angle ϕ in a RC high-pass filter at its lower cut off frequency f_1 is
    (a) Zero (b) 90^o
    (c) 45 (d) π radians
31. The expression for delay time t_d of a low-pass filter is
    (a) $t_d = 0.35/2$ (b) $t_d = 0.11/f_2$
    (c) $t_d = \sqrt{2} / f_2$ (d) $t_d = \pi / f_2$
32. When RC >>T, the output of a low pass filter is
    (a) Constant (b) Proportional to the input
    (c) Derivative of the input (d) Integration of the input

33. The frequency functions G(f) of an first order low pass filter is

(a) $G(f) = \dfrac{1}{1 - j(f_2/f)}$ (b) $G(f) = \dfrac{1}{1 - j(f/f_2)}$

(c) $G(f) = \dfrac{1}{1 + j(f/f_2)}$ (d) $G(f) = \dfrac{1}{1 + j(f_2/f)}$

34. The phase angle ϕ of an RC high-pass filter is expressed as

(a) $\phi = -\tan^{-1}(f/f_1)$ (b) $\phi = -\tan^{-1}(f_1/f)$

(c) $\phi = \tan^{-1}(f_1/f)$ (d) $\phi = \tan^{-1}(f/f_1)$

35. The series capacitor in an RC high-pass filter is called

(a) By-pass capacitor (b) Stabilization capacitor

(c) Filter capacitor (d) Blocking capacitor

36. A ramp waveform $V_i(t) = \alpha t$ during the interval $0<t<T$, is transmitted through an RC low-pass filter with RC<<T. The output waveform is

(a) $V_o(t) = \dfrac{\alpha t^2}{2RC}$ (b) $V_o(t) = \alpha(t - RC)$

(c) $V_o(t) = \alpha t$ (d) $V_o(t) = \alpha RC$

37. A symmetrical square wave is transmitted through a RC high pass filter with RC<<T. The % tilt P in the output waveform can be given by

(a) $P = \pi\left(\dfrac{f_1}{f}\right) \times 100$

(b) The tilt P is same as that suffered by a ramp waveform

(c) $P = \dfrac{T}{2RC} \times 100$

(d) It is not possible to find P

38. An RC high-pass filter acts as a good differentiator when

(a) RC = T (b) RC<<T (c) RC = 20T (d) RC>>T

39. A sinusoidal waveform is transmitted through an RC low pass filter and the phase angle ϕ is found to be -45^{o}. What is the expression for the frequency of the sine wave when this condition is satisfied?

(a) $f_1 = \dfrac{1}{2\pi RC}$ (b) $f_2 = \dfrac{1}{2\pi\sqrt{RC}}$

(c) $f_1 = \dfrac{1}{2\pi\sqrt{RC}}$ (d) $f_2 = \dfrac{1}{2\pi RC}$

40. The gain of the RC high-pass filter is always

(a) Around 1.414
(b) Lies in the range 0.707 and 1.41
(c) Around 0.707
(d) Less than unity

41. A ramp waveform $V_i(t) = \alpha t$ during the interval $0<t<T$, is transmitted through an RC high pass filter with $RC>>T$. The output wavefrom is

(a) $V_o(t) = \frac{\alpha t^2}{2RC}$
(b) $V_o(t) = \alpha t$
(c) $V_o(t) = \alpha(t - RC)$
(d) $V_o(t) = \alpha RC$

42. The frequency function G(f) of a first-order high pass filter is

(a) $G(f) = \frac{1}{1 - j(f_1 / f)}$
(b) $G(f) = \frac{1}{1 - j(f / f_1)}$
(c) $G(f) = \frac{1}{1 + j(f / f_1)}$
(d) $\frac{1}{1 + j(f_1 / f)}$

43. A ramp waveform $V_1(t) = \alpha t$ during the interval $0<t<T$ is transmitted through an RC high pass filter with $RC <<T$. The output wave form is

(a) $V_o(t) = \alpha RC$
(b) $V_o(t) = \alpha t$
(c) $V_o(t) = \alpha(t - RC)$
(d) $V_o(t) = \frac{\alpha t^2}{2RC}$

44. A symmetrical square wave is transmitted through a RC high-pass filter $RC>>T$. Percentage tilt P in the output waveform can be given by

(a) $P = \pi\ (f_1/f) \times 100$
(b) The tilt P is same as that suffered by a ramp waveform
(c) $P = \pi\ (f/f_1) \times 100$
(d) It is not possible to find P

45. The RC low-pass filter acts as a good integrator when

(a) $RC = T/20$
(b) $RC << T$
(c) $RC = T$
(d) $RC >> T$